Graphing and Linear Equations

Hello! This assignment is divided into three days. Explanations and examples are provided for each day. At the end of each day are problems to complete to check for your understanding of the material. Questions/problems should take a total of 20 minutes daily, or 60 minutes weekly.

Day 1: Part I: Graphing

- The location of a point on the graph is provided by an ordered pair which indicates the point’s coordinates...(x coordinate, y coordinate).
- Example 1: What is the ordered pair for point A? Start at (0,0), called the origin. First go right or left to the point, in this case 3 units left. Then move up or down, 2 units up. So A is at (-3,2)

![Graph of point A](image1)

Beware!

- The easiest mistake to make is with coordinates that have zeros in them!!!!
- Example: (-2, 0) means, starting at the origin, go left 2 units, do not go up or down at all! Place your point:

![Graph of point (-2,0)](image2)

However, (0, -2) means don’t go right or left, but go down 2 units!

![Graph of point (0,-2)](image3)

Part II: Examining linear equations

Characteristics of linear equations:

- Here is one of several common forms.. \( y = mx + b \) is called **slope-intercept form** because \( m \) is the slope, and \( b \) is the y-intercept.
- **Slope** is the steepness of the line. Some describe it as rise over run and is a number to describe direction of the line and the steepness.
The y-intercept is the point where the line crosses the y axis. The value of \( b \) is a single number, but the actual point will always be in the form \((0, y)\) because you don’t go right or left (an \( x \) value of 0) but go straight up or down along the vertical y axis. For the positive slope example above, the actual ordered pair for the y-intercept is \((0, 1)\)

**Part III: Rearranging the Equation from Standard form to Slope-intercept form:** Ex. \(3x + 4y = 8\) This equation is in standard form \((Ax + By = C)\). You CANNOT read the slope or y-intercept directly from it.

\[
3x + 4y = 8
\]

\[
-3x - 3x
\]

\[
4y = -3x + 8
\]

\[
\frac{4y}{4} = \frac{-3x + 8}{4}
\]

\[
y = -\frac{3}{4}x + 2
\]

Where \( m \) (slope) = \(-\frac{3}{4}\) and \( b \) (y-intercept) = 2

**Day 2: Writing Linear Equations**

To write a linear equation given a graph of a line, we need to find the slope and y-intercept, and then we can fill in the \( m \) and \( b \) of the slope intercept form of a linear equation: \( y = mx + b \)

**To calculate the slope of a line:**

**Method 1:** Use two ordered pairs from the line (if the coordinates are not given, find two spots on the line that are not)

**Example 1:** Find the slope of a line that passes through \((-2, 1)\) and \((4, -4)\). Think of these two points as; \((x_1, y_1)\) and \((x_2, y_2)\)

\[
m (slope) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 1}{4 - (-2)} = \frac{-5}{6}
\]
Example 2: Find the slope of the line on the graph.

First identify the 2 points you will be using (you may pick any 2 points because slope is always constant along a line). Then use the formula above.

Find 2 points and write their coordinates. I choose (3, 0) and (-3, 4). Label them $(x_1, y_1)$ and $(x_2, y_2)$

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

\[
m = \frac{4 - 0}{-3 - 3}
\]

\[
m = \frac{4}{-6} \text{ (Simplify to } -\frac{2}{3}\text{ )}
\]

Method 2: Using Rise Over Run on the Graph

- Choose two points on the graph. Start at one, count how many spaces up (+) or down (-) to the other. That number is the numerator of your slope. Count right (+) or left(-) to the other point. That number is the denominator of your slope.

Simply look at the line and find the point where it crosses the y axis. This is your intercept. For the line above, it is (0, 4)

Writing the equation of the Line:
Simply fill in the values for the slope \((m)\), and y-intercept \((b)\). Use the line above as an example…. \(m = -\frac{3}{2}\) and \(b = 4\), so the equation is \(y = -\frac{3}{2}x + 4\)

****Either method can be used to find the linear equation….it is just a matter of choice which you use!

**Day 3: Graphing Linear Equations**

When given the equation of a linear function, there are three methods for graphing it:

**Method 1: Making a table of values.**

Ex. Graph \(y = -3x - 3\)

Choose any three \(x\) values—a negative, positive, and 0 are often used…..I randomly pick

<table>
<thead>
<tr>
<th>(x) value</th>
<th>(y) value</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-3</td>
</tr>
<tr>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>1</td>
<td>-6</td>
</tr>
</tbody>
</table>

Substitute each value in for \(x\), and find the matching \(y\) value:

\[y = -3(-1) - 3\]
\[y = 1\]

Do this for each \(x\) value, and fill in the table.

