El Segundo Middle School

Summer Math Packet

For Incoming 7th Graders

Complete the odd numbered problems and check your work using the answer key. Expect an assessment covering these concepts during the first month of school.

Core Connections, Course 1
Checkpoint Materials

A Note to Students (and their Teachers)

Students master different skills at different speeds. No two students learn exactly the same way at the same time. At some point you will be expected to perform certain skills accurately. Most of the Checkpoint problems incorporate skills that you should have been developing in grades 4 and 5. If you have not mastered these skills yet it does not mean that you will not be successful in this class. However, you may need to do some work outside of class to get caught up on them.

Starting in Chapter 1 and finishing in Chapter 9, there are 12 problems designed as Checkpoint problems. Each one is marked with an icon like the one above. After you complete each of the Checkpoint problems, check your answers by referring to this section. If your answers are incorrect, you may need some extra practice to develop that skill. The practice sets are keyed to each of the Checkpoint problems in the textbook. Each has the topic clearly labeled, followed by the answers to the corresponding Checkpoint problem and then some completed examples. Next, the complete solution to the Checkpoint problem from the text is given, and there are more problems for you to practice with answers included.

Remember, looking is not the same as doing! You will never become good at any sport just by watching it, and in the same way, reading through the worked examples and understanding the steps is not the same as being able to do the problems yourself. How many of the extra practice problems do you need to try? That is really up to you. Remember that your goal is to be able to do similar problems on your own confidently and accurately. This is your responsibility. You should not expect your teacher to spend time in class going over the solutions to the Checkpoint problem sets. If you are not confident after reading the examples and trying the problems, you should get help outside of class time or talk to your teacher about working with a tutor.

Checkpoint Topics

1. Using Place Value to Round and Compare Decimals
2. Addition and Subtraction of Decimals
3. Addition and Subtraction of Fractions
4. Addition and Subtraction of Mixed Numbers
5. Multiple Representations of Portions
6. Locating Points on a Number Line and on a Coordinate Graph

7A. Multiplication of Fractions and Decimals
7B. Area and Perimeter of Quadrilaterals and Triangles
8A. Rewriting and Evaluating Variable Expressions
8B. Division of Fractions and Decimals
9A. Displays of Data: Histograms and Box Plots
9B. Solving One-Step Equations

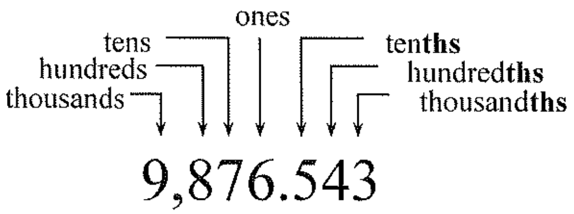
Checkpoint 1

Problem 1-93

Using Place Value to Round and Compare Decimals

Answers to Problem 1-93: a. 17.19, b. 0.230, c. 8.3, d. $>$, e. $>$, f. $<$

First a review of place value:



Example 1: Round 17.23579 to the nearest hundredth.

Solution: We start by identifying the digit in the hundredths place—the 3. The digit to the right of it is 5 or more so hundredths place is increased by one. 17.24

Example 2: Round 8.039 to the nearest tenth.

Solution: Identify the digit in the tenths place– the 0. The digit to the right of it is less than 5 so the tenths place remains the same. 8.0 (the zero must be included)

Example 3: Use the correct inequality sign ($<$, $>$) to compare 23.17 and 23.1089.

Solution: Identify the first place from the left where the digits are different–in this case, the hundredths. The number with the greater digit in this place is the greater number. $23.17 > 23.1089$

Now we can go back and solve the original problem.

a. 17.1936 (hundredths): 9 is the hundredths digit, $3 < 5$. The answer is 17.19.

b. 0.2302 (thousandths): 0 is thousandths digit, $2 < 5$. The answer is 0.230.

c. 8.256 (tenths): 2 is tenths digit, $5 \geq 5$. The answer is 8.3.

d. 47.2___47.197: The tenths place is the first different digit, $2 > 1$ so $47.2 > 47.197$.

e. 1.0032___1.00032: The thousandths place is the first different digit,
$3 > 0$ so $1.0032 > 1.00032$.

f. 0.0089___0.03: The hundredths place is the first different digit,
$0 < 3$ so $0.0089 < 0.03$.

Here are some more to try. For problems 1 through 10, round each number to the indicated place value. In problems 11 through 20, place the correct inequality sign in the blank.

1. 6.256 (tenths)
2. 0.7891 (thousandths)
3. 5.8000 (tenths)
4. 13.62 (tenths)
5. 27.9409 (thousandths)
6. 0.0029 (hundredths)
7. 9.126 (hundredths)
8. 0.6763 (tenths)
9. 33.333 (hundredths)
10. 0.425 (tenths)
11. 13.2___9.987
12. 6.52___74.52
13. 15.444___20.2
14. 12.17___8.8
15. 23.45___234.5
16. 32.168___28.1
17. 8976___0.8976
18. 45.987___48.21
19. 9.345___5.963
20. 7.891___7.812

Answers:

1. 6.3
2. 0.789
3. 5.8
4. 13.6
5. 27.941
6. 0.00
7. 9.13
8. 0.7
9. 33.33
10. 0.4
11. >
12. <
13. <
14. >
15. <
16. >
17. >
18. <
19. >
20. >

Checkpoint 2

Problem 2-90

Addition and Subtraction of Decimals

Answers to problem 2-90: a. 32.25, b. 8.825, c. 27.775, d. 89.097

To add or subtract decimals, write the problem in column form with the decimal points in a vertical column so that digits with the same place value are kept together. Include zeros so that all decimal parts of the number have the same number of digits. Add or subtract as with whole numbers. Place the decimal point in the answer aligned with those in the problem.

