

Name \_\_\_\_\_ Date \_\_\_\_\_

## Summer Math Project for Rising 10th, 11th, & 12th Grade Students

### Due 9/20/19

**Directions:** Answer all 40 questions on this assignment. Each correct answer will receive **two points**. No partial credit will be allowed. Record your answers on this separate answer sheet. **ALL WORK MUST BE SHOWN TO RECEIVE FULL CREDIT.** A correct answer with no work shown will receive only one point.

**Helpful websites:** [www.regentsprep.org](http://www.regentsprep.org)

[www.khanacademy.org](http://www.khanacademy.org)

[www.teachertube.com](http://www.teachertube.com)

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40. \_\_\_\_\_

Below, you will find an overview of each topic on this assignment. For best results and maximum benefit, it is recommended that you read the overview before attempting the questions.

## Overview

### I. Analyzing Statistics (Mean, Median and Mode.)

Using the **mean, median, and mode** are three ways to analyze statistics.

The "**mean**" is the "average", where you add up all the numbers and then divide by the number of numbers.

The "**median**" is the "middle" value in the list of numbers. To find the median, your numbers have to be listed in numerical order.

The "**mode**" is the value that occurs most often. If no number is repeated, there is no mode for the list.

### II. Fraction Operations

To **add (or subtract) fractions**:

-Find the least common denominator

-Write both original fractions as equivalent fractions with the least common denominator.

-Add (or subtract) the numerators.

-Write the result with the denominator.

To **multiply** two fractions:

-Multiply the numerator by the numerator.

-Multiply the denominator by the denominator.

For all real numbers  $a, b, c, d$  where ( $b \neq 0$  and  $d \neq 0$ ), we have  $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$

To **divide** by a fraction, multiply by its reciprocal. (Also referred to as "Keep, Change, Flip.")

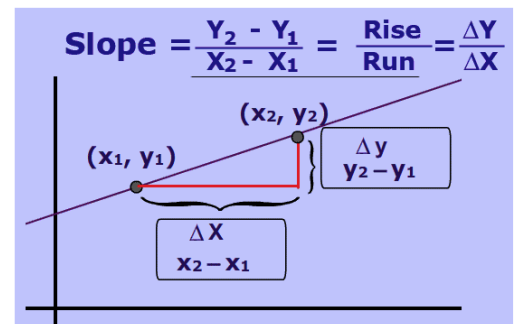
### III. Slope / Rate of Change

The slope (of a line) characterizes the general direction that the line goes.

To calculate the slope, divide the difference of the y-coordinates by the difference of the x-coordinates.

To identify the slope, given a linear equation, the slope is the value in front of the variable (commonly x) given that y is alone.

$y = mx + b$ , where  $m$  is the slope and  $b$  is the y-intercept.



### IV. Simplifying Square Roots

To simplify a square root, find two numbers that multiply to give the number under the radical, where one of those numbers is a perfect square.

Repeat this process until the number under the radical has no perfect square factors, other than 1.

### V. Laws of Exponents

The exponent of a number says **how many times** to use the number **in multiplication**.

Ex)  $3^4 = 3 \cdot 3 \cdot 3 \cdot 3$ .

When working with exponents:

Laws of Exponents		
I. <b>Multiplication</b>	$b^n \cdot b^m = b^{n+m}$	} multiply exponents
II. <b>Power of a power</b>	$(b^n)^m = b^{nm}$	
III. <b>Power of a product</b>	$(bc)^n = b^n c^n$	
IV. <b>Power of a fraction</b>	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	} subtract exponents
V. <b>Division</b>	$\frac{b^m}{b^n} = b^{m-n}$	
	or, alternatively, $= \frac{1}{b^{n-m}}$	

## VI. Graphing (Quadratics)

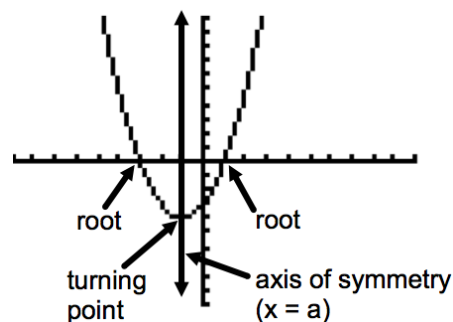
**General form** of a quadratic equation:  $ax^2 + bx + c = 0$ .

$$y = ax^2 + bx + c$$

### Helpful vocabulary:

**Roots** are where the function crosses the x-axis.

Other names for roots are **zeros, x-intercepts, or solutions**.



