# PROPORTIONAL RELATIONSHIPS 4.2.1, 4.2.2, and 4.2.4

A **proportion** is an equation stating the two ratios (fractions) are equal. Two values are in a proportional relationship if a proportion may be set up to relate the values.

For more information, see the Math Notes boxes in Lessons 4.2.3, 4.2.4, and 7.2.2 of the *Core Connections, Course 2* text. For additional examples and practice, see the *Core Connections, Course 2* Checkpoint 9 materials.

## Example 1

The average cost of a pair of designer jeans has increased $15 in 4 years. What is the unit growth rate (dollars per year)?

Solution: The growth rate is $\frac{15 \text{ dollars}}{4 \text{ years}}$. To create a unit rate we need a denominator of "one."
$\frac{15 \text{ dollars}}{4 \text{ years}} = \frac{x \text{ dollars}}{1 \text{ year}}$. Using a Giant One: $\frac{15 \text{ dollars}}{4 \text{ years}} = \boxed{\frac{4}{4}} \cdot \frac{x \text{ dollars}}{1 \text{ year}} \Rightarrow 3.75 \frac{\text{dollars}}{\text{year}}$.

## Example 2

Ryan's famous chili recipe uses 3 tablespoons of chili powder for 5 servings. How many tablespoons are needed for the family reunion needing 40 servings?

Solution: The rate is $\frac{3 \text{ tablespoons}}{5 \text{ servings}}$ so the problem may be written as a proportion: $\frac{3}{5} = \frac{t}{40}$.

One method of solving the proportion is to use the Giant One:

$$\frac{3}{5} = \frac{t}{40} \Rightarrow \frac{3}{5} \cdot \boxed{\frac{8}{8}} = \frac{24}{40} \Rightarrow t = 24$$

Another method is to use cross multiplication:

$$\frac{3}{5} = \frac{t}{40}$$
$$\frac{3}{5} \times \frac{t}{40}$$
$$5 \cdot t = 3 \cdot 40$$
$$5t = 120$$
$$t = 24$$

Finally, since the unit rate is $\frac{3}{5}$ tablespoon per serving, the equation $t = \frac{3}{5}s$ represents the general proportional situation and one could substitute the number of servings needed into the equation: $t = \frac{3}{5} \cdot 40 = 24$. Using any method the answer is 24 tablespoons.

## Example 3

Based on the table at right, what is the unit growth rate (meters per year)?

+2

| height (m) | 15 | 17 |
|---|---|---|
| years | 75 | 85 |

+10

Solution: $\frac{2 \text{ meters}}{10 \text{ years}} = \frac{2 \text{ meters}\cdot\frac{1}{10}}{10 \text{ years}\cdot\frac{1}{10}} = \frac{\frac{1}{5} \text{ meter}}{1 \text{ year}} = \frac{1}{5} \frac{\text{meter}}{\text{year}}$

## Problems

For problems 1 through 10 find the unit rate. For problems 11 through 25, solve each problem.

1. Typing 731 words in 17 minutes (words per minute)

2. Reading 258 pages in 86 minutes (pages per minute)

3. Buying 15 boxes of cereal for \$43.35 (cost per box)

4. Scoring 98 points in a 40 minute game (points per minute)

5. Buying $2\frac{1}{4}$ pounds of bananas cost \$1.89 (cost per pound)

6. Buying $\frac{2}{3}$ pounds of peanuts for \$2.25 (cost per pound)

7. Mowing $1\frac{1}{2}$ acres of lawn in $\frac{3}{4}$ of a hour (acres per hour)

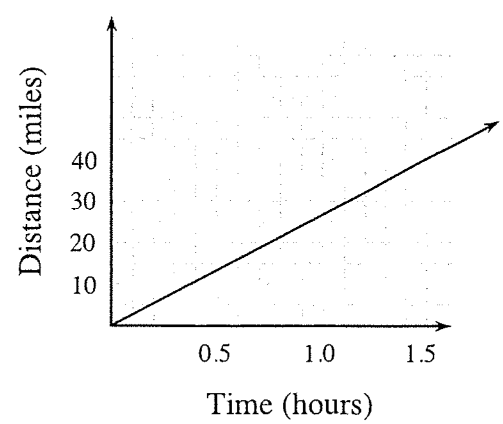
8. Paying \$3.89 for 1.7 pounds of chicken (cost per pound)

9.

| weight (g) | 6 | 8 | 12 | 20 |
|---|---|---|---|---|
| length (cm) | 15 | 20 | 30 | 50 |

What is the weight per cm?

10. For the graph at right, what is the rate in miles per hour?




11. If a box of 100 pencils costs \$4.75, what should you expect to pay for 225 pencils?

12. When Amber does her math homework, she finishes 10 problems every 7 minutes. How long will it take for her to complete 35 problems?

13. Ben and his friends are having a TV marathon, and after 4 hours they have watched 5 episodes of the show. About how long will it take to complete the season, which has 24 episodes?

