### Main Ideas/Questions

#### Parts of a Fraction

- **Numerator**: $\frac{5}{6}$
- **Denominator**

A rational number is a number that can be written as the ratio of two integers.

#### Simplest Form

A fraction than cannot be simplified any further.

**Directions**: Write each fraction in simplest form.

<table>
<thead>
<tr>
<th>Example</th>
<th>Simplified Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{4}{30}$</td>
<td>$\frac{2}{15}$</td>
</tr>
<tr>
<td>$\frac{2}{14}$</td>
<td>$\frac{1}{7}$</td>
</tr>
<tr>
<td>$\frac{15}{40}$</td>
<td>$\frac{3}{8}$</td>
</tr>
<tr>
<td>$\frac{22}{34}$</td>
<td>$\frac{-11}{17}$</td>
</tr>
<tr>
<td>$\frac{5}{62}$</td>
<td>$\frac{-5}{31}$</td>
</tr>
<tr>
<td>$\frac{18}{24}$</td>
<td>$\frac{3}{4}$</td>
</tr>
<tr>
<td>$\frac{9}{108}$</td>
<td>$\frac{1}{12}$</td>
</tr>
<tr>
<td>$\frac{14}{30}$</td>
<td>$\frac{7}{15}$</td>
</tr>
<tr>
<td>$\frac{-6}{40}$</td>
<td>$\frac{-3}{20}$</td>
</tr>
</tbody>
</table>

#### Improper Form

A fraction in which the numerator is larger than the denominator.

#### Mixed Numbers

A number paired with a proper fraction.

**Directions**: Write each improper fraction as a mixed number.

<table>
<thead>
<tr>
<th>Example</th>
<th>Mixed Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{16}{4}$</td>
<td>$4$</td>
</tr>
<tr>
<td>$\frac{-28}{16}$</td>
<td>$\frac{-12}{8} = \frac{-3}{2}$</td>
</tr>
<tr>
<td>$\frac{18}{5}$</td>
<td>$3\frac{3}{5}$</td>
</tr>
<tr>
<td>$\frac{23}{4}$</td>
<td>$5\frac{3}{4}$</td>
</tr>
<tr>
<td>$\frac{11}{6}$</td>
<td>$1\frac{5}{6}$</td>
</tr>
<tr>
<td>$\frac{-42}{9}$</td>
<td>$-4\frac{6}{9} = -4\frac{2}{3}$</td>
</tr>
</tbody>
</table>

**Directions**: Write each mixed number as an improper fraction.

<table>
<thead>
<tr>
<th>Example</th>
<th>Improper Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2\frac{2}{9}$</td>
<td>$\frac{20}{9}$</td>
</tr>
<tr>
<td>$-5\frac{1}{3}$</td>
<td>$-\frac{16}{3}$</td>
</tr>
<tr>
<td>$3\frac{3}{7}$</td>
<td>$\frac{24}{7}$</td>
</tr>
<tr>
<td>$-4\frac{1}{4}$</td>
<td>$-\frac{17}{4}$</td>
</tr>
<tr>
<td>$6\frac{1}{8}$</td>
<td>$\frac{49}{8}$</td>
</tr>
<tr>
<td>$-11\frac{1}{2}$</td>
<td>$-\frac{23}{2}$</td>
</tr>
</tbody>
</table>

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### Adding & Subtracting Fractions

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td><strong>Examples</strong></td>
<td></td>
</tr>
<tr>
<td>22. $\frac{1}{12} + \frac{5}{12} = \frac{6}{12}$</td>
<td>23. $\frac{11}{15} - \frac{4}{15} = \frac{7}{15}$</td>
</tr>
<tr>
<td>$= \frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td>24. $\frac{5}{3} + \frac{2}{3} = \frac{7}{3}$ (or $2\frac{1}{3}$)</td>
<td>25. $-\frac{5}{8} + \frac{4}{8}$</td>
</tr>
<tr>
<td>$= \frac{7}{3}$</td>
<td>$= -\frac{9}{8}$</td>
</tr>
<tr>
<td></td>
<td>(or $-1\frac{1}{8}$)</td>
</tr>
<tr>
<td>26. $-\frac{5}{6} + \left(-\frac{1}{6}\right)$</td>
<td>27. $-3\frac{3}{4} - \left(-2\frac{1}{4}\right)$</td>
</tr>
<tr>
<td>$= -\frac{12}{6}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= -\frac{6}{4} = -\frac{3}{2}$</td>
</tr>
<tr>
<td></td>
<td>(or $-1\frac{1}{2}$)</td>
</tr>
</tbody>
</table>