## VII. System of Equations

**System of Equations** – two or more equations.

**Solutions** are points/coordinates.

To solve:

1. Solve one of the equations (the “less involved” one) for one of the variables.
2. Plug what you found into the other equation. (For example, if you solve one equation for  $y$ , where every  $y$  appears in the other equation, write (substitute) what you found  $y$  to equal.
3. Solve for the variable. (Your equation will now contain just one variable.)
4. Plug the value(s) you get for the variable (you solved for) into either of the given equations to determine the value for the other variable.

**Note:** If this question is a multiple choice question, you can just plug in the choices to see which coordinates are the solution(s).

## VIII. Factoring

Factor – means to break something down in terms of a **product**.

### Methods:

1. **GCF** – Greatest Common Factor. Always try first. Puts in simpler form.
2. **Factor by Grouping** – Four terms. Split down the middle. Take the GCF of the first two terms. Take the GCF of the last two terms. What is in the parenthesis should match. Collect terms.
3. **Trinomials when  $a = 1$** . ( $ax^2 + bx + c$ ) - Set up two sets of parentheses. Need to find numbers that multiply to give you the last term, the “ $c$ ” term, and combine, by + or -, to give you the middle term, the “ $b$ ” term.
4. **Trinomials when  $a \neq 1$** . Use the “borrow payback” or “AC” method.
5. **Difference between two perfect squares**. Factors will be conjugates.

**Conjugates** – same numbers only the sign in the middle is different. Ex)  $-2x - 3$  and  $-2x + 3$  are conjugates.

6. **Factor completely** means factor something until it can't be factored/simplified any further. Usually, more than one factoring technique is used.

## IX. Quadratic Formula

- Used to **solve (any) quadratic equations**. (To find the solutions, roots, zeroes.)

- Equation must be in **general form**:  $ax^2 + bx + c = 0$ .

- Very useful. Especially when roots are not whole numbers or the polynomial is not factorable.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. Brian correctly used a method of completing the square to solve the equation  $x^2 + 7x - 11 = 0$ . Brian's first step was to rewrite the equation as  $x^2 + 7x = 11$ . He then added a number to both sides of the equation. Which number did he add?

- 1)  $\frac{7}{2}$
- 2)  $\frac{49}{4}$
- 3)  $\frac{49}{2}$
- 4) 49

**Work/Explanation:**

2. Which step can be used when solving  $x^2 - 6x - 25 = 0$  by completing the square?

- 1)  $x^2 - 6x + 9 = 25 + 9$
- 2)  $x^2 - 6x - 9 = 25 - 9$
- 3)  $x^2 - 6x + 36 = 25 + 36$
- 4)  $x^2 - 6x - 36 = 25 - 36$

**Work/Explanation:**

3. If  $x^2 = 12x - 7$  is solved by completing the square, one of the steps in the process is

- 1)  $(x - 6)^2 = -43$
- 2)  $(x + 6)^2 = -43$
- 3)  $(x - 6)^2 = 29$
- 4)  $(x + 6)^2 = 29$

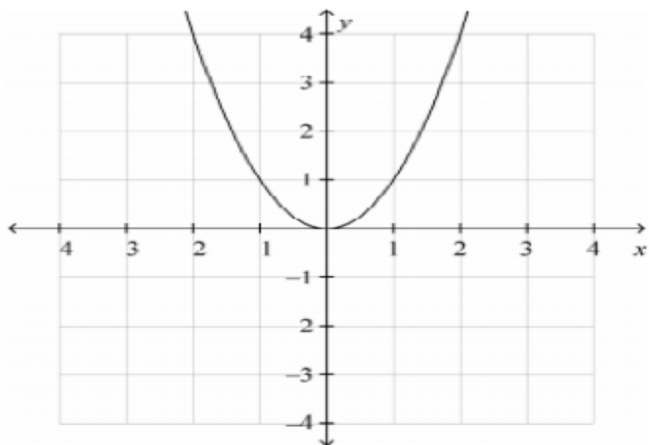
**Work/Explanation:**

4. Which transformation of  $y = f(x)$  moves the graph 7 units to the left and 3 units down?

- 1)  $y = f(x + 7) - 3$
- 2)  $y = f(x + 7) + 3$
- 3)  $y = f(x - 7) - 3$
- 4)  $y = f(x - 7) + 3$

**Work/Explanation:**

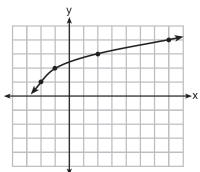
5. Identify the vertex of the graph. Tell whether it is a minimum or maximum.



