

**INSTRUCTIONS:** Where applicable, put your solutions in interval notation. Do not use any calculator (except on Topics 15:#5 & 24:#6). Please do all work on separate paper and do the problems in order. **SHOW ALL WORK!** We've given you the answers—we care about the process that gets you there.

### Topic 1: Fractional and Negative Exponents

Simplify, using positive only exponents.

$$1. 2\left(\frac{2}{2-x}\right)\left[\frac{-2}{(2-x)^2}\right]^3 \quad 2. \frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}} \quad 3. \frac{\frac{1}{2}(2x+5)^{\frac{3}{2}}}{\frac{3}{2}} \quad 4. \left(\frac{1}{x^{-2}} + \frac{4}{x^{-1}y^{-1}} + \frac{1}{y^{-2}}\right)^{\frac{1}{2}}$$

### Topic 2: Domain

Find the domain of the following functions.

$$1. y = \frac{x^2 - 5x - 6}{x^2 - 3x - 18} \quad 2. y = \frac{\sqrt{2x-9}}{2x+9} \quad 3. y = \sqrt{x^2 - 5x - 14} \quad 4. y = \log(2x - 12)$$

### Topic 3: Solving Inequalities

Write the following absolute value function as a piece-wise function.

$$1. y = |2x - 4|$$

Solve the following absolute value inequalities.

$$2. |x - 3| \leq 4 \quad 3. |3x - 4| > -2$$

Solve the following quadratic inequalities.

$$4. x^2 - 3x \geq 10 \quad 5. x^3 + 4x^2 - x \geq 4 \quad 6. 2\sin^2 x \geq \sin x, \quad 0 \leq x < 2\pi$$

Solve the following rational inequality.

$$7. \frac{2x-1}{3x-2} \leq 1$$

### Topic 4: Even and Odd Functions

Show algebra to determine if the relation is even, odd, or neither.

$$1. f(x) = 2x^2 - 7 \quad 2. f(x) = -4x^3 - 2x \quad 3. f(x) = 4x^2 - 4x + 4$$

### Topic 5: Function Transformation

If  $f(x) = x^2 - 1$ , describe in words what the following would do to the graph of :

$$1. f(x) - 4 \quad 2. f(x - 4) \quad 3. -f(x + 2)$$

$$4. 5f(x) + 3 \quad 5. f(2x) \quad 6. |f(x)|$$

### Topic 6: Factor theorem ( $p$ over $q$ method/synthetic division)

Use the  $p$  over  $q$  method and synthetic division to factor the polynomial  $P(x)$ . Then solve  $P(x) = 0$ .

1.  $P(x) = x^3 - 6x^2 + 3x + 10$

### Topic 7: Special Factorization

Factor completely.

1.  $27x^3 - 125y^3$

2.  $x^4 + 11x^2 - 80$

3.  $2x^2 + 50y^2 - 20xy$

4.  $x^2 + 12x + 36 - 9y^2$

5.  $(x - 3)^2(2x + 1)^3 + (x - 3)^3(2x + 1)^2$

6.  $(3x + 4)^{-3}(2x - 5)^3 + (3x + 4)^{-2}(2x - 5)^2$

7.  $\frac{1}{10}(2x + 1)^{5/2} - \frac{1}{6}(2x + 1)^{3/2}$

### Topic 8: Solving by Factoring or Quadratic Formula

Solve each equation.

1.  $x^2 + 6x + 4 = 0$

2.  $2x^2 - (x + 2)(x - 3) = 12$

3.  $x - 10\sqrt{x} + 9 = 0$

4.  $\frac{1}{x^2} - \frac{1}{x} = 6$

5.  $x^3 + 2x^2 - 3x - 6 = 0$

### Topic 9: Asymptotes

For each function, find the equations of both the vertical and horizontal asymptote(s), if they exist.

1.  $y = \frac{x + 4}{x^2 + 1}$

2.  $y = \frac{x^2 - 9}{x^3 + 3x^2 - 18x}$

3.  $y = \frac{2x^3}{x^3 - 1}$

### Topic 10: Complex Fractions

Simplify.

