The Quadratic formula is a way to solve a quadratic function for the value of x. In order to solve an equation using the quadratic formula, the equation must be set equal to zero. Sometimes a quadratic is not easy to factor or maybe you do not want to factor.

- Set the equation in the form \( ax^2 + bx + c = 0 \)
- identify the \( a \), \( b \), and \( c \)
- substitute them into the equation and solve.

**What does that “±” symbol mean?**

It means that there are two solutions. The quadratic equation could be written as two separate equations: 

\[
 x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

The \( a \) will always be the coefficient with the \( x^2 \). The \( b \) is always the coefficient with the \( x \) and the \( c \) is always the constant.

**Example:** Solve \( x^2 + x - 6 = 0 \)

<table>
<thead>
<tr>
<th>Set Equation = 0</th>
<th>( x^2 + x - 6 = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>( b )</td>
</tr>
<tr>
<td>( a = 1, b = 1 ) and ( c = -6 )</td>
<td>( x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-6)}}{2(1)} )</td>
</tr>
<tr>
<td>Step 2: Substitute the ( a ), ( b ), and ( c ) into the quadratic formula.</td>
<td>( x = \frac{-1 \pm 1 + 24}{2} = \frac{-1 \pm 25}{2} = \frac{-1 \pm 5}{2} )</td>
</tr>
<tr>
<td>Step 3: Simplify the right hand side of the equation.</td>
<td>( x = \frac{-1 + 5}{2} ) and ( x = \frac{-1 - 5}{2} )</td>
</tr>
<tr>
<td>Step 4: Create two separate equations, one with the “+” sign and one with the “−” sign.</td>
<td>( x = \frac{4}{2} = 2 ) and ( x = \frac{-6}{2} = -3 )</td>
</tr>
<tr>
<td>Step 5: Solve each of the equations.</td>
<td>The solution to the equation is ( x = 2 ) or ( x = -3 )</td>
</tr>
</tbody>
</table>

Finally, we need to check our answer to make sure they work. We substitute each value back into the original equation to make sure it work.

\[
\begin{array}{ccc}
  x^2 + x - 6 &=& 0 \\
  (2)^2 + (2) - 6 &=& 0 \\
  4 + 2 - 6 &=& 0 \\
  0 &=& 0 & \checkmark \\
  x^2 + x - 6 &=& 0 \\
  (-3)^2 + (-3) - 6 &=& 0 \\
  9 - 3 - 6 &=& 0 \\
  0 &=& 0 & \checkmark 
\end{array}
\]
Our solutions are correct: \( x = 2 \) or \( x = -3 \). These values can also be used to find the factors of \( x^2 + x - 6 = 0 \). To find the factors, take the answers and rewrite them so they are equal to zero. Take the first answer \( x = 2 \), and subtract 2 from both sides. This becomes \( x - 2 = 0 \). Take \( x = -3 \), and add 3 to both sides. This becomes \( x + 3 = 0 \).

\[
x^2 + x - 6 = 0 \text{ is factored into } (x - 2)(x + 3).
\]

Example 2: Solve \( 5x^2 + 6x + 1 = 0 \)

<table>
<thead>
<tr>
<th>Set Equation = 0</th>
<th>( 5x^2 + 6x + 1 = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1: Identify the ( a, b, ) and ( c )</td>
<td>( a = 5, b = 6 ) and ( c = 1 )</td>
</tr>
<tr>
<td>Step 2: Substitute the ( a, b, ) and ( c ) into the quadratic formula.</td>
<td>( x = \frac{-6 \pm \sqrt{36 - 20}}{10} = \frac{-6 \pm \sqrt{16}}{10} = \frac{-6 \pm 4}{10} )</td>
</tr>
<tr>
<td>Step 3: Simplify the right hand side of the equation.</td>
<td>( x = \frac{-6 + 4}{10} = -0.2 ) and ( x = \frac{-6 - 4}{10} = -1 )</td>
</tr>
<tr>
<td>Step 4: Create two separate equations, one with the “+” sign and one with the “-” sign.</td>
<td>( x = \frac{-6 + 4}{10} = -0.2 ) or ( x = \frac{-6 - 4}{10} = -1 )</td>
</tr>
<tr>
<td>Step 5: Solve each of the equations.</td>
<td>( x = \frac{-6 + 4}{10} = -0.2 ) or ( x = \frac{-6 - 4}{10} = -1 )</td>
</tr>
<tr>
<td>So the solution to the equation is ( x = -0.2 ) or ( x = -1 )</td>
<td></td>
</tr>
</tbody>
</table>

Finally, we need to check our answer to make sure they work. We substitute each value back into the original equation to make sure it work.

\[
\begin{align*}
5x^2 + 6x + 1 &= 0 \\
5 \left( \frac{1}{5} \right)^2 + 6 \left( \frac{1}{5} \right) + 1 &= 0 \\
5 \left( \frac{1}{25} \right) - \frac{6}{5} + 1 &= 0 \\
\frac{1}{5} - \frac{6}{5} + 1 &= 0 \\
0 &= 0
\end{align*}
\]

\[
\begin{align*}
5x^2 + 6x + 1 &= 0 \\
5 \left( -\frac{1}{5} \right)^2 + 6 \left( -\frac{1}{5} \right) + 1 &= 0 \\
5 - 6 + 1 &= 0 \\
0 &= 0
\end{align*}
\]

We know that our solutions are correct. We had \( x = -1 \) and \( x = -\frac{1}{5} \). To find the factors, we take each of those solutions and rewrite them so that they equal 0. If we take \( x = -1 \) and rewrite it by adding 1 to both sides we get \( x + 1 = 0 \).