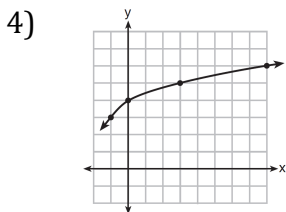
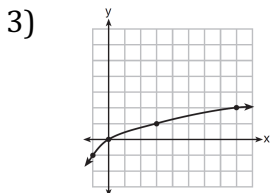
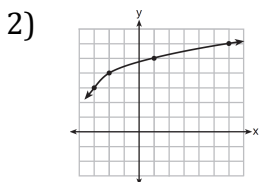
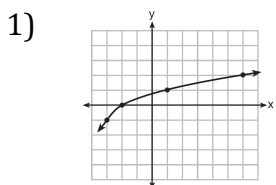
- 1) (0,0); maximum
- 2) (0,1); maximum
- 3) (0,0); minimum
- 4) (0,1); minimum

Work/Explanation:

6. The graph of  $y = f(x)$  is shown below.



What is the graph of  $y = f(x + 1) - 2$ ?



Work/Explanation:

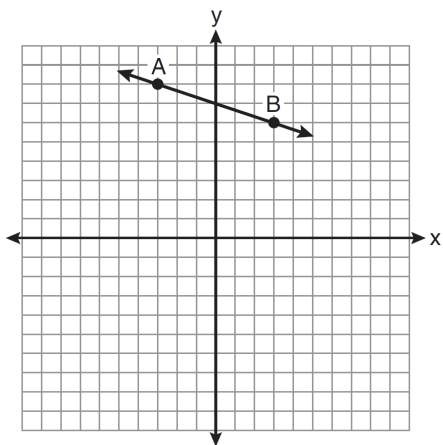
7. What is the slope of the line containing the points (3,4) and (-6,10)?

- 1)  $\frac{1}{2}$
- 2) 2
- 3)  $-\frac{2}{3}$
- 4)  $-\frac{3}{2}$

Work/Explanation:

8. What is the slope of the line passing through the points A and B, as shown on the graph below?

- 1) -3
- 2)  $-\frac{1}{3}$
- 3) 3
- 4)  $\frac{1}{3}$



Work/Explanation:

9. Which equation represents a line that is parallel to the y-axis?

- 1)  $x = 5$
- 2)  $x = 5y$
- 3)  $y = 5$
- 4)  $y = 5x$

Work/Explanation:

10. The graph of the equation  $y = -2$  is a line

- 1) parallel to the x-axis
- 2) parallel to the y-axis
- 3) passing through the origin
- 4) passing through the point (-2, 0)

Work/Explanation:

11. Fred is given a rectangular piece of paper. If the length of Fred's piece of paper is represented by  $2x - 6$  and the width is represented by  $3x - 5$ , then the paper has a total area represented by

- 1)  $5x - 11$
- 2)  $6x^2 - 28x + 30$
- 3)  $10x - 22$
- 4)  $6x^2 - 6x - 11$

**Work/Explanation:**

12. When factored completely,  $x^3 - 13x^2 - 30x$  is

- 1)  $x(x + 3)(x - 10)$
- 2)  $x(x - 3)(x - 10)$
- 3)  $x(x + 2)(x - 15)$
- 4)  $x(x - 2)(x + 15)$

**Work/Explanation:**

13. If the area of a rectangle is expressed as  $x^4 - 9y^2$ , then the product of the length and the width of the rectangle could be expressed as

- 1)  $(x - 3y)(x + 3y)$
- 2)  $(x^2 - 3y)(x^2 + 3y)$
- 3)  $(x^2 - 3y)(x^2 - 3y)$
- 4)  $(x^4 + y)(x - 9y)$

**Work/Explanation:**

14. When factored completely, the expression  $p^4 - 81$  is equivalent to

- 1)  $(p^2 + 9)(p^2 - 9)$
- 2)  $(p^2 - 9)(p^2 - 9)$
- 3)  $(p^2 + 9)(p + 3)(p - 3)$
- 4)  $(p + 3)(p - 3)(p + 3)(p - 3)$

**Work/Explanation:**

15. Which equation has the same solutions as

$$2x^2 + x - 3 = 0$$

- 1)  $(2x - 1)(x + 3) = 0$
- 2)  $(2x + 1)(x - 3) = 0$
- 3)  $(2x - 3)(x + 1) = 0$
- 4)  $(2x + 3)(x - 1) = 0$

