

Carteret  
High  
School

Calculus

Summer Project, 2018

Name: \_\_\_\_\_

Date: \_\_\_\_\_

## CALCULUS SUMMER PACKET

Copy and complete all steps of each problem in a well-organized notebook and draw graphs on grid paper. Be neat and attempt all problems. You should be able to do every problem **without a calculator** (except graphing problems where you might use calculator to create table of values or might want to use the calculator to check your work). You would be **tested on these concepts** sometimes during the first week of class.

1. Are the following statements true? If not, change them to make them true.

$$a. \frac{2k}{2k+4} = \frac{k}{k+4}$$

$$b. \frac{1}{p} + \frac{1}{q} = \frac{1}{p+q}$$

$$c. \frac{x}{2} + \frac{y}{2} = \frac{x+y}{4}$$

$$d. 3\left(\frac{x}{y}\right) = \frac{3x}{3y}$$

$$e. 3\left(\frac{a+b}{c}\right) = \frac{3a+b}{c}$$

$$f. \left(\frac{x+b}{x}\right)\left(\frac{x+d}{x}\right) = \left(\frac{1+b}{1}\right)\left(\frac{1+d}{1}\right)$$

2. Simplify

$$a. \frac{\frac{x}{2}}{\frac{x}{4}}$$

$$b. h \div \frac{x+h}{h}$$

$$c. \frac{\sqrt{x-2} + \frac{5}{\sqrt{x-2}}}{x-2}$$

$$d. \frac{x^3}{x^{-5}}$$

$$e. \frac{2x^3}{y^{-5}} \cdot \frac{y^2}{3x^7}$$

$$f. \frac{x^2 - 4x - 5}{x^2 + 2x + 1}$$

$$g. \frac{x-4}{8-2x}$$

$$h. (x-2)^3$$

$$i. \frac{1}{x^3} \cdot \frac{3}{x^5}$$

$$k. \frac{\frac{1}{x} - \frac{1}{5}}{\frac{1}{x^2} - \frac{1}{25}}$$

$$l. \frac{1}{x+h} - \frac{1}{x}$$

$$m. \frac{\frac{1}{3+x} - \frac{1}{3}}{x}$$

$$n. \frac{x^3 - 8}{x-2}$$

$$o. \frac{5-x}{x^2 - 25}$$

$$p. \frac{2x^2 + 5x - 12}{x^2 - 16}$$

3. Solve for z

$$a. xz + y = 1 + z$$

$$b. 2x^2z + 2yz = 5z + 2x$$

$$c. 3x^2yz + 2xy^2 = 2yz$$

4. Solve the equation for all real values of x.

$$a. 4x^2 - 21x - 18 = 0$$

$$b. x^4 - 9x^2 + 8 = 0$$

$$c. \frac{2}{x+1} = \frac{x-2}{2}$$

$$d. \frac{1}{x} + x = 4$$

$$e. \sqrt{x-1} - \frac{5}{\sqrt{x-1}} = 0$$

$$f. (x-5)^2 - 9 = 0$$

$$g. |x-3| = 7$$

$$h. 27^{2x} = 9^{x-3}$$

$$i. 5^{(x+1)} = \frac{1}{25}$$

$$j. \log x = 3$$

$$k. \log_3 x^2 = 2 \log_3 4$$

$$l. \ln(2x) - \ln(x-3) = 0$$

5. Write as a single fraction with the denominator in the factored form.

$$a. \frac{7x^2 + 5x}{x^2 + 1} - \frac{5x}{x^2 - 6}$$

$$b. 20\left(\frac{2}{x+1} - \frac{3}{x}\right)$$

6. Graph the following equation  $y = x^3 - x$  and answer the following questions.

- Is the point (3,2) on the graph?
- Is the point (2,6) on the graph?
- Is the function odd, even or neither?
- Find the x and y – intercepts.

7. Factor completely

a.  $3x^3 + 192$

b.  $9x^2 - 3x - 2$

c.  $2\sqrt{x} - 6x^{\frac{3}{2}}$

d.  $\sin x + \tan x$

e.  $\frac{2x}{3\sqrt{x}}$

f.  $e^{-x} - xe^{-x} + 2x^2e^{-x}$

8. Find the equation of the line that passes through the point (2, 4) and is parallel to the line  $2x + 3y - 8 = 0$ .

9. Find the equation of the line that is perpendicular to the line  $2x + 3y - 8 = 0$  at the point (1, 2).

10. The line with slope 5 that passes through the point (-1, 3) intersects the x-axis at a point. What are the coordinates of this point?

11. What are the coordinates of the point at which the line passing through the points (1, -3) and (-2, 4) intersects the y-axis?

12. A 20 foot ladder rests against a building 15 feet from the floor. How far does the ladder extend from the base of the wall? What angle does the ladder make with the ground?