Example 1: Add: $37.68+5.2+125$

Solution:

$$\begin{array}{r} 37.68 \\ 5.2\mathbf{0} \\ +\underline{125.\mathbf{00}} \\ 167.88 \end{array}$$

Example 2: Subtract: $17-8.297$

Solution:

$$\begin{array}{r} 17.\mathbf{000} \\ \underline{-8.297} \\ 8.703 \end{array}$$

Now we can go back and solve the original problem.

a. $$\begin{array}{r} 2.95 \\ 18.30 \\ +\underline{11.00} \\ 32.25 \end{array}$$

b. $$\begin{array}{r} 9.200 \\ -\underline{0.375} \\ 8.825 \end{array}$$

c. $$\begin{array}{r} 0.275 \\ +\underline{27.500} \\ 27.775 \end{array}$$

d. $$\begin{array}{r} 90.000 \\ -\underline{0.903} \\ 89.097 \end{array}$$

Here are some more to try. Add or subtract the decimal numbers below.

1. $38.72+6.7$
2. $3.93+2.82$
3. $4.7+7.9$
4. $3.8-2.406$
5. $8.63-4.6$
6. $42.1083+14.73$
7. $0.647-0.39$
8. $58.3+79.84$
9. $2.037+0.09387$
10. $9.38-7.5$
11. $14-7.432$
12. $8.512-6.301$
13. $4.2-1.764$
14. $2.07-0.523$
15. $15+27.4+1.009$
16. $47.9+68.073$
17. $9.999+0.001$
18. $18-9.043$
19. $87.43-15.687-28.0363$
20. $347.68+28.00476+84.3$

Answers:

1. 45.42
2. 6.75
3. 12.6
4. 1.394
5. 4.03
6. 56.8383
7. 0.257
8. 138.14
9. 2.13087
10. 1.88
11. 6.568
12. 2.211
13. 2.436
14. 1.547
15. 43.409
16. 115.973
17. 10
18. 8.957
19. 43.7067
20. 459.98476

Checkpoint 3

Problem 3-132

Addition and Subtraction of Fractions

Answers to problem 3-132: a. $\frac{19}{20}$, b. $\frac{3}{8}$, c. $\frac{11}{9}=1\frac{2}{9}$, d. $\frac{7}{12}$

To add or subtract two fractions that are written with the same denominator, simply add or subtract the numerators and then simplify if possible. For example: $\frac{5}{9}+\frac{1}{9}=\frac{6}{9}=\frac{2}{3}$.

If the fractions have different denominators, a common denominator must be found. One way to find the lowest common denominator (or least common multiple) is to use a table as shown below.

The multiples of 3 and 5 are shown in the table at right. 15 is the least common multiple and a lowest common denominator for fractions with denominators of 3 and 5.

3	6	9	12	**15**	18
5	10	**15**	20	25	30

After a common denominator is found, rewrite the fractions with the same denominator (using the Giant One, for example).

Example 1: $\frac{1}{5}+\frac{2}{3}$

Solution: $\frac{1}{5}+\frac{2}{3}\Rightarrow\frac{1}{5}\cdot\boxed{\frac{3}{3}}+\frac{2}{3}\cdot\boxed{\frac{5}{5}}\Rightarrow\frac{3}{15}+\frac{10}{15}=\frac{13}{15}$

Example 2: $\frac{5}{6}-\frac{1}{4}$

Solution: $\frac{5}{6}-\frac{1}{4}\Rightarrow\frac{5}{6}\cdot\boxed{\frac{2}{2}}-\frac{1}{4}\cdot\boxed{\frac{3}{3}}\Rightarrow\frac{10}{12}-\frac{3}{12}=\frac{7}{12}$

Now we can go back and solve the original problem.

a. $\frac{3}{4}+\frac{1}{5}\Rightarrow\frac{3}{4}\cdot\frac{5}{5}+\frac{1}{5}\cdot\frac{4}{4}\Rightarrow\frac{15}{20}+\frac{4}{20}\Rightarrow\frac{19}{20}$

b. $\frac{5}{8}-\frac{1}{4}\Rightarrow\frac{5}{8}-\frac{1}{4}\cdot\frac{2}{2}\Rightarrow\frac{5}{8}-\frac{2}{8}\Rightarrow\frac{3}{8}$

c. $\frac{2}{3}+\frac{5}{9}\Rightarrow\frac{2}{3}\cdot\frac{3}{3}+\frac{5}{9}\Rightarrow\frac{6}{9}+\frac{5}{9}\Rightarrow\frac{11}{9}\Rightarrow1\frac{2}{9}$

d. $\frac{3}{4}-\frac{1}{6}\Rightarrow\frac{3}{4}\cdot\frac{3}{3}-\frac{1}{6}\cdot\frac{2}{2}\Rightarrow\frac{9}{12}-\frac{2}{12}\Rightarrow\frac{7}{12}$

Here are some more to try. Compute each sum or difference. Simplify if possible.

1. $\frac{3}{8}+\frac{3}{8}$
2. $\frac{7}{9}-\frac{1}{9}$
3. $\frac{1}{3}+\frac{3}{8}$
4. $\frac{3}{4}-\frac{1}{2}$
5. $\frac{5}{9}-\frac{1}{3}$
6. $\frac{1}{4}+\frac{2}{3}$
7. $\frac{17}{20}-\frac{4}{5}$
8. $\frac{1}{6}+\frac{1}{3}$
9. $\frac{6}{7}-\frac{3}{4}$
10. $\frac{14}{15}-\frac{1}{3}$
11. $\frac{3}{9}+\frac{3}{4}$
12. $\frac{3}{4}-\frac{2}{3}$
13. $\frac{7}{8}-\frac{5}{12}$
14. $\frac{3}{4}+\frac{9}{10}$
15. $\frac{12}{18}-\frac{2}{3}$
16. $\frac{3}{7}-\frac{1}{5}$
17. $\frac{4}{25}+\frac{3}{5}$
18. $\frac{4}{6}-\frac{11}{24}$
19. $\frac{5}{8}+\frac{3}{8}$
20. $\frac{7}{8}+\frac{7}{12}$

Answers:

1. $\frac{6}{8}=\frac{3}{4}$
2. $\frac{6}{9}=\frac{2}{3}$
3. $\frac{17}{24}$
4. $\frac{1}{4}$
5. $\frac{2}{9}$
6. $\frac{11}{12}$
7. $\frac{1}{20}$
8. $\frac{3}{6}=\frac{1}{2}$
9. $\frac{3}{28}$
10. $\frac{9}{15}=\frac{3}{5}$
11. $\frac{13}{12}=1\frac{1}{12}$
12. $\frac{1}{12}$
13. $\frac{11}{24}$
14. $\frac{33}{20}=1\frac{13}{20}$
15. 0
16. $\frac{8}{35}$
17. $\frac{19}{25}$
18. $\frac{5}{24}$
19. $\frac{8}{8}=1$
20. $\frac{35}{24}=1\frac{11}{24}$

Checkpoint 4

Problem 4-81

Addition and Subtraction of Mixed Numbers

Answers to problem 4-81: a. $10\frac{1}{6}$, b. $4\frac{1}{30}$, c. $5\frac{2}{15}$, d. $1\frac{1}{3}$

To add or subtract two mixed numbers, you can either add or subtract their parts, or you can change the mixed numbers into fractions greater than one.

Example 1: Compute the sum: $8\frac{3}{4}+4\frac{2}{5}$

Solution: This addition example shows adding the whole number parts and the fraction parts separately. The answer is adjusted because the fraction part is greater than one.

$$\begin{array}{rl} 8\frac{3}{4}=8+\frac{3}{4}\cdot\frac{5}{5} & = \quad 8\frac{15}{20} \\ +4\frac{2}{5}=4+\frac{2}{5}\cdot\frac{4}{4} & \underline{=+4\frac{8}{20}} \\ & 12\frac{23}{20}=13\frac{3}{20} \end{array}$$

Example 2: Compute the difference: $2\frac{1}{6}-1\frac{4}{5}$

Solution: This subtraction example shows changing the mixed numbers to fractions greater than one and then computing in the usual way.

$$\begin{array}{l} 2\frac{1}{6}-1\frac{4}{5}\Rightarrow\frac{13}{6}-\frac{9}{5} \\ \quad\Rightarrow\frac{13}{6}\cdot\frac{5}{5}-\frac{9}{5}\cdot\frac{6}{6} \\ \quad\Rightarrow\frac{65}{30}-\frac{54}{30}=\frac{11}{30} \end{array}$$

Now we can go back and solve the original problem.

a. (Using the method of Example 1.)

$$\begin{array}{rl} 5\frac{1}{2}=5+\frac{1}{2}\cdot\frac{3}{3}= & 5\frac{3}{6} \\ +\underline{4\frac{2}{3}}=4+\frac{2}{3}\cdot\frac{2}{2}= & +\underline{4\frac{4}{6}} \\ & 9\frac{7}{6}=10\frac{1}{6} \end{array}$$

b. (Using the method of Example 2.)

$$\begin{array}{l} 1\frac{5}{6}+2\frac{1}{5}\Rightarrow\frac{11}{6}+\frac{11}{5} \\ \quad\Rightarrow\frac{11}{6}\cdot\frac{5}{5}+\frac{11}{5}\cdot\frac{6}{6} \\ \quad\Rightarrow\frac{55}{30}+\frac{66}{30}=\frac{121}{30}=4\frac{1}{30} \end{array}$$

c. (Using the method of Example 1.)

$$\begin{array}{rl} 9\frac{1}{3}=9+\frac{1}{3}\cdot\frac{5}{5}= & 9\frac{5}{15} \\ -\underline{4\frac{1}{5}}=4+\frac{1}{5}\cdot\frac{3}{3}= & -\underline{4\frac{3}{15}} \\ & 5\frac{2}{15} \end{array}$$

d. (Using the method of Example 2.)

$$\begin{array}{l} 10-8\frac{2}{3}\Rightarrow\frac{10}{1}-\frac{26}{3} \\ \quad\Rightarrow\frac{10}{1}\cdot\frac{3}{3}-\frac{26}{3} \\ \quad\Rightarrow\frac{30}{3}-\frac{26}{3}=\frac{4}{3}=1\frac{1}{3} \end{array}$$

Here are some more to try. Compute each sum or difference. Simplify if possible.

1. $2\frac{1}{3}+3\frac{1}{4}$
2. $7\frac{1}{2}-2\frac{14}{15}$
3. $3\frac{6}{7}-1\frac{2}{3}$
4. $2\frac{3}{5}+5\frac{1}{4}$
5. $9\frac{5}{6}+1\frac{23}{30}$
6. $8\frac{3}{5}-\frac{8}{9}$
7. $6-1\frac{2}{3}$
8. $4\frac{1}{4}-3\frac{1}{3}$
9. $11\frac{1}{3}-2\frac{5}{6}$
10. $2\frac{7}{8}+\frac{23}{24}$
11. $5\frac{7}{12}+8$
12. $7\frac{3}{8}-6\frac{2}{5}$
13. $3\frac{4}{5}+5\frac{2}{3}$
14. $4\frac{3}{4}+1\frac{13}{14}$
15. $7\frac{1}{8}-7\frac{1}{12}$
16. $4\frac{3}{8}+3\frac{5}{24}$
17. $6\frac{1}{4}-3\frac{4}{5}$
18. $10\frac{1}{3}-6\frac{4}{7}$
19. $4\frac{4}{9}+3\frac{5}{6}$
20. $3\frac{13}{20}-2\frac{27}{40}$

Answers:

1. $\frac{67}{12}=5\frac{7}{12}$
2. $\frac{137}{30}=4\frac{17}{30}$
3. $\frac{46}{21}=2\frac{4}{21}$
4. $\frac{157}{20}=7\frac{17}{20}$
5. $\frac{348}{30}=11\frac{18}{30}=11\frac{3}{5}$
6. $\frac{347}{45}=7\frac{32}{45}$
7. $\frac{13}{3}=4\frac{1}{3}$
8. $\frac{11}{12}$
9. $\frac{51}{6}=8\frac{3}{6}=8\frac{1}{2}$
10. $\frac{92}{24}=3\frac{20}{24}=3\frac{5}{6}$
11. $\frac{163}{12}=13\frac{7}{12}$
12. $\frac{39}{40}$
13. $\frac{142}{15}=9\frac{7}{15}$
14. $\frac{187}{28}=6\frac{19}{28}$
15. $\frac{1}{24}$
16. $\frac{182}{24}=7\frac{14}{24}=7\frac{7}{12}$
17. $\frac{49}{20}=2\frac{9}{20}$
18. $\frac{79}{21}=3\frac{16}{21}$
19. $\frac{149}{18}=8\frac{5}{18}$
20. $\frac{39}{40}$

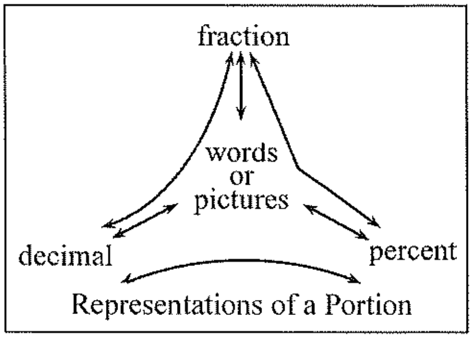
Checkpoint 5

Problem 5-103

Multiple Representations of Portions

Answers to problem 5-103: a. $23\%, \frac{23}{100}$; b. $\frac{7}{10}, 0.7, 70\%$; c. $\frac{19}{100}, 0.19$; d. $68\%, 0.68$

Portions of a whole may be represented in various ways as represented by this web. Percent means "per hundred" and the place value of a decimal will determine its name. Change a fraction in an equivalent fraction with 100 parts to name it as a percent.



Representations of a Portion

Example 1: Write the given portion as a fraction and as a percent.

0.3

Solution: The digit 3 is in the tenths place so $0.3 = \textit{three-tenths} = \frac{3}{10}$. On a diagram or a hundreds grid, 3 parts out of 10 is equivalent to 30 parts out of 100 so $\frac{3}{10} = \frac{30}{100} = 30\%$.

Example 2: Write the given portion as a fraction and as a decimal.

35%

Solution: $35\% = \frac{35}{100} = 0.35$

Now we can go back and solve the original problem.

a. 0.23 is *twenty-three-hundredths* or $\frac{23}{100} = 23\%$.

b. seven-tenths is $\frac{7}{10} = \frac{7}{10} \cdot \frac{10}{10} = \frac{70}{100} = 70\%$

c. $19\% = \frac{19}{100} = 0.19$

d. $\frac{17}{25} = \frac{17}{25} \cdot \frac{4}{4} = \frac{68}{100} = 0.68 = 68\%$

Here are some more to try. For each portion of a whole, write it as a percent, fraction, and a decimal.

1. 7%
2. 0.33
3. $\frac{3}{4}$
4. $\frac{1}{5}$
5. 0.15
6. 14%
7. $\frac{11}{25}$
8. 43%
9. $\frac{3}{5}$
10. 0.05
11. 99%
12. 37%
13. $\frac{3}{10}$
14. 0.66
15. $\frac{13}{20}$
16. 26%
17. 0.52
18. 1.0
19. 51%
20. $\frac{78}{100}$

Answers:

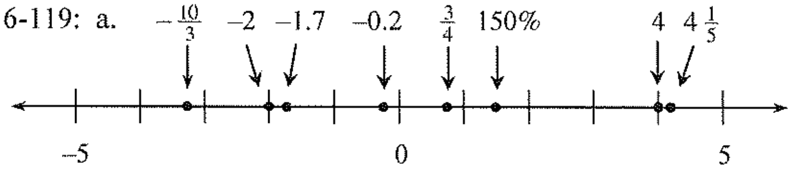
1. $\frac{7}{100}$, 0.07
2. 33%, $\frac{33}{100}$
3. 75%, 0.75
4. 20%, 0.2
5. 15%, $\frac{15}{100}=\frac{3}{20}$
6. $\frac{14}{100}=\frac{7}{50}$, 0.14
7. 44%, 0.44
8. $\frac{43}{100}$, 0.43
9. 60%, 0.6
10. 5%, $\frac{5}{100}=\frac{1}{20}$
11. $\frac{99}{100}$, 0.99
12. $\frac{37}{100}$, 0.37
13. 30%, 0.3
14. 66%, $\frac{66}{100}=\frac{33}{50}$
15. 65%, 0.65
16. $\frac{26}{100}=\frac{13}{50}$, 0.26
17. 52%, $\frac{52}{100}=\frac{13}{25}$
18. 100%, $\frac{100}{100}=\frac{1}{1}$
19. $\frac{51}{100}$, 0.51
20. 78%, 0.78

Checkpoint 6

Problem 6-119

Locating Points on a Number Line and on a Coordinate Graph

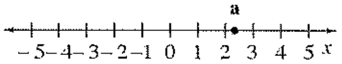
Answers to problem 6-119: a.



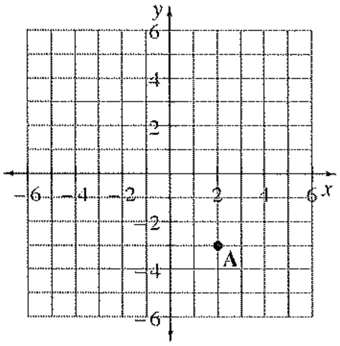
b. A = (0, 6), B = (2, 2), C = (1, –4), D = (–1, –6), E = (–6, 0), F = (–5, 3)

Points on a number line represent the locations of numbers. Numbers to the right of 0 are positive; to the left of 0, they are negative. For vertical lines, normally the top is positive.

Point **a** at right approximates the location of $2\frac{1}{3}$.



Two perpendicular intersecting number lines (or axes) such as the ones at right create a coordinate system for locating points on a graph. Points are located using a pair of numbers (x, y), or coordinates, where x represents the horizontal direction and y represent the vertical direction. In this case "A" represents the point $(2, -3)$.



