Graphing Linear Inequalities and Systems of Inequalities

Day 1…. Background:

Do you remember that a single variable inequality, such as $x > 2$ has many solutions? 2, 3, 4, 5, 6, ....... which is represented on a number line as:

![Number Line with Shading]

The shading indicates that all of the numbers greater than 2 are part of the solution set. Similarly, for a linear inequality that involves 2 variables, the line is graphed on a coordinate plane as usual, but a shaded section will indicate where many solutions exist. Ex.

![Graph of Linear Inequality]

Ex. The point (3, 1) lies in the shaded area, and it is part of the solution set because if you substitute those values into the inequality:

$$3 \leq 1 + 2$$

$$3 \leq 3$$ and this IS a true statement. As you can see, an infinite number of points are in the shaded area, and all are solutions.

So, before we learn how to graph an inequality, there are a few important symbols to review:

- $<$ less than
- $>$ greater than
- $\leq$ less than or equal to
- $\geq$ greater than or equal to

| 3 < 4 | 6 > 5 | 2 ≤ 2 | 1 ≤ 2 | 5 ≥ 5 | 5 ≥ 4 |

Graphing a Linear Inequality:

<table>
<thead>
<tr>
<th>Steps</th>
<th>Example</th>
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<tbody>
<tr>
<td>1. Change the equation to slope-intercept form $y = mx + b$</td>
<td><img src="image" alt="Graph of Linear Inequality" /></td>
</tr>
<tr>
<td>2. Graph the equation (called a boundary line)</td>
<td></td>
</tr>
</tbody>
</table>
| 3. If $<$ or $>$ use a dashed line  
If $\leq$ or $\geq$ use a solid line | |
| 4. If $y > mx + b$ or $y \geq mx + b$ shade above the line  
If $y < mx + b$ or $y \leq mx + b$ shade below the line | |
| 5. To check if the shading is correct, pick a point | |
Point in the shaded area. Substitute the $x$ and $y$ values of the point into the inequality. If the inequality is TRUE, the shading is correct because your test point indicates that is where the solutions are. If it is FALSE, the shaded area is incorrect and the opposite side of the line should be shaded.

I randomly choose $(3,0)$ which is in the shaded area.

\[
y \leq 2x - 3
\]

Is $0 \leq 2(3) - 3$ ?  
Is $0 \leq 6 - 3$ ?  
Is $0 \leq 3$ ?

Yes! True statement, so the shaded area contains the solutions.

Problems Day 1:

1) Which ordered pairs are solutions to the inequality $y \leq 5x - 4$ ?
   a) $(6, 0)$  
   b) $(0, 4)$  
   c) $(1, 1)$  
   d) $(1, 3)$

2) The graph below shows which linear inequality?

Graph the following inequalities:

3) $y \leq -5x - 1$

4) $y > x - 5$

5) $x + 4y \geq 6$

6) $x < -2$
Day 2 Graphing Systems of Linear Inequalities

A system of inequalities just means two linear inequalities, and the solution to a system is simply an area on the graph that contains the solution areas of BOTH the individual inequalities meaning the overlapping shaded areas.

Example 1: Determine if the ordered pair (3, -3) is a solution to the system

\[ 3x + y > 5 \]
\[ 2x - y \geq 10 \]

Substitute:
\[ 3(3) + 3 > 5 \]
\[ 2(3) - (-3) \geq 10 \]

Is it a true statement? Is \(12 > 5\)? Is \(9 \geq 10\)?

Yes, it is a solution. No! It is not a solution!

Therefore, even though the ordered pair is a solution to the first inequality, it is not to the second inequality and is NOT a solution to the system!

Example 2: Graph the solution to this system of linear inequalities:

\[ y \geq 2x - 1 \]
\[ y < x + 1 \]

<table>
<thead>
<tr>
<th>Steps</th>
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<tbody>
<tr>
<td>1. Graph the first linear inequality using the steps from Day 1.</td>
<td></td>
</tr>
<tr>
<td>2. Graph the second linear inequality.</td>
<td></td>
</tr>
<tr>
<td>3. Shade the overlapping solution area</td>
<td></td>
</tr>
</tbody>
</table>
darker, or using a different color.  
4. Choose a test point in that area to check that it in fact contains the solutions to BOTH inequalities.  
Ex. I choose (0, 0) . Substitute the x and y values in both:
\[ y \geq 2x - 1 \]
Is \( 0 \geq 2(0) - 1 \)?
Is \( 0 \geq 0 - 1 \)?
Yes! \( 0 \geq -1 \) So (0, 0) is a Solution for the first inequality. Now check the test point in the second inequality:
\[ y < x + 1 \]
Is \( 0 < 0 + 1 \)?
Yes! \( 0 < 1 \) This is a statement so our test point is also a Solution to the 2\(^{nd}\) inequality. Since we showed that it is a solution to BOTH Inequalities, then we know the overlapping shaded area correctly represents the solution to this system.

Problems Day 2:

1) Is the ordered pair (7, 1) a solution to this system? Show your mathematical reasoning in making your decision (as in Example 1 above).

\[ 4x - y < 10 \quad -2x + 2y > -8 \]

2) Which of the following is the graph of the system of linear inequalities?
\[ y \geq 3x - 2 \quad y \leq -\frac{1}{2}x + 3 \]
Day 3   Real World Problems

The power of linear inequalities is in solving real world problems. When you are translating words into mathematical symbols, here are a few of the most commonly used phrases:

<table>
<thead>
<tr>
<th>Expression</th>
<th>Inequality</th>
<th>Example</th>
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<tbody>
<tr>
<td>No more than</td>
<td>≤</td>
<td>• You can be no more than 12 years old to trick or treat ( x \leq 12 ) if ( x ) = age</td>
</tr>
<tr>
<td>At most</td>
<td></td>
<td>• You can only be at most 12 to trick or treat</td>
</tr>
<tr>
<td>No less than</td>
<td>≥</td>
<td>• You can be no less than 18 to vote ( x \geq 18 ) if ( x ) = age</td>
</tr>
<tr>
<td>At least</td>
<td></td>
<td>• You must be at least 18 to vote</td>
</tr>
</tbody>
</table>

Now, let’s try answering a real world problem!