13. Find  $f(1) - f(5)$  given  $f(x) = |x - 3| - 5$ .

14. Find  $f(x+2) - f(2)$  given  $f(x) = x^2 - 3x + 4$ .

15. Find  $f(x+h)$  for  $f(x) = x^2 - 2x - 3$ .

16. Find  $\frac{f(x+h) - f(x)}{h}$  for  $f(x) = 8x^2 + 1$ .

17. Find  $\frac{f(x+h) - f(x)}{h}$  for  $f(x) = \frac{1}{x}$ .

18. Determine whether the following functions are even, odd, or neither.

a.  $f(x) = x^5 - x$

b.  $f(x) = x^6 - 8x^4 + 2x^2$

c.  $f(x) = 3x^3 - 1$

19. If  $f(x) = \{(3,5), (2,4), (1,7)\}$   $g(x) = \sqrt{x-3}$  determine each of the following:  
 $h(x) = \{(3,2), (4,3), (1,6)\}$   $k(x) = x^2 + 5$

- a.  $(f+h)(1)$  b.  $(k-g)(5)$   
 c.  $f(h(3))$  d.  $(g \circ k)(7)$   
 e.  $f^{-1}(x)$  f.  $k^{-1}(x)$   
 g.  $\frac{1}{f(x)}$  h.  $(kg)(x)$

20. Graph the functions.

- a.  $f(x) = \begin{cases} 1 & x \leq 0 \\ -1 & x > 0 \end{cases}$  b.  $f(x) = \begin{cases} 2x & (-\infty, -1) \\ 2x^2 & [-1, 2) \\ -x+3 & [2, \infty) \end{cases}$  c.  $f(x) = \sqrt{16-x^2}$

21. Given  $f(x) = x-3$  and  $g(x) = \sqrt{x}$ , complete the following:

- a.  $f(g(x)) =$  b.  $g(f(x)) =$  c.  $f(f(x)) =$

22. Given  $f(x) = \frac{1}{x-5}$  and  $g(x) = x^2 - 5$ , complete the following:

- a.  $f(g(7)) =$  b.  $g(f(v)) =$  c.  $g(g(x)) =$

23. Find all intercepts and asymptotes

- a.  $y = \frac{x^2 + 3x}{(3x+1)^2}$  b.  $y = \frac{x^2 - 4}{x^2 - x - 12}$  c.  $y = \frac{3x-1}{2x^2 + x - 6}$

24. Find the surface area of a box of height  $h$  whose base dimensions are  $p$  and  $q$  and satisfies the following conditions.

- a. The box is closed.
- b. The box has an open top.
- c. The box has an open top and a square base with side length  $p$ .

25. A seven foot ladder, leaning against a wall, touches the wall  $x$  feet above the ground. Write an expression in terms of  $x$  for the distance from the foot of the ladder to the base of the wall.

26. A piece of wire 5 inches long is to be cut into two pieces. One piece is  $x$  inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Find an expression for the total area made up by the square and the circle as a function of  $x$ .

27. Solve the following for the principal values of the indicated variable.

a.  $3 \cos x - 1 = 2$

b.  $2 \sin(2x) - \sqrt{3} = 0$

c.  $\tan^2 x - 1 = 0$

28. Evaluate (use unit circle)

a.  $\cos(0)$

b.  $\sin(0)$

c.  $\tan\left(\frac{\pi}{2}\right)$

d.  $\cos\frac{\pi}{4}$

e.  $\sin\frac{\pi}{2}$

f.  $\sin \pi$

g.  $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

h.  $\tan^{-1}(-1)$

i.  $\cos^{-1}\left(\frac{1}{2}\right)$

j.  $\sec^{-1}(\sqrt{2})$

k.  $\cos^{-1}(-1)$

l.  $\sec\left(\frac{\pi}{2}\right)$

m.  $\tan\left(-\frac{\pi}{6}\right)$

n.  $\sin\left(\frac{5\pi}{3}\right)$

o.  $\csc\left(-\frac{9\pi}{4}\right)$

29. Express  $y$  in terms of  $x$

a.  $\ln y = x + 2$

b.  $\ln y = 2 \ln x + \ln 10$

c.  $\ln y = 4 \ln x + 3$

d.  $x = \ln\left(\frac{e^{x^2}}{4y}\right)$

30. Solve for  $x$

a.  $\ln e^3 = x + 1$

b.  $\ln e^x = 4$

c.  $\ln x = -\ln x$

d.  $\ln 1 - \ln e = x$

e.  $\ln 6 + \ln x - \ln 2 = 3$

f.  $\ln(x+5) = \ln(x-1) - \ln(x+1)$

31. Multiply the following:

a.  $(\cos x)(3)$

b.  $(\tan x)(2x)$

c.  $(\sec x)(-5x^2)$

d.  $(3 \sin x)(-2 \cos x)$

e.  $(\sin x)(\sin x)$

f.  $(2x \tan x)(3x \sin x)$

32. Graph each equation in separate graphs. State its domain and range.

a.  $y = \sin x$

b.  $y = \csc x$

c.  $y = \cos x$

d.  $y = \sec x$

e.  $y = \tan x$

f.  $y = \cot x$

g.  $y = \sqrt{x}$

h.  $y = \sqrt[3]{x}$

i.  $y = |x+3| - 2$

j.  $y = e^x$

k.  $y = \ln x$

l.  $x^2 + y^2 = 16$