### Applications

28. Amaya is making bracelets. She has a piece of string that is $12\frac{1}{4}$ inches long. She cuts off a piece that is $8\frac{3}{4}$ inches. How much string does she have left? $12\frac{1}{4} - 8\frac{3}{4} = \frac{49}{4} - \frac{35}{4} = \frac{14}{4} = \frac{7}{2} = 3\frac{1}{2}$ in

29. Marcus is skiing. He is $860\frac{1}{10}$ feet up the mountain. He descends to $450\frac{7}{10}$ feet. What is his change in elevation? $860\frac{1}{10} - 450\frac{7}{10} = \frac{8601}{10} - \frac{4507}{10} = \frac{4094}{10} = 409\frac{4}{10} = 409\frac{2}{5}$ ft

30. Bill has $23\frac{4}{5}$ gallons of water in his aquarium. He adds $18\frac{3}{5}$ more gallons. How much water is in his aquarium? $23\frac{4}{5} + 18\frac{3}{5} = \frac{119}{5} + \frac{93}{5} = \frac{212}{5} = 42\frac{2}{5}$ gal
### Main Ideas/Questions

1. Write all mixed numbers as improper fractions.
2. Simplify up and down and along the diagonals (if possible).
3. Multiply the numerators to get the new numerator.
4. Multiply the denominators to get the new denominator.
5. Simplify (if needed).

### Examples

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td><strong>Examples</strong></td>
<td></td>
</tr>
<tr>
<td>1. $2 \times -\frac{7}{9} = \frac{2}{1} \cdot -\frac{7}{9}$</td>
<td>2. $1\frac{1}{7} \times \frac{1}{10} = \frac{8}{4} \cdot \frac{1}{10}$</td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>3. $-1\frac{2}{3} \times -\frac{2}{3} = -\frac{5}{3} \cdot -\frac{5}{3}$</td>
<td>4. $2\frac{1}{2} \cdot 1\frac{1}{2} = \frac{5}{2} \cdot \frac{3}{2}$</td>
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</tr>
<tr>
<td>5. $-3\frac{1}{4} \cdot \frac{3}{7} = -\frac{22}{1} \cdot \frac{3}{4}$</td>
<td>6. $-2 \times \frac{3}{4} = -\frac{2}{1} \cdot \frac{7}{4}$</td>
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<tr>
<td>7. $2\frac{1}{4} \cdot 2\frac{2}{3} = \frac{9}{4} \cdot \frac{8}{3}$</td>
<td>8. $-\frac{1}{2} \times 1\frac{4}{5} = -\frac{1}{2} \cdot \frac{9}{5}$</td>
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</tbody>
</table>
9. \[ -\frac{2}{3} \cdot \frac{8}{3} = -\frac{1}{3} \cdot \frac{8}{3} = -\frac{8}{9} \]
   \[= -\frac{56}{9} \cdot (-\frac{3}{4}) \]

10. \[ -\frac{4}{9} \cdot -\frac{9}{10} = -\frac{22}{9} \cdot -\frac{19}{10} = -\frac{11}{9} \cdot -\frac{19}{5} = \frac{209}{45} \cdot (4 \frac{29}{45}) \]

11. \[ \frac{5}{6} \cdot \frac{2}{5} = \frac{5}{6} \cdot \frac{12}{5} = \frac{1}{1} \cdot \frac{2}{1} = \frac{2}{1} = 2 \]

12. \[ -\frac{5}{9} \cdot -\frac{7}{12} = \frac{14}{9} \cdot -\frac{19}{12} = \frac{1}{9} \cdot -\frac{19}{6} = \frac{133}{54} \cdot (2 \frac{25}{54}) \]

13. Nikki just exercised for 180 minutes. Three-fifths of those minutes were spent running. How many minutes did she spend running?
   \[180 \cdot \frac{3}{5} = \frac{180}{1} \cdot \frac{3}{5} = \frac{36}{1} \cdot \frac{3}{1} = 108 \text{ min} \]

14. One batch of cookies requires \(2\frac{3}{4}\) cups of sugar. How much sugar does Angela need if she wants to make \(1\frac{1}{2}\) batches?
   \[2\frac{3}{4} \cdot 1\frac{1}{2} = \frac{8}{3} \cdot \frac{3}{2} = \frac{4}{1} \cdot = 4 \text{ cups} \]

15. When she got in her car to drive to work, Mia had 11\(\frac{1}{2}\) gallons of gas in her tank. If she used \(\frac{1}{8}\) of the gas in her tank to get to work, how much gas does she have left?
   \[11\frac{1}{2} \cdot \frac{1}{8} = \frac{23}{2} \cdot \frac{1}{8} = \frac{23}{16} = 1\frac{7}{16} \]
   \[11\frac{1}{2} - 1\frac{7}{16} = \frac{23}{2} - \frac{23}{16} = \frac{184}{16} - \frac{23}{16} = \frac{161}{16} = 10\frac{1}{16} \text{ gal} \]