Now we can go back and solve the original problem.

a. –2 is two units left of 0 and 4 is four units right of 0. –1.7 is larger than –2 so it is slightly to the right of –2. $\frac{3}{4}$ is halfway between $\frac{1}{2}$ and 1. –0.2 is slightly smaller than 0 so it is slightly to the left of that number. $-\frac{10}{3} = -3\frac{1}{3}$ so it is $\frac{1}{3}$ of the way from –3 to –4. $4\frac{1}{5}$ is $\frac{1}{5}$ of the way from 4 to 5. 150% = 1.5 so it is halfway between 1 and 2. See the number line graph above.

b. A is at the intersection of 0 on the x-axis and 6 of the y-axis. Its coordinates are $(0, 6)$.

B is at the intersection of 2 on the x-axis and 2 of the y-axis. Its coordinates are $(2, 2)$.

C is at the intersection of 1 on the x-axis and –4 of the y-axis. Its coordinates are $(1, -4)$.

D is at the intersection of –1 on the x-axis and –6 of the y-axis. Its coordinates are $(-1, -6)$.

E is at the intersection of –6 on the x-axis and 0 of the y-axis. Its coordinates are $(-6, 0)$.

F is at the intersection of –5 on the x-axis and 3 of the y-axis. Its coordinates are $(-5, 3)$.

Here are some more to try. Indicate the approximate location of each set of numbers on a number line.

1. $2, 3, 2\frac{1}{3}, 2\frac{9}{10}$

2. 12, 12.6, 14, 13.3

3. 0, 20%, 67%, 100%

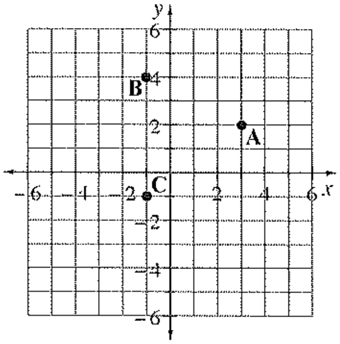
4. $1, -2, -\frac{1}{2}, \frac{3}{4}$

5. −1, −3, −1.3, −3.3

6. $7, 8, 7\frac{1}{8}, 7.8$

In problem 7, write the coordinates (x, y) of each point. In problem 8, draw a set of axes and graph the given points.

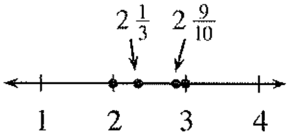
7.



8. A = (1, 4)
B = (−2, 1)
C = (4, −1)

Answers:

1.



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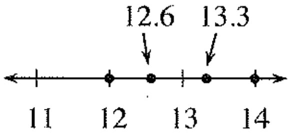

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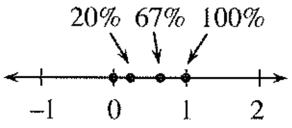

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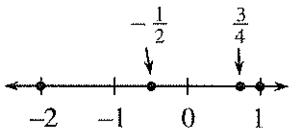
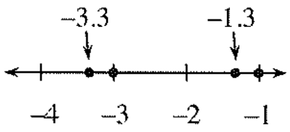


5.



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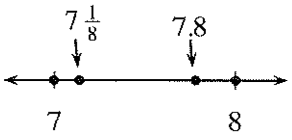
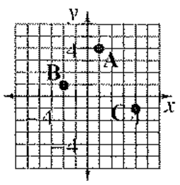


7. A = (3, 2)
B = (−1, 4)
C = (−1, −1)

8.



Checkpoint 7A

Problem 7-67

Multiplication of Fractions and Decimals

Answers to problem 7-67: a. $\frac{4}{15}$, b. $\frac{1}{5}$, c. $5\frac{5}{6}$, d. $2\frac{8}{9}$, e. 12.195, f. 0.000245

To multiply fractions, multiply the numerators and then multiply the denominators. To multiply mixed numbers, change each mixed number to a fraction greater than one before multiplying. In both cases, simplify by looking for factors than make "one."

To multiply decimals, multiply as with whole numbers. In the product, the number of decimal places is equal to the total number of decimal places in the multiplied numbers. Sometimes zeros need to be added to place the decimal point.

Example 1: Multiply $\frac{3}{8}\cdot\frac{4}{5}$

Solution:

$$\frac{3}{8}\cdot\frac{4}{5}\Rightarrow\frac{3\cdot4}{8\cdot5}\Rightarrow\frac{3\cdot\cancel{4}}{2\cdot\cancel{4}\cdot5}\Rightarrow\frac{3}{10}$$

Example 2: Multiply $3\frac{1}{3}\cdot2\frac{1}{2}$

Solution:

$$3\tfrac{1}{3}\cdot2\tfrac{1}{2}\Rightarrow\frac{10}{3}\cdot\frac{5}{2}\Rightarrow\frac{10\cdot5}{3\cdot2}\Rightarrow\frac{5\cdot\cancel{2}\cdot5}{3\cdot\cancel{2}}\Rightarrow\frac{25}{3}\text{ or }8\tfrac{1}{3}$$

Note that we are simplifying using Giant Ones but no longer drawing the Giant One.