1.  $\frac{\frac{3}{x} - \frac{4}{y}}{\frac{4}{x} - \frac{3}{y}}$

2.  $\frac{1 - \frac{2}{3x}}{x - \frac{4}{9x}}$

3.  $\frac{\frac{x^2 - y^2}{xy}}{\frac{x + y}{y}}$

4.  $\frac{x^{-3} - x}{x^{-2} - 1}$

5.  $\frac{\frac{4}{x - 5} + \frac{2}{x + 2}}{\frac{2x}{x^2 - 3x - 10}} + 3$

### Topic 11: Composition of Functions

If  $f(x) = x^2$ ,  $g(x) = 2x - 1$ , and  $h(x) = 2^x$ , find the following:

1.  $f(g(2))$

2.  $h(f(-1))$

3.  $g\left(f\left(h\left(\frac{1}{2}\right)\right)\right)$

4.  $g(f(x))$

5.  $g(g(x))$

6.  $f(h(x))$

## Topic 12: Rationalizing Denominators and Numerators

For problem #1 rationalize the denominator. For problem #2 rationalize the numerator.

1.  $\frac{3}{\sqrt[4]{(3x)^3}}$

2.  $\frac{5\sqrt{2} + \sqrt{5}}{5}$

## Topic 13: Solving Rational Equations

Solve equation for  $x$ .

1.  $\frac{x-5}{x+1} = \frac{3}{5}$

2.  $\frac{60}{x} - \frac{60}{x-5} = \frac{2}{x}$

3.  $\frac{2}{x+5} + \frac{1}{x-5} = \frac{16}{x^2-25}$

4.  $\frac{x}{x-2} + \frac{2x}{4-x^2} = \frac{5}{x+2}$

5.  $\frac{2x+3}{x-1} = \frac{10}{x^2-1} + \frac{2x-3}{x+1}$

## Topic 14: Right Triangle Trigonometry

1. If  $\cos \theta = -\frac{5}{13}$ ,  $\theta$  in quadrant II,  
find  $\sin \theta$  and  $\tan \theta$ .

2. If  $\cot \theta = 3$ ,  $\theta$  in quadrant III,  
find  $\sin \theta$  and  $\cos \theta$ .

3. A kite is 100m above the ground. If there are 200m of string out, what is the angle (in radians) between the string and the horizontal? (Assume that the string is perfectly straight.)

## Topic 15: Solving Trigonometric Equations

Solve each equation on the interval  $[0, 2\pi)$ . Please use a calculator to complete #5.

1.  $\cos^2 x = \cos x$

2.  $4 \sin^2 x = 1$

3.  $2 \sin^2 x + \sin x = 1$

4.  $2 \sin x \cos x + \sin x = 0$

5.  $8 \cos^2 x - 2 \cos x = 1$

6.  $\sin^2 x - \cos^2 x = 0$

## Topic 16: Logarithms

Simplify.

1.  $\log_2 5 + \log_2(x^2 - 1) - \log_2(x - 1)$

2.  $2 \log_4 9 - \log_2 3$

3.  $3^{2 \log_3 5}$

## Topic 17: Solving for Indicated Variable

Solve each equation for the indicated variable.

1.  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ , for  $a$

2.  $A = 2\pi r^2 + 2\pi rh$ , for  $r > 0$

3.  $\frac{2x}{4\pi} + \frac{1-x}{2} = 0$ , for  $x$

### Topic 18: Equations of Lines

Determine the equation of each line:

1. the line through  $(-1, 3)$  and  $(2, -4)$
2. the line through  $(-1, 2)$  and perpendicular to the line  $2x - 3y + 5 = 0$
3. the line through  $(2, 3)$  and the midpoint of the line segment from  $(-1, 4)$  to  $(3, 2)$

### Topic 19: Equations of Circles

In #1 and #2, for the circle  $x^2 + y^2 + 6x - 4y + 3 = 0$ , find:

1. the center and radius
2. the equation of the tangent at  $(-2, 5)$
3. A curve is traced by a point  $P(x, y)$  which moves such that its distance from the point  $A(-1, 1)$  is three times its distance from the point  $B(2, -1)$ . Determine the equation of the curve.