16. What is the area of a triangle that has a base of 8 inches and a height of 3\(\frac{3}{2}\) inches?
   \[A = \frac{1}{2} \cdot 8 \cdot 3\frac{3}{2} = \frac{1}{2} \cdot \frac{8}{1} \cdot \frac{17}{2} = \frac{1}{1} \cdot \frac{4}{1} \cdot \frac{17}{2} = \frac{68}{2} = 13\frac{3}{2} \text{ in}^2 \]

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### Main Ideas/Questions

<table>
<thead>
<tr>
<th></th>
<th>Notes/Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Write all mixed numbers as improper fractions.</td>
</tr>
<tr>
<td>2</td>
<td>Change the division symbol to multiplication and FLIP the second fraction to its reciprocal (KISS!)</td>
</tr>
<tr>
<td>3</td>
<td>Multiply the numerators to get the new numerator. Multiply the denominators to get the new denominator.</td>
</tr>
<tr>
<td>4</td>
<td>Simplify (if needed).</td>
</tr>
</tbody>
</table>

### Dividing Fractions

#### Examples

1. \(-3 + \dfrac{1}{4} = \dfrac{-3}{1} - \dfrac{1}{4}\)
   
   \[= \boxed{12}\]

2. \(\dfrac{8}{5} + 2\dfrac{2}{3} = \dfrac{8}{5} + \dfrac{8}{3}\)
   
   \[= \dfrac{8}{5} \cdot \dfrac{3}{8}\]
   
   \[= \boxed{3}\]

3. \(\dfrac{1}{7} + \dfrac{1}{4} = \dfrac{8}{7} + \dfrac{1}{4}\)
   
   \[= \dfrac{8}{7} \cdot \dfrac{4}{1}\]
   
   \[= \boxed{-4\dfrac{3}{7}}\]

4. \(12 + 1\dfrac{1}{2} = 12 + \dfrac{3}{2}\)
   
   \[= \dfrac{12}{1} \cdot \dfrac{2}{3}\]
   
   \[= \boxed{8}\]

5. \(3\dfrac{1}{3} \div 4\dfrac{1}{21} = \dfrac{10}{3} \div \dfrac{4}{21}\)
   
   \[= \dfrac{10}{3} \cdot \dfrac{21}{4}\]
   
   \[= \boxed{3\dfrac{5}{2}} \quad (17\dfrac{1}{2})\]

6. \(8\dfrac{1}{4} \div 6 = \dfrac{33}{4} \div \dfrac{6}{1}\)
   
   \[= \dfrac{33}{4} \cdot \dfrac{1}{6}\]
   
   \[= \boxed{1\dfrac{1}{8}} \quad (1\dfrac{3}{8})\]

7. \(2\dfrac{1}{4} + 1\dfrac{1}{6} = \dfrac{9}{4} + \dfrac{7}{6}\)
   
   \[= \dfrac{9}{4} \cdot \dfrac{7}{1}\]
   
   \[= \boxed{2\dfrac{7}{14}} \quad (1\dfrac{13}{14})\]

8. \(-\dfrac{3}{4} + 2\dfrac{2}{5} = -\dfrac{3}{4} \div \dfrac{12}{5}\)
   
   \[= -\dfrac{3}{4} \cdot \dfrac{5}{12}\]
   
   \[= \boxed{-\dfrac{5}{16}}\]

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9. \(-3\frac{3}{7} + \frac{6}{11}\) = \(-\frac{24}{7} + \frac{6}{11}\)  
\[= -\frac{24}{7} \cdot \frac{11}{11}\] 
\[= \frac{244}{7} \cdot (6\frac{3}{7})\] 
10. \(7\frac{1}{7} + 4\) = \(\frac{50}{7} \cdot \frac{5}{4}\)  
\[= \frac{50}{7} \cdot \frac{4}{5}\] 
\[= \frac{25}{14} \cdot (1\frac{11}{14})\] 
11. \(-20 + \frac{5}{9}\) = \(-\frac{20}{1} + \frac{5}{9}\)  
\[= \frac{36}{1}\] 
12. \(10\frac{5}{6} + \frac{5}{8}\) = \(\frac{65}{6} \cdot \frac{5}{8}\)  
\[= \frac{65}{6} \cdot \frac{8}{5}\] 
\[= \frac{52}{3} \cdot (17\frac{1}{3})\] 

### Applications

13. Bill has a ladder that is \(8\frac{1}{2}\) feet tall. The ladder has 6 equal spaces between the rungs. How large is each space?  
\[8\frac{1}{2} \div 6 = \frac{17}{2} \cdot \frac{1}{6}\] 
\[= \frac{17}{12} \cdot \left[\frac{5}{12 \text{ ft}}\right]\] 
\[= 1\frac{5}{12} \text{ ft} \left(1\frac{1}{12} \text{ in}\right)\] 