Example 3: Multiply $12.5\cdot0.36$

Solution:

$$\begin{array}{rl} 12.5 & \text{(one decimal place)} \\ \times\underline{0.36} & \text{(two decimal places)} \\ 750 & \\ \underline{3750} & \\ 4.500 & \text{(three decimal places)} \end{array}$$

Now we can go back and solve the original problem.

a. $\frac{2}{3}\cdot\frac{2}{5}\Rightarrow\frac{2\cdot2}{3\cdot5}\Rightarrow\frac{4}{15}$

b. $\frac{7}{10}\cdot\frac{2}{7}\Rightarrow\frac{\cancel{7}\cdot\cancel{2}}{5\cdot\cancel{2}\cdot\cancel{7}}\Rightarrow\frac{1}{5}$

c. $2\frac{1}{3}\cdot2\frac{1}{2}\Rightarrow\frac{7}{3}\cdot\frac{5}{2}\Rightarrow\frac{7\cdot5}{3\cdot2}\Rightarrow\frac{35}{6}$ or $5\frac{5}{6}$

d. $1\frac{1}{3}\cdot2\frac{1}{6}\Rightarrow\frac{4}{3}\cdot\frac{13}{6}\Rightarrow\frac{2\cdot\cancel{2}\cdot13}{3\cdot\cancel{2}\cdot3}\Rightarrow\frac{26}{9}$ or $2\frac{8}{9}$

e.

$$\begin{array}{r} 2.71 \\ \times\underline{4.5} \\ 1355 \\ \underline{10840} \\ 12.195 \end{array}$$

f.

$$\begin{array}{r} 0.35 \\ \times\underline{0.0007} \\ 0.000245 \end{array}$$

Here are some more to try. Multiply the fractions and decimals below.

1. $0.08 \cdot 4.7$
2. $0.21 \cdot 3.42$
3. $\frac{4}{7} \cdot \frac{1}{2}$
4. $\frac{5}{6} \cdot \frac{3}{8}$
5. $\frac{8}{9} \cdot \frac{3}{4}$
6. $\frac{7}{10} \cdot \frac{3}{4}$
7. $3.07 \cdot 5.4$
8. $6.57 \cdot 2.8$
9. $\frac{5}{6} \cdot \frac{3}{20}$
10. $2.9 \cdot 0.056$
11. $\frac{6}{7} \cdot \frac{4}{9}$
12. $3\frac{1}{7} \cdot 1\frac{2}{5}$
13. $\frac{2}{3} \cdot \frac{5}{9}$
14. $\frac{3}{5} \cdot \frac{9}{13}$
15. $2.34 \cdot 2.7$
16. $2\frac{1}{3} \cdot 4\frac{4}{5}$
17. $4\frac{3}{5} \cdot \frac{1}{2}$
18. $\frac{3}{8} \cdot \frac{5}{9}$
19. $0.235 \cdot 0.43$
20. $421 \cdot 0.00005$

Answers:

1. 0.376
2. 0.7182
3. $\frac{2}{7}$
4. $\frac{5}{16}$
5. $\frac{2}{3}$
6. $\frac{21}{40}$
7. 16.578
8. 18.396
9. $\frac{1}{8}$
10. 0.1624
11. $\frac{8}{21}$
12. $4\frac{2}{5}$
13. $\frac{10}{27}$
14. $\frac{27}{65}$
15. 6.318
16. $11\frac{1}{5}$
17. $2\frac{3}{10}$
18. $\frac{5}{24}$
19. 0.10105
20. 0.02105

Checkpoint 7B

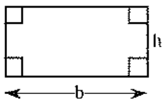
Problem 7-109

Area and Perimeter of Quadrilaterals and Triangles

Answers to problem 7-109: a. 40 cm^2, 26 cm; b. 88.5 in.^{2}, 51 in.; c. 459 cm^2, 90 cm;
d. 132 m^2, 51m

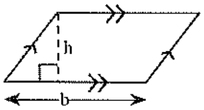
Area is the number of square units in a flat region. The formulas to calculate the areas of several kinds of quadrilaterals or triangles are:

RECTANGLE



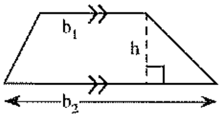
$A = bh$

PARALLELOGRAM



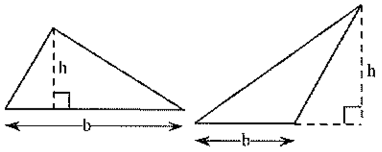
$A = bh$

TRAPEZOID



$A = \frac{1}{2}(b_1 + b_2)h$

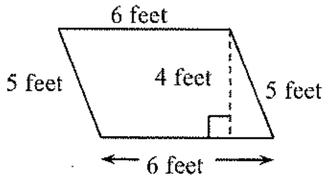
TRIANGLE



$A = \frac{1}{2}bh$

Perimeter is the number of units needed to surround a region. To calculate the perimeter of a quadrilateral or triangle, add the lengths of the sides.

Example 1:
Compute the area and perimeter.

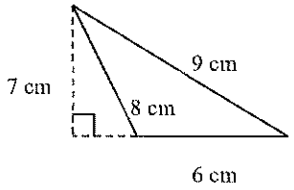


parallelogram

$A = bh = 6 \cdot 4 = 24$ feet^2

$P = 6 + 6 + 5 + 5 = 22$ feet

Example 2:
Compute the area and perimeter.



triangle

$A = \frac{1}{2}bh = \frac{1}{2} \cdot 6 \cdot 7 = 21$ cm^2

$P = 6 + 8 + 9 = 23$ cm

Now we can go back and solve the original problem.

a. rectangle: $A = bh = 8 \cdot 5 = 40$ cm^2; $P = 8 + 8 + 5 + 5 = 26$ cm

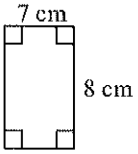
b. triangle: $A = \frac{1}{2}bh = \frac{1}{2} \cdot 23 \cdot 7.7 = 88.5$ in.^{2}; $P = 11 + 17 + 23 = 51$ in.

c. parallelogram: $A = bh = 27 \cdot 17 = 459$ cm^2; $P = 18 + 18 + 27 + 27 = 90$ cm

d. trapezoid: $A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(10 + 20) \cdot 8.8 = 132$ m^2; $P = 9 + 10 + 12 + 20 = 51$ m

Here are some more to try. Find the area and perimeter of each figure.

1.