**Example 1:** Mr. Green wants to plant new trees in his yard this spring. He plans to spend no more than $720 on his project. The large trees cost $120 and the small trees cost $80. Can he plant 4 large trees and 5 small trees and stay within his budget?

**Part A:** Write an inequality to model this situation.

| Step 1: Identify your variables | Let \( x \) = the number of small trees  
|                                | \( y \) = the number of large trees   |
| Step 2: Represent information in the problem | \( 80x \) = amount spent on small trees (the cost per tree times the number of trees)  
|                                | \( 120y \) = amount spent on large trees |
| Step 3: Choose the Inequality   | He can spend **no more than**, so we will use \( \leq \)  |
Step 4: Write the inequality

\[ 80x + 120y \leq 720 \]

(the $ spent on small trees + the $ spent on large trees cannot be any more than $720)

Part B: Represent this inequality on a graph

Most word problem inequalities can most easily be graphed using the x- and y-intercept method because the easiest way to model them is in standard form \( Ax + By = C \)

- To find the x-intercept:
  Substitute 0 in for \( y \) in the equation:
  \[ 80x + 120(0) = 720 \]
  (In the context of the problem, the question this answers is: if he plants no large trees and plants all small trees, how many can he plant and stay in his budget?)

  Solve for \( x \):
  \[ 80x = 720 \]
  \[ x = 9 \]

- To find the y-intercept:
  Substitute 0 in for \( x \) in the equation:
  \[ 80(0) + 120y = 720 \]
  (In the context of the problem, the question this answers is: if he plants no large trees and plants all small trees, how many can he plant and stay in his budget?)

  Solve for \( y \):
  \[ 120y = 720 \]
  \[ y = 6 \]

- Plot the points representing the two ordered pairs (x an y intercepts)
  (9, 0) and (0, 6)

- Choose the boundary line (solid since the symbol is \( \leq \)) and the shade the appropriate solution area (below the line because he must stay under $720)
Part C: Answer the original question: Can he plant 4 large trees and 5 small trees and stay within his budget?

There are two methods to answer this:

**Method 1: Graphically**

Plot the point (5, 4) on the graph. Is it in the solution area? If so, then yes, he can plant that combination of trees. ***In this case, however, the point (5, 4) lies outside of the shaded area....you try using the graph above....So, NO, it would cost more than $720 to plant 4 large trees and 5 small trees, so he can't do it!!!

**Method 2: Algebraically**

Similar to the test point example in the box above, substitute 5 in for \( x \) and 4 in for \( y \) to see if the inequality statement is true with these values:

\[
80(5) + 120(4) \leq 720
\]

\[
400 + 480 \leq 720
\]

NO! \( 880 \leq 720 \) is NOT a true statement, so he could not plant that combination of trees...he would be over his budget of $720!

Part D: An additional question:

If he had his heart set on planting the 4 large trees, what is the greatest number of small trees that he could plant and stay within his budget?

This can be solved using EITHER method below:

**Method 1: Graphically:**

- Choose a test point to make sure you have shaded the correct area:

  Try (3, 2) which is in the shaded area and randomly chosen. This means: can he plant 3 small trees and 2 large trees and stay under his $720 budget limit?

  Is \( 80(3) + 120(2) \leq 720 \) ?

  Is \( 240 + 240 \leq 720 \) ?

  Yes! \( 480 \leq 720 \) is True, so the correct area is shaded to show the solution area.
Since 4 large is the y value, go up to 4 on the y axis, go over to the right until you hit the boundary line.....what is the x value at that point? 3 small trees...

Method 2: Algebraically:

If we know he wants to plant 4 large trees, substitute 4 in the inequality for the y value, and solve for x, which is the number of small trees:

\[
\begin{align*}
80x + 120(4) & \leq 720 \\
80x + 480 & \leq 720 \\
\underline{-480} & \underline{-480} \\
80x & \leq 240 \\
\frac{80x}{80} & \leq \frac{240}{80} \\
x & \leq 3
\end{align*}
\]

The largest integer (a tree cannot be a decimal) that makes this inequality true is 3, so the greatest number of small trees that he can plant with the 4 large trees is 3.

Problem Day 3:

The Junior Beta Club collected food and clothing to ship to an area of the country devastated by recent tornados. Each package of food costs $18 to ship while each package of clothing costs $14 to ship. They collected $126 to be used to pay for shipping costs. Up until now, they have received enough donations for 2 food packages and 5 clothing packages. Do they have enough money to send these packages?

Part A: If x = the number of food packages, and y= the number of clothing packages, which of the following inequalities correctly represents the situation?

a) 18x + 14y \leq 126  
 b) 14x + 18y \leq 126  
 c) 18x + 14y \geq 126  
 d) 14x + 18y \geq 126

Part B: Which graph represents this situation?
Part C: Using EITHER the graphing OR algebraic method described in the example, answer the question: **Does the club have enough money to send 2 food and 5 clothing packages?** Explain in detail how you reached your conclusion....

Part D: If the Red Cross stresses that food is more drastically needed, and the club receives more food donations (enough for 4 packages), what is the greatest number of clothing packages that they can ship with the 4 food packages and have enough money to do it? Explain your answer using mathematics (Use either method...graphing or algebraic)