**Work/Explanation:**

<p>16. Which expression is equivalent to <math>x^4 - 12x^2 + 36</math>?</p> <p>1) <math>(x^2 - 6)(x^2 - 6)</math></p> <p>2) <math>(x^2 + 6)(x^2 + 6)</math></p> <p>3) <math>(6 - x^2)(6 + x^2)</math></p> <p>4) <math>(x^2 + 6)(x^2 - 6)</math></p>	<p><b><u>Work/Explanation:</u></b></p>
<p>17. When factored completely, <math>x^3 + 3x^2 - 4x - 12</math> equals</p> <p>1) <math>(x+2)(x-2)(x-3)</math></p> <p>2) <math>(x+2)(x-2)(x+3)</math></p> <p>3) <math>(x^2 - 4)(x+3)</math></p> <p>4) <math>(x^2 - 4)(x-3)</math></p>	<p><b><u>Work/Explanation:</u></b></p>
<p>18. What are the factors of <math>x^2 - 10x - 24</math>?</p> <p>1) <math>(x - 4)(x+6)</math></p> <p>2) <math>(x - 4)(x - 6)</math></p> <p>3) <math>(x - 12)(x + 2)</math></p> <p>4) <math>(x + 12)(x - 2)</math></p>	<p><b><u>Work/Explanation:</u></b></p>
<p>19. When factored completely, the expression <math>3x^2 - 9x + 6</math> is equivalent to</p> <p>1) <math>(3x - 3)(x + 2)</math></p> <p>2) <math>(3x + 3)(x + 2)</math></p> <p>3) <math>3(x + 1)(x - 2)</math></p> <p>4) <math>3(x - 1)(x - 2)</math></p>	<p><b><u>Work/Explanation:</u></b></p>
<p>20. Factored completely, the expression <math>12x^4 + 10x^3 - 12x^2</math> is equivalent to</p> <p>1) <math>x^2(4x + 6)(3x - 2)</math></p> <p>2) <math>2(2x^2 + 3x)(3x^2 - 2x)</math></p> <p>3) <math>2x^2(2x - 3)(3x + 2)</math></p> <p>4) <math>2x^2(2x + 3)(3x - 2)</math></p>	<p><b><u>Work/Explanation:</u></b></p>
<p>21. The equation for the volume of a cylinder is <math>V = \pi r^2 h</math>. The positive value of <math>r</math>, in terms of <math>h</math> and <math>V</math>, is</p> <p>1) <math>r = \sqrt{\frac{V}{\pi h}}</math></p> <p>2) <math>r = \sqrt{V\pi h}</math></p> <p>3) <math>r = 2V\pi h</math></p> <p>4) <math>r = \frac{V}{2\pi}</math></p>	<p><b><u>Work/Explanation:</u></b></p>



22. Which value of  $p$  is the solution of  $5p - 1 = 2p + 20$ ?

- 1)  $\frac{19}{7}$
- 2)  $\frac{19}{3}$
- 3) 3
- 4) 7

**Work/Explanation:**

23. The formula for the volume of a cone is  $V = \frac{1}{3} \pi r^2 h$ .  
The radius,  $r$ , of the cone may be expressed as

- 1)  $\sqrt{\frac{3V}{\pi h}}$
- 2)  $\sqrt{\frac{V}{3\pi h}}$
- 3)  $\sqrt[3]{\frac{V}{\pi h}}$
- 4)  $\frac{1}{3} \sqrt{\frac{V}{\pi h}}$