14. The tax on a \$600 vase is \$54. What should be the tax on a \$1700 vase?

## WRITING EQUATIONS FOR WORD PROBLEMS (THE 5-D PROCESS) 5.3.4 and 5.3.5

At first students used the 5-D Process to solve problems. However, solving complicated problems with the 5-D Process can be time consuming and it may be difficult to find the correct solution if it is not an integer. The patterns developed in the 5-D Process can be generalized by using a variable to write an equation. Once you have an equation for the problem, it is often more efficient to solve the equation than to continue to use the 5-D Process. Most of the problems here will not be complex so that you can practice writing equations using the 5-D Process. The same example problems previously used are used here except they are now extended to writing and solving equations.

### Example 1

A box of fruit has three times as many nectarines as grapefruit. Together there are 36 pieces of fruit. How many pieces of each type of fruit are there?

**Describe:** Number of nectarines is three times the number of grapefruit.
Number of nectarines plus number of grapefruit equals 36.

| | **Define** | | **Do** | **Decide** |
|---|---|---|---|---|
| | # of Grapefruit | # of Nectarines | Total Pieces of Fruit | 36? |
| Trial 1: | 11 | 33 | 44 | too high |
| Trial 2: | 10 | 30 | 40 | too high |

After several trials to establish a pattern in the problem, you can generalize it using a variable. Since we could try any number of grapefruit, use $x$ to represent it. The pattern for the number of nectarines is three times the number of grapefruit, or $3x$. The total pieces of fruit is the sum of column one and column two, so our table becomes:

| **Define** | | **Do** | **Decide** |
|---|---|---|---|
| # of Grapefruit | # of Nectarines | Total Pieces of Fruit | 36? |
| $x$ | $3x$ | $x + 3x$ | $= 36$ |

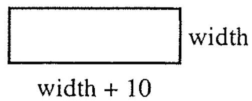
Since we want the total to agree with the check, our equation is $x + 3x = 36$. Simplifying this yields $4x = 36$, so $x = 9$ (grapefruit) and then $3x = 27$ (nectarines).

**Declare:** There are 9 grapefruit and 27 nectarines.

## Example 2

The perimeter of a rectangle is 120 feet. If the length of the rectangle is 10 feet more than the width, what are the dimensions (length and width) of the rectangle?

**Describe/Draw:**




| | Define | | Do | Decide |
|---|---|---|---|---|
| | Width | Length | Perimeter | 120? |
| Trial 1: | 10 | 25 | $(10 + 25) \cdot 2 = 70$ | too low |
| Trial 2: | 20 | 30 | 100 | too low |

Again, since we could guess any width, we labeled this column $x$. The pattern for the second column is that it is 10 more than the first: $x + 10$. The perimeter is found by multiplying the sum of the width and length by 2. Our table now becomes:

| Define | | Do | Decide |
|---|---|---|---|
| Width | Length | Perimeter | 120? |
| $x$ | $x + 10$ | $(x + x + 10) \cdot 2$ | $= 120$ |

Solving the equation:

$$
\begin{aligned}
(x + x + 10) \cdot 2 &= 120 \\
2x + 2x + 20 &= 120 \\
4x + 20 &= 120 \\
4x &= 100
\end{aligned}
$$

So $x = 25$ (width) and $x + 10 = 35$ (length).

**Declare:** The width is 25 feet and the length is 35 feet.

## Example 3

Jorge has some dimes and quarters. He has 10 more dimes than quarters and the collection of coins is worth \$2.40. How many dimes and quarters does Jorge have?

**Describe:** The number of quarters plus 10 equals the number of dimes.
The total value of the coins is \$2.40.

| | Define | | | | Do | Decide |
|---|---|---|---|---|---|---|
| | Quarters | Dimes | Value of Quarters | Value of Dimes | Total Value | \$2.40? |
| Trial 1: | 10 | 20 | 2.50 | 2.00 | 4.50 | too high |
| Trial 2: | 8 | 18 | 2.00 | 1.80 | 3.80 | too high |
| | $x$ | $x + 10$ | $0.25x$ | $0.10(x + 10)$ | $0.25x + 0.10(x + 10)$ | |

Since you need to know both the number of coins and their value, the equation is more complicated. The number of quarters becomes $x$, but then in the table the Value of Quarters column is $0.25x$. Thus the number of dimes is $x + 10$, but the value of dimes is $0.10(x + 10)$. Finally, to find the numbers, the equation becomes $0.25x + 0.10(x + 10) = 2.40$.

Solving the equation:

$$\begin{aligned} 0.25x + 0.10x + 1.00 &= 2.40 \\ 0.35x + 1.00 &= 2.40 \\ 0.35x &= 1.40 \\ x &= 4.00 \end{aligned}$$

**Declare:** There are 4 quarters worth \$1.00 and 14 dimes worth \$1.40 for a total value of \$2.40.

## Problems

Start the problems using the 5-D Process. Then write an equation. Solve the equation.

1. A wood board 100 centimeters long is cut into two pieces. One piece is 26 centimeters longer than the other. What are the lengths of the two pieces?

2. Thu is five years older than her brother Tuan. The sum of their ages is 51. What are their ages?

3. Tomás is thinking of a number. If he triples his number and subtracts 13, the result is 305. Of what number is Tomás thinking?

4. Two consecutive numbers have a sum of 123. What are the two numbers?

5. Two consecutive even numbers have a sum of 246. What are the numbers?

6. Joe's age is three times Aaron's age and Aaron is six years older than Christina. If the sum of their ages is 149, what is Christina's age? Joe's age? Aaron's age?