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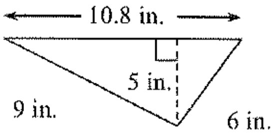

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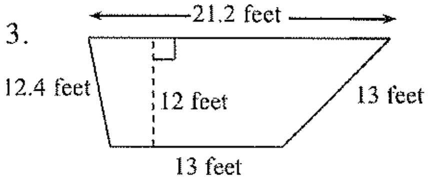

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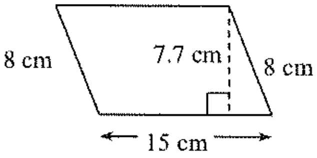

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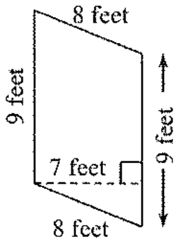

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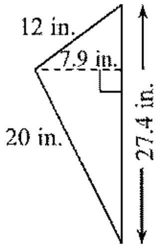

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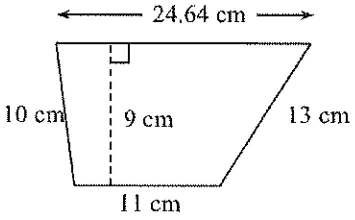

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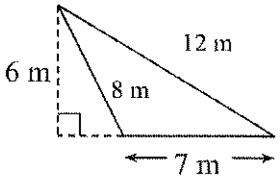

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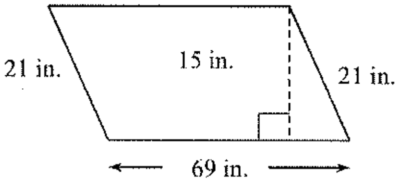

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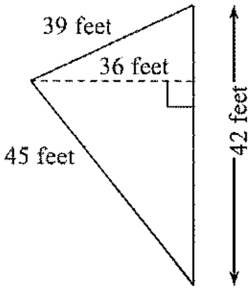

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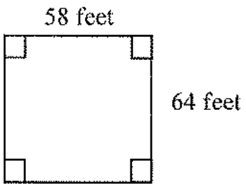

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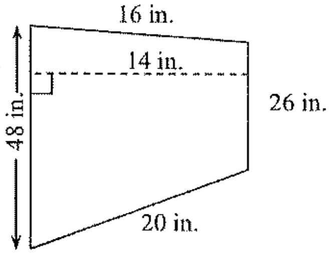

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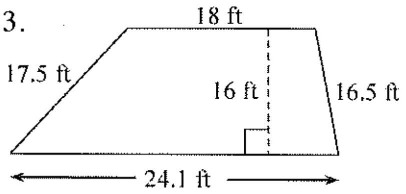

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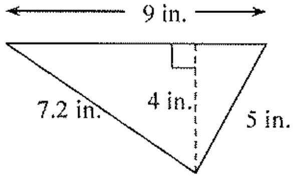

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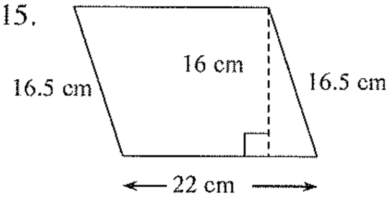

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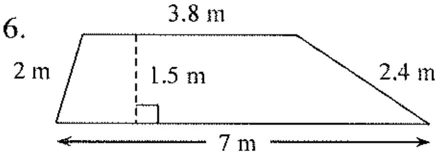


Answers:

1. Area = 56 cm^2
Perimeter = 30 cm

2. Area = 27 in.^{2}
Perimeter = 25.8 in.

3. Area = 205.2 feet^2
Perimeter = 59.6 feet

4. Area = 115.5 cm^2
Perimeter = 46 cm

5. Area = 63 feet^2
Perimeter = 34 feet

6. Area = 108.23 in.^{2}
Perimeter = 59.4 in.

7. Area = 106.38 cm^2
Perimeter = 58.64 cm

8. Area = 21 m^2
Perimeter = 27 m

9. Area = 1035 in.^{2}
Perimeter = 180 in.

10. Area = 756 feet^2
Perimeter = 126 feet

11. Area = 3712 feet^2
Perimeter = 244 feet

12. Area = 518 in.^{2}
Perimeter = 110 in.

13. Area = 336.8 feet^2
Perimeter = 76.1 feet

14. Area = 18 in.^{2}
Perimeter = 21.2 in.

15. Area = 352 cm^2
Perimeter = 77 cm

16. Area = 8.1 m^2
Perimeter = 15.2 m

Checkpoint 8A

Problem 8-60

Rewriting and Evaluating Variable Expressions

Answers to problem 8-60: a. $6+3x$; b. $5x+4y$; c. $5(x+2)$; d. $6(4+3y)$; e. 54; f. 17

Expressions may be rewritten by using the Distributive Property: $a(b+c)=a\cdot b+a\cdot c$. This equation demonstrates how expressions with parenthesis may be rewritten without parenthesis. Often this is called multiplying. If there is a common factor, expressions without parenthesis may be rewritten with parenthesis. This is often called factoring.

To evaluate a variable expression for particular values of the variables, replace the variables in the expression with their known numerical values (this process is called substitution) and simplify using the rules for order of operations.

Example 1: Multiply and then simplify $3(2x+y)-x$.

Solution: First rewrite using the Distributive Property and then combine like terms.

$3(2x+y)-x$

$3\cdot 2x+3\cdot y-x$

$6x+3y-x$

$5x+3y$

Example 2: Factor $12x+8$.

Solution: First, look for the greatest common factor in each term. Rewrite each term using that greatest common factor and then use the Distributive Property.

$12x+8$

$4\cdot 3x+4\cdot 2$

$4(3x+2)$

Example 3: Evaluate $3x^2+5x$ for $x=7$.

Solution: $3x^2+5x$

$3\cdot 7^2+5\cdot 7$

$3\cdot 7\cdot 7+5\cdot 7$

$147+35$

182

Now we can go back and solve the original problem.

a. $3(2+x)$

$3\cdot 2+3\cdot x$

$6+3x$

b. $2(x+2y)+3x$

$2x+4y+3x$

$5x+4y$

c. $5x+10$

$5\cdot x+5\cdot 2$

$5(x+2)$

d. $24+18y$

$6\cdot 4+6\cdot 3y$

$6(4+3y)$

e. $6y^2$

$6\cdot 3^2$

$6\cdot 3\cdot 3$

54

f. $4x+5y$

$4\cdot \frac{1}{2}+5\cdot 3$

$2+15$

17

Here are some more to try. In problems 1 through 16 rewrite each expression and in problems 17 through 24 evaluate each expression using the given value(s) for the variables.