### Topic 20: The Difference Quotient

Simplify  $\frac{f(x+h) - f(x)}{h}$ , where:

1.  $f(x) = 2x + 3$
2.  $f(x) = \frac{1}{x+1}$
3.  $f(x) = x^2$

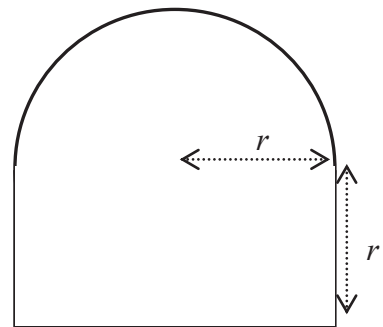
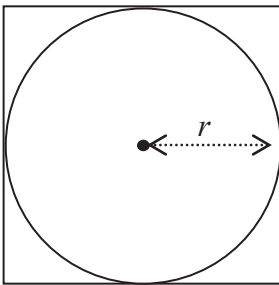
### Topic 21: Inverse Functions

Find the inverse of each function:

1.  $f(x) = 2x + 3$
2.  $f(x) = \frac{x+2}{5x-1}$

### Topic 22: Area

1. Find the ratio of the area inside the square but outside the circle to the area of the square in the picture below.
2. Find a formula for the perimeter of a window of the shape in the picture below.



3. A water tank has the shape of a cone (like an ice cream cone without the ice cream). The tank is 10m high and has a radius of 3m at the top. If the water is 5m deep (in the middle) what is the surface area of the top of the water?

### Topic 23: Trigonometric Identities

You should know the following identities:

1.  $\sin(-x) = -\sin x$

2.  $\cos(-x) = \cos x$

3.  $\sin^2 x + \cos^2 x = 1$

4.  $\sin 2x = 2 \sin x \cos x$

5.  $\cos 2x = \cos^2 x - \sin^2 x$

6.  $\cos 2x = 2 \cos^2 x - 1$

7.  $\cos 2x = 1 - 2 \sin^2 x$

8.  $\left| \cos \frac{x}{2} \right| = \sqrt{\frac{1 + \cos x}{2}}$

9.  $\left| \sin \frac{x}{2} \right| = \sqrt{\frac{1 - \cos x}{2}}$

10.  $\cos(x + y) = \cos x \cos y - \sin x \sin y$

11.  $\sin(x + y) = \sin x \cos y + \cos x \sin y$

### Topic 24: Vectors

Please use a calculator to complete #6.

1. Let  $\mathbf{u} = \langle 5, -4 \rangle$ ,  $\mathbf{v} = \langle 1, -2 \rangle$ . Find  $2\mathbf{u} - \mathbf{v}$ .

2. Find a unit vector in the direction of  $\mathbf{v} = \langle 3, -1 \rangle$  and write your answer in component form.

3. Given that  $P = \langle 4, -1 \rangle$  and  $Q = \langle 7, -2 \rangle$ , find the component form and magnitude of the vector  $\overrightarrow{PQ}$ .

4. Determine whether the vectors  $\mathbf{u}$  and  $\mathbf{v}$  are parallel, orthogonal, or neither.

$$\mathbf{u} = \langle 5, 3 \rangle, \mathbf{v} = \langle -10/4, -3/2 \rangle$$

5. Find  $\mathbf{a} \cdot \mathbf{b}$ , where  $\mathbf{a} = 12\mathbf{i} - 4\mathbf{j}$ , and  $\mathbf{b} = -3\mathbf{i} + \mathbf{j}$ .

6. Find the angle between the given vectors (in degrees) to the nearest tenth of a degree.

$$\mathbf{u} = \langle 5, -4 \rangle, \mathbf{v} = \langle 1, -2 \rangle$$

### Topic 25: Parametric and Polar

Eliminate the parameter,  $t$ .

1.  $x = t - 3, y = \frac{2}{t}$

2.  $x = 9 \cos t, y = 9 \sin t$

3. Find the rectangular coordinates of the point with the polar coordinates  $(-2, \frac{5\pi}{6})$ .

4. Find two polar coordinate pairs (in radians) to describe the point with rectangular coordinates  $(-2, -2)$ .

5. Convert the rectangular equation to polar form:  $3x + 2y = 4$

6. Convert the polar equation to rectangular form:  $r = 2 \sin \theta - 4 \cos \theta$ . Please leave your answer in standard form.

