Factoring Special Products

(DAY 1) Factoring Perfect Square Trinomials

There are some trinomials that can be factored as a perfect square. Examples of perfect squares are $1^2=1$, $2^2=4$, $3^2=9$, $4^2=16$, $5^2=25$, $6^2=36$, $7^2=49$, ........etc.

For a trinomial to be factored as a perfect square:

- The first term must be a perfect square
- The last term must be a perfect square
- The middle term has to be 2 times the product of the square root of the first and second term

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

Where $a =$ square root of first term and $b =$ square root of last term.

Example 1: Factor $4x^2 - 28x + 49$

Step 1: Check to see if the first term is a perfect square. What is $\sqrt{4x^2} \? \sqrt{4x^2} = 2x \ because 2x \cdot 2x = 4x^2$. $2x = a$

Step 2: Check to see if the last term is a perfect square. What is $\sqrt{49} \? \sqrt{49} = 7 \ because 7 \cdot 7 = 49$. $7 = b$.

Step 3: Check middle term to see if $2ab$ equals the middle term. $2 \cdot 2x \cdot 7 = 28x$

Step 4: This is in the form $a^2 - 2ab + b^2 = (a - b)^2$ so the answer is $(2x - 7)^2$, since $a = 2x$ and $b = 7$.

Answer: $(2x - 7)^2$

Example 2: Factor $x^2 + 10x + 25$

Step 1: Check to see if the first term is a perfect square. What is $\sqrt{1x^2} \? \sqrt{1x^2} = 1x = x \ because x \cdot x = x^2$. $x = a$

Step 2: Check to see if the last term is a perfect square. What is $\sqrt{25} \? \sqrt{25} = 5 \ because 5 \cdot 5 = 25$. $b = 5$.

Step 3: Check middle term to see if $2ab$ equals the middle term. $2 \cdot 5 \cdot x = 10x$

Step 4: This is in the form $a^2 + 2ab + b^2 = (a + b)^2$ so the answer is $(x + 5)^2$, since $a = x$ and $b = 5$.

Answer: $(x + 5)^2$

YOUR TURN

Directions: Factor each of the following:

1. $16t^2 - 40t + 25$
2. $4c^2 + 12c + 9$
3. \( y^2 - 8y + 16 \)

4. \( h^2 - 20h + 100 \)

5. \( p^2 + 2p + 1 \)

(Day 2) Differences of Squares

There are some cases where you can use a technique called differences of squares to factor the polynomial.

In order to use differences of squares the following must be true:

- There are two terms
- First and last term are perfect squares
- There is subtraction in the problem

\[
a^2 - b^2 = (a - b)(a + b) \text{ or } (a + b)(a - b) \text{ where } a \text{ is the } \sqrt{a^2} \text{ and } b \text{ is the } \sqrt{b^2}.
\]

Example 1: Factor \( x^2 - 25 \)

Step 1: Check to see if there is a GCF (greatest common factor) and factor it out. There is no GCF for this problem.

Step 2: Can you get the square root of the first term? \( \sqrt{x^2} = x \). So, \( x = a \)

Step 3: Can you get the square root of the second term? \( \sqrt{25} = 5 \). So, \( 5 = b \)

Step 4: The answer is \((a - b)(a + b) = (x - 5)(x + 5)\)

Example 2: Factor \( 16x^2 - 9 \)

Step 1: Check to see if there is a GCF (greatest common factor) and factor it out. There is no GCF for this problem.