14. After a party in her class, Mrs. Jones has \(3\frac{3}{5}\) pounds of fruit left. She splits the fruit into \(\frac{2}{5}\) pound groups. How many groups will she have?  
\[3\frac{3}{5} \div \frac{2}{5} = \frac{18}{5} \cdot \frac{5}{2}\] 
\[= 9 \text{ groups}\] 

15. A group of friends are running a relay race that covers \(3\frac{1}{2}\) miles. If each person runs \(\frac{1}{4}\) mile, how many runners are there?  
\[3\frac{1}{2} \div \frac{1}{4} = \frac{7}{2} \cdot \frac{4}{1}\] 
\[= 14 \text{ runners}\] 

16. Fifteen songs that are all the same length are performed during a chorus concert. If the concert lasts 27 minutes and 30 seconds, how long is each song?  
\[27\frac{1}{2} \div 15 = \frac{55}{2} \cdot \frac{1}{15}\] 
\[= \frac{11}{6} \cdot \left[\frac{5}{10} \text{ min}\right]\] 
\[= 1\text{ min}, 50\text{ sec}\] 

Summary:
### APPLICATIONS

13. Nick was scuba diving at \(-32\frac{1}{2}\) feet. If he descends another \(8\frac{3}{5}\) feet, what is his location?

\[
-32\frac{1}{2} - 8\frac{3}{5} = -\frac{65}{2} - \frac{43}{5} = \frac{-325}{10} - \frac{86}{10} = \frac{-411}{10} = -41\frac{1}{10}\text{ ft}
\]

14. Melanie is going to remove a \(6\frac{1}{2}\) inch section from a piece of yarn that is \(15\frac{3}{4}\) inches long. How much yarn is left?

\[
15\frac{3}{4} - 6\frac{1}{2} = \frac{63}{4} - \frac{13}{2} = \frac{63}{4} - \frac{26}{4} = \frac{37}{4} = 9\frac{1}{4}\text{ in}
\]

15. Caroline's baby sister sleeps \(\frac{5}{8}\) of the day. How many hours does she sleep?

\[
24 \cdot \frac{5}{8} = \frac{24 \cdot 5}{8} = \frac{3}{1} \cdot \frac{5}{1} = 15\text{ hrs}
\]

16. Lucy brought \(2\frac{3}{4}\) batches of cookies to share with her coworkers. By the end of the day \(\frac{5}{6}\) had been eaten. How much of the cookies are left?

\[
2\frac{3}{4} \cdot \frac{5}{6} = \frac{11}{4} \cdot \frac{5}{6} = \frac{55}{24} = 2\frac{7}{24}\]

\[
2\frac{3}{4} - 2\frac{7}{24} = \frac{11}{4} - \frac{55}{24} = \frac{66}{24} - \frac{55}{24} = \frac{11}{24}\text{ batch left}
\]

17. Josiah is building a birdhouse and has a piece of wood that is \(18\) feet long. He needs \(\frac{17}{8}\) foot long pieces. How many smaller pieces can he cut?

\[
18 \div \frac{17}{8} = \frac{18}{1} \div \frac{17}{8} = \frac{18}{1} \cdot \frac{8}{17} = \frac{144}{17} = 9\frac{3}{17}\text{ pieces}
\]

18. A square has a side length of \(5\frac{1}{4}\) inches. What is the area of the square? \((A = s \cdot s)\)

\[
5\frac{1}{4} \cdot 5\frac{1}{4} = \frac{21}{4} \cdot \frac{21}{4} = \frac{441}{16} = 27\frac{9}{16}\text{ inches}^2
\]

19. Kevin and Justin wanted to see who could run the furthest in \(10\) minutes. Kevin ran \(1\frac{3}{8}\) miles and Justin ran \(1\frac{2}{5}\) miles. How much further did Kevin run?

\[
1\frac{3}{8} - 1\frac{2}{5} = \frac{11}{8} - \frac{7}{5} = \frac{55}{40} - \frac{42}{40} = \frac{13}{30}\text{ miles}
\]

20. Melissa practiced piano for \(\frac{5}{6}\) of an hour on Monday and \(\frac{3}{4}\) of an hour on Tuesday. How many more minutes did she practice on Tuesday compared to Monday?

\[
\frac{5}{6} - \frac{5}{6} = \frac{16}{18} - \frac{15}{18} = \frac{1}{18}\text{ hours}
\]

\[
\frac{1}{18} \cdot 60 = \frac{1}{3} \cdot 10 = \frac{10}{3} = 3\frac{1}{3}\text{ minutes}
\]