**Exponential Functions and Modeling**

Let’s start looking at exponential functions, by looking at exponential functions and linear functions side by side.

<table>
<thead>
<tr>
<th>General Form</th>
<th>Linear Model</th>
<th>Exponential Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = ax + b )</td>
<td>( f(x) = a(x) )</td>
<td>( f(x) = a(b)^x )</td>
</tr>
</tbody>
</table>

Meaning of Parameters \( a \) and \( b \)

- \( a \) is the slope of the line or the constant rate of change; \( b \) is the y-intercept of the \( f(x) \) value at \( x = 0 \).
- \( a \) is the y-intercept or the \( f(x) \) value when \( x = 0 \); \( b \) is the base or the constant quotient of change.

Example

- \( f(x) = 2x + 3 \)
- \( f(x) = 3(2)^x \)

Rule for Finding \( f(x + 1) \) from \( f(x) \)

- Starting at \((0, 3)\), for find \( f(x + 1) \), add 2 to \( f(x) \).
- Starting at \((0, 3)\), for find \( f(x + 1) \), multiply \( f(x) \) by 2.

Table

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

Graph

As you can see the graph of an exponential is a curve, while a linear graph is a line. If you look at the table, you can see with the linear table, the \( y \)'s are going up by adding 2 to the previous term, while with the exponential table, after the second term, the remaining terms were found by multiplying the previous term by 2.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
</tr>
</tbody>
</table>

Let’s create a table for \( f(x) = 3^x \). Just looking at this equation, I know that after the first two terms, I can find the next term by multiplying the previous term by 3. I also know that anything to the zero power equals 1, and anything to the 1st power is itself. (If the equation was \( f(x) = 2(3^x) \) then all of the \( f(x) \) values would be multiplied by 2.)
Let’s look at another example: \( f(x) = 3(4^x) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) = 3(4^x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>48</td>
</tr>
<tr>
<td>3</td>
<td>192</td>
</tr>
<tr>
<td>4</td>
<td>768</td>
</tr>
<tr>
<td>5</td>
<td>3074</td>
</tr>
</tbody>
</table>

Multiply the previous term by 4, then multiply that answer by 3.

Notice that the \( y \)-values increase very quickly. This is a characteristic of an exponential equation.

Here’s an example where the table is given and you need to create the equation.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>160</td>
</tr>
</tbody>
</table>

Create the equation in the form \( f(x) = a(b)^x \). Looking at the table the \( y \)-value is being multiplied by 2, so \( b = 2 \). \( a \) must be equal to 5 because \( ab^0 = 5 \). (Any number/variable raised to the zero power is 1.)

The equation for this table is \( f(x) = 5(2)^x \).

YOUR TURN:

1. \( f(x) = 6(8)^x \)
   a. When \( x = 0 \), \( f(x) = \)
   b. The next term in the sequence is found by multiplying the previous term by \( \)\( \), then \( \).

2. \( f(x) = \frac{1}{2}(10)^x \)
   a. When \( x = 0 \), \( f(x) = \)
   b. The next term in the sequence is found by multiplying the previous term by \( \)\( \), then \( \).

For the next two problems complete the table.

3. \( f(x) = 4(3)^x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) = 4(3)^x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

4. \( f(x) = 5(2)^x \)

<table>
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<tr>
<td>0</td>
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<td></td>
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<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>
Now that you can create a table of values, let’s create graphs of exponential equations.

\[ f(x) = 5(2)^x \]

When \( x = 0, y = 5 \) or \( (0, 5) \). When \( x = 1, y = 10 \) or \( (1, 10) \). Start by graphing these two points.

When you substitute \( x = 2 \) and then \( x = 3 \), the next points would be \( (2,20) \) and \( (3,40) \). Notice that the \( x \)-values are increasing by 1, but the \( y \)-values are multiplied by 2. Finally you need to sketch a curve through the points.

Notice that as the graph approaches the \( x \)-axis, it looks like it should touch. However, it will never touch the \( x \)-axis, because the function \( f(x) = 5(2)^x \) can never be 0 or have a negative value. You should also notice that the graph is a curve.

As you can see, the key to graphing an exponential equation is to find the first two terms and know that the “\( b \)” in the problem is what each previous \( y \)-value needs to be multiplied by to get the next \( y \)-value.

Let’s look at another example, before you try a few.

\[ y = 8 \left( \frac{1}{2} \right)^x \]

After the first two terms, multiply the \( y \)-term by \( \frac{1}{2} \) to get the next value. If \( x = 0 \), the first point will be \( (0,8) \). When \( x = 1 \), the point is \( (1,4) \).

Plot those two points. The rule is to add one to the \( x \)-value and take half of the previous \( y \)-value.

**YOUR TURN:** Graph each of the following equations on the grid provided.

1. \( y = (3)^x \)
2. \( y = 6 \left( \frac{1}{2} \right)^x \)
Exponential Growth and Decay
Exponential functions are useful for describing real world situations.

Note that the two formulas are identical except one uses addition and the other uses subtraction. Growth uses addition and decay uses subtraction, so it should be easy to remember which formula goes with which type of problem.

Example 1: An abandoned cat named Riley, was found and taken to a local animal shelter. When he arrived, he had 12 fleas on him. If left untreated, the fleas will increase at the rate of 80% each week. If Riley was not treated for fleas, how many fleas would you expect to be on Riley at the end of 6 weeks?

What you know:
- Initial number of fleas on Riley: \( a = 12 \)
- Growth rate: \( r = 80\% = 0.8 \)
- Time that will pass: \( x = 6 \) weeks

Growth problem: \( y = a(1 + r)^x \)

Substituting in the values, we have
\[
\begin{align*}
y &= 12(1 + 0.8)^6 \\
y &= 12(1.8)^6 \\
y &= 12(34) \\
y &= 408 \text{ fleas}
\end{align*}
\]

Example 2: The number of wolves in the wild in the northern section of a county is decreasing at the rate of 3.5% per year. An environmental group counted 80 wolves in the area. If the rate of decrease remains the same, how many wolves can you expect to have in 15 years?

What you know:
- Initial number of wolves: \( a = 80 \)
- Decreasing rate: \( r = 3.5\% = 0.035 \)
- Time that will pass: \( x = 15 \) years

Decay problem: \( y = a(1 - r)^x \)

\[
\begin{align*}
y &= 80(1 - 0.035)^{15} \\
y &= 80(0.965)^{15} \\
y &= 80(0.59) \\
y &\approx 47.2 \approx 47 \text{ wolves}
\end{align*}
\]

YOUR TURN:
1. \( y = 1200(1 + 0.3)^x \)
   - A. Does this function represent exponential growth or exponential decay?
   - B. What is your initial value?
   - C. What is the rate of growth or rate of decay?
2. \( y = 55(1 - 0.02)^x \)
   
   A. Does this function represent exponential growth or exponential decay?
   
   B. What is your initial value?
   
   C. What is the rate of growth or rate of decay?

Solve each of the following problems.

3. The first year of a charity walk event had an attendance of 500. The attendance increases by 5% each year.
   A. Write an exponential growth function to represent this situation.
   B. How many people will attend in the 10th year?

4. Your car cost $42,500, when you purchased it in 2015. The value of the car decreases by 15% annually.
   A. Write an exponential decay function to represent this situation.
   B. How much will your car be worth in 2022 (7 years)? Round your answer to the nearest dollar.