### Topic 26: Sequences and Series

Find the explicit form of the  $n$ th term of each sequence.

1.  $-5, -3, -1, 1, 3, \dots$

2.  $0.5, -1, 2, -4, 8, \dots$

Find the sum of the finite or infinite series.

3.  $\sum_{n=3}^5 n^2 - 3n$

4.  $\sum_{n=0}^{\infty} 4\left(\frac{1}{3}\right)^n$

## Topic 27: Limits

Evaluate the following limits.

1.  $\lim_{x \rightarrow -2} \frac{x^3 + 8}{x^2 - 4}$

2.  $\lim_{x \rightarrow 0} \frac{x}{x^2}$

3.  $\lim_{h \rightarrow 0} \frac{1}{h} \left( \frac{1}{2+h} - \frac{1}{2} \right)$

4.  $\lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{\pi - x}$

5.  $\lim_{x \rightarrow 1} f(x)$ , when  $f(x) = \begin{cases} \frac{x^2 - 1}{x - 1}, & x \neq 1 \\ 4, & x = 1 \end{cases}$

## Topic 28: Basic Differentiation

Find the derivative of each function.

1.  $y = \sqrt{3 - x + x^2}$

2.  $y = (4x + 1)(1 - x)^3$

3.  $y = \frac{1 + x^2}{1 - x^2}$

## AP Calculus BC Summer Packet SOLUTIONS

**Topic 1:** 1)  $\frac{-(2-x)^5}{2}$       2)  $\frac{2}{\sqrt[4]{(x-4)}}$       3)  $\frac{1}{3(2x+5)^{3/2}}$       4)  $\frac{1}{\sqrt{x^2 + 4xy + y^2}}$

**Topic 2:** 1)  $(-\infty, -3) \cup (-3, 6) \cup (6, \infty)$     2)  $[9/2, \infty)$     3)  $(-\infty, -2] \cup [7, \infty)$     4)  $(6, \infty)$

**Topic 3:** 1)  $y = \begin{cases} 2x - 4, & x \geq 2 \\ 4 - 2x, & x < 2 \end{cases}$       2)  $[-1, 7]$       3)  $(-\infty, \infty)$       4)  $(-\infty, -2] \cup [5, \infty)$

5)  $[-4, -1] \cup [1, \infty)$       6)  $[0] \cup [\pi/6, 5\pi/6] \cup [\pi, 2\pi)$       7)  $\left(-\infty, \frac{2}{3}\right) \cup [1, \infty)$

**Topic 4:** 1) even      2) odd      3) neither

**Topic 5:** 1) translated 4 units down    2) translated 4 units to the right  
3) reflected over the  $x$ -axis and translated 2 units left  
4) stretched vertically by a factor of 5 and translated up 3 units  
5) stretched horizontally by a factor of  $\frac{1}{2}$   
6) no change  $(-\infty, -1) \cup (1, \infty)$ , on  $[-1, 1]$  the graph would be reflected over the  $x$ -axis

**Topic 6:** 1)  $x = -1, 2, \text{ or } 5$

**Topic 7:** 1)  $(3x - 5y)(9x^2 + 15xy + 25y^2)$     2)  $(x^2 + 16)(x^2 - 5)$     3)  $2(x - 5y)^2$

4)  $(x + 6 + 3y)(x + 6 - 3y)$     5)  $(x - 3)^2(2x + 1)^2(3x - 2)$

6)  $\frac{(2x - 5)^2(5x - 1)}{(3x + 4)^3}$     7)  $\frac{(2x + 1)^{3/2}(3x - 1)}{15}$

**Topic 8:** 1)  $x = -3 \pm \sqrt{5}$       2)  $x = -3$  or  $2$       3)  $x = 1$  or  $81$       4)  $x = -\frac{1}{2}$  or  $\frac{1}{3}$   
 5)  $x = -2, \sqrt{3}, -\sqrt{3}$