**Work/Explanation:**

24. Which value of  $x$  is the solution of the equation

$$2(x - 4) + 7 = 3?$$

- 1) 1
- 2) 2
- 3) 6
- 4) 0

**Work/Explanation:**

25. If  $3(x + 2) - 2(x + 1) = 8$ , the value of  $x$  is

- 1) 1
- 2)  $\frac{1}{5}$
- 3) 5
- 4) 4

**Work/Explanation:**

26. What is the solution set of the equation  $\frac{x}{5} + \frac{x}{2} = 14$ ?

- 1) {4}
- 2) {10}
- 3) {20}
- 4) {49}

**Work/Explanation:**

<p>27. What is the solution set of the equation <math>3x^2 = 48</math> ?</p> <ol style="list-style-type: none"> <li>1) <math>\{-2, -8\}</math></li> <li>2) <math>\{2, 8\}</math></li> <li>3) <math>\{4, -4\}</math></li> <li>4) <math>\{4, 4\}</math></li> </ol>	<p><b><u>Work/Explanation:</u></b></p>
<p>28. The solution of the equation <math>(x + 3)^2 = 7</math> is</p> <ol style="list-style-type: none"> <li>1) <math>3 \pm \sqrt{7}</math></li> <li>2) <math>7 \pm \sqrt{3}</math></li> <li>3) <math>-3 \pm \sqrt{7}</math></li> <li>4) <math>-7 \pm \sqrt{3}</math></li> </ol>	<p><b><u>Work/Explanation:</u></b></p>
<p>29. What is the solution set for the equation <math>x^2 - 5x + 6 = 0</math> ?</p> <ol style="list-style-type: none"> <li>1) <math>\{-6, 1\}</math></li> <li>2) <math>\{6, -1\}</math></li> <li>3) <math>\{-2, -3\}</math></li> <li>4) <math>\{2, 3\}</math></li> </ol>	<p><b><u>Work/Explanation:</u></b></p>
<p>30. The solutions of <math>x^2 = 16x - 28</math> are</p> <ol style="list-style-type: none"> <li>1) <math>-2</math> and <math>-14</math></li> <li>2) <math>2</math> and <math>14</math></li> <li>3) <math>-4</math> and <math>-7</math></li> <li>4) <math>4</math> and <math>7</math></li> </ol>	<p><b><u>Work/Explanation:</u></b></p>
<p>31. Which verbal expression represents <math>2(n - 6)</math> ?</p> <ol style="list-style-type: none"> <li>1) two times <math>n</math> minus six</li> <li>2) two times six minus <math>n</math></li> <li>3) two times the quantity <math>n</math> less than six</li> <li>4) two times the quantity six less than <math>n</math></li> </ol>	<p><b><u>Work/Explanation:</u></b></p>
<p>32. When <math>5\sqrt{20}</math> is written in simplest radical form, the result is <math>k\sqrt{5}</math>. What is the value of <math>k</math>?</p> <ol style="list-style-type: none"> <li>1) 20</li> <li>2) 10</li> <li>3) 7</li> <li>4) 4</li> </ol>	<p><b><u>Work/Explanation:</u></b></p>

33. What is  $3\sqrt{250}$  expressed in simplest radical form?

- 1)  $5\sqrt{10}$
- 2)  $8\sqrt{10}$
- 3)  $15\sqrt{10}$
- 4)  $75\sqrt{10}$

Work/Explanation:

34. Expressed in simplest radical form, the product of

$\sqrt{6} \cdot \sqrt{15}$  is

- 1  $9\sqrt{10}$
- 2  $3\sqrt{15}$
- 3  $\sqrt{90}$
- 4  $3\sqrt{10}$

Work/Explanation:

35. To watch a varsity basketball game, spectators must buy a ticket at the door. The cost of an adult ticket is \$3.00 and the cost of a student ticket is \$1.50. If the number of adult tickets sold is represented by  $a$  and student tickets sold by  $s$ , which expression represents the amount of money collected at the door from the ticket sales?

- 1)  $4.50as$
- 2)  $4.50(a + s)$
- 3)  $(3.00a)(1.50s)$
- 4)  $3.00a + 1.50s$

Work/Explanation:

36. The expression  $\frac{12w^9y^3}{-3w^3y^3}$  is equivalent to

- 1)  $-4w^6$
- 2)  $-4w^3y$
- 3)  $9w^6$
- 4)  $9w^3y$

Work/Explanation:

37. Which expression represents  $(3x^2y^4)(4xy^2)$  in simplest form?

- 1)  $12x^2y^8$
- 2)  $12x^2y^6$
- 3)  $12x^3y^8$
- 4)  $12x^3y^6$

Work/Explanation:

38. A cube, with faces numbered 1 to 6, is rolled, and a penny is tossed at the same time. How many elements in the sample space consist of an even number and a tail?

- 1) 12
- 2) 2
- 3) 3
- 4) 4

**Work/Explanation:**

39. 1. Given set  $S = 16, 3, 19, 3, 9$ . Identify the median, mode and mean.

- 1) Median = 5, Mode = 10, Mean = 3.
- 2) Median = 9, Mode = 3, Mean = 10.
- 3) Median = 3, Mode = 9, Mean = 10.
- 4) Median = 2, Mode = None, Mean = 4

**Work/Explanation:**

40.

Julie averaged 85 on the first three tests of the semester in her mathematics class. If she scores 93 on each of the remaining tests, her average will be 90. Which equation could be used to determine how many tests,  $T$ , are left in the semester?

- (1)  $\frac{255 + 93T}{3T} = 90$
- (2)  $\frac{255 + 90T}{3T} = 93$
- (3)  $\frac{255 + 93T}{T + 3} = 90$
- (4)  $\frac{255 + 90T}{T + 3} = 93$

**Work/Explanation:**