Step 2: Can you get the square root of the first term? \( \sqrt{16x^2} = 4x \). So, \( 4x = a \)

Step 3: Can you get the square root of the second term? \( \sqrt{9} = 3 \). So, \( 3 = b \)

Step 4: The answer is \((a - b)(a + b) = (4x - 3)(4x + 3)\)

Example 3: Factor \( 75x^2 - 12 \)

Step 1: Check to see if there is a GCF (greatest common factor) and factor it out. The GCF for this problem is 3. You must factor 3 out first. \( 75x^2 - 12 = 3(25x^2 - 4) \)

Step 2: Can you get the square root of the first term? \( \sqrt{25x^2} = 5x \). So, \( 5x = a \)

Step 3: Can you get the square root of the second term? \( \sqrt{4} = 2 \). So, \( 2 = b \)

Step 4: The answer is \((a - b)(a + b) = (5x - 2)(5x + 2)\)
YOUR TURN
Directions: Factor each of the following

1. $18c^3 - 8c$
2. $x^4 - 81$

3. $25x^2 - 1$
4. $y^2 - 144$

5. $25m^2 - 9n^2$

(Day 3) Intro to Solving Quadratics

To solve quadratics, you must first factor out the quadratic and then set the factored out quadratic equal to zero.

Here are the steps required for Solving Quadratics by Factoring:

Step 1: Write the equation in the correct form. To be in the correct form, you must remove all parentheses from each side of the equation by distributing, combine all like terms, and finally set the equation equal to zero with the terms written in descending order.

Step 2: Use a factoring strategies to factor the problem.

Step 3: Use the Zero Product Property and set each factor containing a variable equal to zero.

Step 4: Solve each factor that was set equal to zero by getting the x on one side and the answer on the other side.

Example 1: Please solve $x^2 + 16 = 10x$

Step 1: Write the equation in proper form: $x^2 + 16 = 10x$
Subtract $10x$ on from both sides and put it in order from greatest exponent to least. $\rightarrow x^2 - 10x + 16 = 0$

Step 2: Use factoring strategies to factor problem: $x^2 - 10x + 16$
Find two numbers that multiply to equal 16 and add to equal $-10$. Those numbers are $-8$ and $-2$. Therefore, the factored form is $(x - 8)(x - 2) = 0$

Step 3: Use the Zero Product Property $(x - 8) = 0$ or $(x - 2) = 0$

Step 4: Solve each factor by solving for $x$.
$x - 8 = 0$, add 8 on both sides; $x = 8$. $x - 2 = 0$, add 2 to both sides; $x = 2$

Answer: 2, 8 (You should but both of these answers back in the original equation to see if they work.

Example 2: Please solve $6x^2 - 5x - 4$

Step 1: Write the equation in proper form: It’s already in proper form.

Step 2: Use factoring strategies to factor problem: $6x^2 - 5x - 4$:
Begin by multiplying the first and the last term: $(6)(-4) = 24$. Now find two numbers that you can multiply that will equal $-24$ and will add to equal $-5$. Those two numbers are $-8$ and $3$. 

Now rewrite the middle term using those two numbers: 6x² + 3x – 8x – 4. Separate the polynomial by grouping the first two terms and the last two terms. (Remember that when grouping the last two terms since the first term in that pair is negative, you will need to factor out the negative sign, which will change the sign of the next term.) (6x² + 3x) – (8x + 4). Now take out the GCF from both of the terms, which will give you, 3x(2x+1) – 4(2x+1). Finally take out the term that both of the terms share leaving your with (2x+1)(3x−4). So the factored form becomes: (2x+1)(3x−4) = 0

**Step 3:** Use the Zero Product Property: 2x+1 = 0 or 3x−4 = 0

**Step 4:** Solve for x in both of those equations. 2x+1 = 0, Subtract 1 from both sides, so you have 2x = −1 and now divide by 2, leaving you with x = \( \frac{1}{2} \). Now take 3x−4 = 0 and add 4 to both sides, so you have 3x = 4.

Divide both sides by 3, leaving you with x = \( \frac{4}{3} \).

Answer: \( \frac{4}{3} \) and \( \frac{1}{2} \). (You should put both of these answers back in the original equation to see if they work.)

**YOUR TURN**

**Directions:** Solve each of the following for x. You should check your answers to make sure they are correct.

1. \( x^2 + 8x = -7 \)  
2. \( x^2 - 11x + 24 = 0 \)

CHECK  

CHECK

3. \( 2x^2 + 3x - 9 = 0 \)  
4. \( 5x^2 - 12 = 4x \)

CHECK  

CHECK