**Topic 9:** 1) VA: none; HA:  $y = 0$       2) VA:  $x = 0, x = -6$ ; HA:  $y = 0$   
 3) VA:  $x = 1$ ; HA:  $y = 2$

**Topic 10:** 1)  $\frac{3y-4x}{4y-3x}$       2)  $\frac{3}{3x+2}, x \neq 0, \frac{2}{3}$       3)  $\frac{x-y}{x}, x \neq -y$   
 4)  $\frac{1+x^2}{x}, x \neq -1, 1$       5)  $\frac{6x-2}{3x^2-7x-30}$

**Topic 11:** 1)  $9$       2)  $2$       3)  $3$       4)  $2x^2-1$       5)  $4x-3$       6)  $2^{2x}$

**Topic 12:** 1)  $\frac{\sqrt[4]{3x}}{x}$       2)  $\frac{9}{5\sqrt{2}-\sqrt{5}}$

**Topic 13:** 1)  $x = 14$       2)  $x = -145$       3)  $x = 7$       4)  $x = \frac{5}{2} \pm \frac{i\sqrt{15}}{2}$       5) no solution

**Topic 14:** 1)  $\sin \theta = \frac{12}{13}; \tan \theta = \frac{-12}{5}$       2)  $\cos \theta = \frac{-3\sqrt{10}}{10}; \sin \theta = \frac{-\sqrt{10}}{10}$       3)  $\theta = \frac{\pi}{6}$

**Topic 15:** 1)  $x = 0, \frac{\pi}{2}, \frac{3\pi}{2}$       2)  $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$       3)  $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$   
 4)  $x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$       5)  $x = \frac{\pi}{3}, \frac{5\pi}{3}, 1.823, 4.460$       6)  $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

**Topic 16:** 1)  $\log_2 5(x+1), x > 1$       2)  $\log_2 3$       3)  $25$

**Topic 17:** 1)  $a = \frac{bcx}{bc-cy-bz}$       2)  $r = \frac{\sqrt{h^2\pi^2 + 2\pi A}}{2\pi} - \frac{h}{2}$       3)  $x = \frac{\pi}{\pi-1}$

**Topic 18:** 1)  $y = -\frac{7}{3}x + \frac{2}{3}$       2)  $y = -\frac{3}{2}x + \frac{1}{2}$       3)  $y = 3$

**Topic 19:** 1) center is at  $(-3, 2)$ ; radius =  $\sqrt{10}$       2)  $y = -\frac{1}{3}x + \frac{13}{3}$   
 3)  $8x^2 - 38x + 8y^2 + 20y + 43 = 0$

**Topic 20:** 1)  $2$       2)  $\frac{-1}{(x+h+1)(x+1)}$       3)  $2x+h$

**Topic 21:** 1)  $f^{-1}(x) = \frac{x-3}{2}$       2)  $f^{-1}(x) = \frac{x+2}{5x-1}$

**Topic 22:** 1)  $\frac{4-\pi}{4}$                       2)  $\pi r + 4r$                       3)  $\frac{9}{4}\pi m^2$

**Topic 24:** 1)  $\langle 9, -6 \rangle$     2)  $\langle \frac{3\sqrt{10}}{10}, \frac{-\sqrt{10}}{10} \rangle$     3)  $\langle 3, -1 \rangle; \sqrt{10}$     4) parallel    5) -40    6)  $24.8^\circ$

**Topic 25:** 1)  $y = \frac{2}{x+3}$                       2)  $x^2 + y^2 = 81$                       3)  $(\sqrt{3}, -1)$

4)  $(2\sqrt{2}, \frac{5\pi}{4}); (-2\sqrt{2}, \frac{\pi}{4})$     5)  $r = \frac{4}{3\cos\theta + 2\sin\theta}$     6)  $(x+2)^2 + (y-1)^2 = 5$

**Topic 26:** 1)  $a_n = 2n - 7$     2)  $a_n = (-1)(-2)^{n-2}$     3) 14    4) 6

**Topic 27:** 1) -3    2)  $\infty$  or does not exist    3)  $-\frac{1}{4}$     4) 1    5) 2

**Topic 28:** 1)  $y' = \frac{-1+2x}{2\sqrt{3-x+x^2}}$     2)  $y' = (1-x)^2(1-16x)$     3)  $y' = \frac{4x}{(1-x^2)^2}$