<table>
<thead>
<tr>
<th>(x) value</th>
<th>(y) value</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>1</td>
<td>-6</td>
</tr>
</tbody>
</table>

Graph the three ordered pairs: (-1,1) (0, -3) and (1, -6)

Connect the three points by drawing a line:

Put arrows at the ends.
Method 2: Using Slope and Y-intercept

Ex. 1) Graph \( y = 3x + 2 \)

1) Graph the y-intercept at (0,2)
2) Write the slope as a fraction...
3) The numerator is 3, which is the y change. Go up since it is positive.
4) The denominator is 1, which is the x change. Go right since it is positive.
5) Place your point.

Ex. 2) A slope of \(-\frac{1}{2}\) can be interpreted as EITHER \(-\frac{1}{2}\) OR \(\frac{1}{-2}\). 

If you are moving from the y-intercept using \(-\frac{1}{2}\) you would move 1 down (since y is negative) and 2 right (since x is positive).

If you are moving from the y-intercept using \(\frac{1}{-2}\) you would move 1 up (since y is positive) and 2 left (since x is negative)

****You can choose either interpretation...depending on the location of the y-intercept one might be handier over the other if you happen to be running off the grid.

Example:

Graph \( y = -\frac{1}{3}x + 3 \) Start at the y-intercept....(0, 3).

Think of \(-\frac{1}{3}\) as \(-\frac{1}{3}\) and move 1 unit down, and 3 right from the y intercept. Note your point will be on the line. Now, go back to the y intercept, and interpret it as \(\frac{1}{-3}\) and move 1 unit up, and 3 units right. You will be on the line!!

Method 3: Using X and Y-intercepts
Remember, we only need two points to graph a line. With that idea, we take advantage of the two "convenient" points which can easily be solved algebraically from the equation of the line: namely, the x-intercept \((x, 0)\) and y-intercept \((0, y)\).

****We usually use this method when the equation given to us in standard form, and when the coefficients are factors of the constant

Ex. Graph \(3x + 4y = 12\) This is standard form, and 3 and 4 are factors of 12.

- **Find the x intercept by substituting 0 in for “y”**
  \[
  3x + 4(0) = 12 \\
  3x + 0 = 12 \\
  3x = 12 \\
  x = 4
  \]
  So, the x intercept is \((4, 0)\).

- **Find the y intercept by substituting 0 in for “x”**
  \[
  3(0) + 4y = 12 \\
  0 + 4y = 12 \\
  4y = 12 \\
  y = 3
  \]
  So, the y-intercept is \((0, 3)\)

- **Graph both intercept points, and connect with a line.**

YOUR TURN Day 1 Problems

1) State the ordered pairs \((, ,)\) for each point:

\[
\begin{align*}
A &= \\
B &= \\
C &= \\
D &= \\
\end{align*}
\]
YOUR TURN Day 2

2) Identify the slope and y-intercept given the following equation:
   \[ y = 2x + 9 \quad \text{slope} = \quad \text{Y-intercept} = \]

3) Based on your knowledge of positive/negative slopes, and y-intercepts, which of the following equations could NOT be the equation of the graph below? There are multiple answers.
   
   a) \( y = x - 2 \)
   b) \( y = x + 2 \)
   c) \( y = -\frac{3}{2}x + 2 \)
   d) \( y = \frac{3}{2}x + 2 \)

4) Which is the equation of the graph below?

   a) \( y = 5 \)
   b) \( y = -5 \)
   c) \( x = 5 \)
   d) \( x = -5 \)

5) Identify the slope and y-intercept given the following equation: SHOW WORK.

   \[ 2x - 3y = 6 \]

   Slope= \quad \text{Y-intercept} = \quad \]

YOUR TURN Day 2

1) Find the slope of the line through the given points. You MUST show work to get credit.
   \((-7, 7), (-1, 10)\)

   A) \( \frac{1}{2} \)
   B) 0
   C) 2
   D) undefined
3) What is the y-intercept of the line above?
4) Write the equation of the above in the form \( y = mx + b \)
5) Find the slope of the line on the graph below:

6) What is the y-intercept of the line above?
7) Write the equation of this line in the form \( y = mx + b \)

YOUR TURN Day 3

1) Complete the table of values and graph the equation \( y = -2x + 2 \)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
</tbody>
</table>
2) Complete the table of values and graph the equation $y = \frac{1}{5}x + 5$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>-5</td>
<td></td>
</tr>
</tbody>
</table>

Hint: the values of 5 and -5 were chosen strategically due to the denominator of 5

3) Choose three $x$ values, and complete the table. Graph the equation $y = 4x + 3$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
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</table>

For the next three problems, graph each equation using the slope and $y$-intercept:
4) \( y = \frac{4}{3}x - 4 \)

5) \( y = -3x - 3 \)

6) \( y = -\frac{4}{3}x - 1 \)

7) Graph the line of this equation using the x and y intercepts: \( 4x + 3y = -12 \). Graph and label each intercept.

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5) \( y = -3x - 3 \)

7) Graph the line of this equation using the x and y intercepts: \( 4x + 3y = -12 \). Graph and label each intercept.