\( x = -\frac{1}{5} \) becomes just a little harder to set equal to zero. We need to multiply both sides of this equation by 5 to get rid of the denominator, so we will have \( 5x = -\frac{1}{5} \Rightarrow 5x = -1 \).

Now we can take that equation, \( 5x = -1 \), and add 1 to both sides. We now have \( 5x + 1 = 0 \).

\[
5x^2 + 6x + 1 = 0 \text{ is factored into } (x + 1)(5x + 1)
\]
YOUR TURN

For the next three problems, identify the $a$, $b$, and $c$.

1. $7x^2 + 22x + 3$
   $a =$ $b =$ $c =$

2. $2x^2 + 7x + 6$
   $a =$ $b =$ $c =$

Identify the $a$, $b$, and $c$ and then use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to find the solution to the problems. Make sure you check your answers.

3. $x^2 + 16x + 28 = 0$

Identify the $a$, $b$, and $c$ and then use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to find the solution to the problems. Finally, use your solutions to write the factors of the quadratic.

4. $x^2 - 2x - 24 = 0$
Graphing Quadratics

Now we are going to look at graphing quadratics. Graphing functions is another way that you can find the factors to a quadratic. You have graphed linear functions, so you know that we can graph by creating a table. Let’s begin by looking at graphing \( y = 2x^2 - 2x - 12 \). When we graph quadratics, we should use at least 5 or 6 points in our table. Sometimes you need more and sometime less. Substitute in values for \( x \) and solve for \( y \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>-8</td>
</tr>
<tr>
<td>0</td>
<td>-12</td>
</tr>
<tr>
<td>1</td>
<td>-12</td>
</tr>
<tr>
<td>2</td>
<td>-8</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ y = 2x^2 - 2x - 12 \]
\[ y = 2(-2)^2 - 2(-2) - 12 = 2(4) + 4 - 12 = 8 + 4 - 12 = 0 \]
\[ y = 2(-1)^2 - 2(-1) - 12 = 2(1) + 2 - 12 = 2 + 2 - 12 = -8 \]
\[ y = 2(0)^2 - 2(0) - 12 = 2(0) + 0 - 12 = 0 + 0 - 12 = -12 \]
\[ y = 2(1)^2 - 2(1) - 12 = 2(1) - 2 - 12 = 2 - 2 - 12 = -12 \]
\[ y = 2(2)^2 - 2(2) - 12 = 2(4) - 4 - 12 = 8 - 8 - 12 = -8 \]
\[ y = 2(3)^2 - 2(3) - 12 = 2(9) - 6 - 6 = 18 - 6 - 12 = 0 \]

Once we have our table, we now need to plot our points.

- **the points do not create a straight line**
- **the graph is symmetrical** If you folded the graph vertically (hotdog style) you would see that you can get the points to line up.
- **Connect the points.** The graph has a “U” shape. Quadratic function always have this type of shape.
- **If the “a” coordinate is positive, the graph will open upward.**
- **If the “a” coordinate is negative, the graph will open downward.** (The \( a \) will always be the coefficient with the \( x^2 \).)
- The zeros, where the graph crosses the x-axis or where \( x = 0 \) are the “factors” of the quadratic. This is another way that you can factor a quadratic.
Let’s look at another one: \( y = -x^2 + 2x + 3 \). Again, we will begin by creating a table of values. Remember you’re going to need between 5 and 6 values in your table. When picking your values, you are looking for the \( y \) to either change their signs or start to repeat. (Remember all of these equations will form a “U”.

\[
\begin{array}{c|c}
 x & y \\
-2 & -5 \\
-1 & 0 \\
0 & 3 \\
1 & 4 \\
2 & 3 \\
3 & 0 \\
\end{array}
\]

\[
y = -x^2 + 2x + 3 \\
y = -(2)^2 + 2(-2) + 3 = -4 - 4 + 3 = -5 \\
y = -(1)^2 + 2(-1) + 3 = -1 - 2 + 3 = 0 \\
y = 0^2 + 2(0) + 3 = 0 + 0 + 3 = 3 \\
y = -(1)^2 + 2(1) + 3 = -1 + 2 + 3 = 4 \\
y = -(2)^2 + 2(2) + 3 = -4 + 4 + 3 = 3 \\
y = -(3)^2 + 2(3) + 3 = -9 + 6 + 3 = 0 \\
\]

Now plot these points and connect them. Notice this time the graph is opening downward, because the \( a \), term of the quadratic is negative.

YOUR TURN For each of the following problems, create a table of value, plot the points, and then connect the points to create the graph of the quadratic.

1. \( y = 3x^2 \)
2. \( y = -2x^2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

3. \( y = 2x^2 + 8x + 5 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
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</table>