1. $3(3x-4)$
2. $2(x+y)+y$
3. $5(b-4)$
4. $3(x+y)$
5. $4x+8$
6. $5m+10n$
7. $12y+16x$
8. $7x+21$
9. $2(y+4)+3(x+2)$
10. $4(8+x)+3(y-5)$
11. $12-4y+2x$
12. $6y+36x$
13. $5(x+y)$
14. $42+7x+14y$
15. $15x+3y+9$
16. $3(x-4)+2(y+7)$
17. $3x-5$ if $x=4$
18. $4(y-2)$ if $y=8$
19. $3x-5y$ if $x=4, y=2$
20. $5(x-y)$ if $x=7, y=2$
21. $3x^2+2x$ if $x=5$
22. $3y(y+2)$ if $y=4$
23. $2(x+y)+\frac{y+2}{x}$ if $y=4, x=2$
24. $2(x+12+y)-(\frac{2}{3}\cdot\frac{y}{x})$ if $x=2, y=3$

Answers:

1. $9x-12$
2. $2x+3y$
3. $5b-20$
4. $3x+3y$
5. $4(x+2)$
6. $5(m+2n)$
7. $4(3y+4x)$
8. $7(x+3)$
9. $2y+3x+14$
10. $4x+3y+17$
11. $2(6-2y+x)$
12. $6(y+6x)$
13. $5x+5y$
14. $7(6+x+2y)$
15. $3(5x+y+3)$
16. $3x+2y+2$
17. 7
18. 24
19. 2
20. 25
21. 85
22. 72
23. 15
24. 33

Checkpoint 8B

Problem 8-120

Division of Fractions and Decimals

Answers to problem 8-120: a. $\frac{3}{4}$, b. $\frac{1}{12}$, c. 9, d. $\frac{7}{10}$, e. 22.85, f. 780

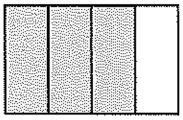
Division of fractions can be shown using an area model or a Giant One. Division using the invert and multiply method is based on a Giant One.

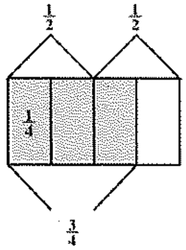
To divide decimals, change the divisor to a whole number by multiplying by a power of 10. Multiply the dividend by the same power of 10 and place the decimal directly above in the answer. Divide as you would with whole numbers. Sometimes extra zeros may be necessary for the number being divided.

Example 1: Use an area model to find $\frac{3}{4}\div\frac{1}{2}$.

Solution: $\frac{3}{4}\div\frac{1}{2}$ means, in $\frac{3}{4}$, how many $\frac{1}{2}$s are there?

Start with $\frac{3}{4}$.





In $\frac{3}{4}$ there is one full $\frac{1}{2}$ shaded and half of another one (that is half of one half).

So $\frac{3}{4}\div\frac{1}{2}=1\frac{1}{2}$
(one and one half halves).

Example 2: Use a Giant One to find $1\frac{1}{3}\div 1\frac{1}{2}$.

Solution: Write the division problem as a fraction and then use a Giant One to change the denominator into "one."

$$1\tfrac{1}{3}\div 1\tfrac{1}{2}\Rightarrow\frac{1\frac{1}{3}}{1\frac{1}{2}}\Rightarrow\frac{\frac{4}{3}}{\frac{3}{2}}\cdot\boxed{\frac{\frac{2}{3}}{\frac{2}{3}}}\Rightarrow\frac{\frac{8}{9}}{1}\Rightarrow\frac{8}{9}$$

Note that this method leads to the invert and multiply method: $\frac{4}{3}\div\frac{3}{2}\Rightarrow\frac{4}{3}\cdot\frac{2}{3}\Rightarrow\frac{8}{9}$.

Example 3: Find $53.6\div 0.004$.

Solution: Multiply both numbers by 1000 (move the decimal 3 places) to change the divisor into a whole number. Place the new decimal location from the dividend directly above in the answer and then divide.

$$0.004\overline{)53.6}\Rightarrow 4\overset{.}{\overline{)53600.}}\Rightarrow 4\overset{13400.}{\overline{)53600.}}$$

Now we can go back and solve the original problem.

a. $\frac{3}{8} \div \frac{1}{2} \Rightarrow \frac{3}{8} \cdot \frac{2}{1} \Rightarrow \frac{3 \cdot \cancel{2}}{4 \cdot \cancel{2}} \Rightarrow \frac{3}{4}$

b. $\frac{1}{3} \div 4 \Rightarrow \frac{1}{3} \cdot \frac{1}{4} \Rightarrow \frac{1}{12}$

c. $1\frac{1}{2} \div \frac{1}{6} \Rightarrow \frac{3}{2} \cdot \frac{6}{1} \Rightarrow \frac{3 \cdot 3 \cdot \cancel{2}}{\cancel{2} \cdot 1} = \frac{9}{1} \Rightarrow 9$

d. $\frac{7}{8} \div 1\frac{1}{4} \Rightarrow \frac{7}{8} \div \frac{5}{4} \Rightarrow \frac{7}{8} \cdot \frac{4}{5} \Rightarrow \frac{7 \cdot \cancel{4}}{2 \cdot \cancel{4} \cdot 5} \Rightarrow \frac{7}{10}$

e. $1.2\overline{)27.42} \Rightarrow 12.\overline{)274.2} \Rightarrow \begin{array}{r} 22.85 \\ 12\overline{)274.20} \\ \underline{24} \\ 34 \\ \underline{24} \\ 10\,2 \\ \underline{9\,6} \\ 60 \\ \underline{60} \\ 0 \end{array}$

f. $0.025\overline{)19.5} \Rightarrow 25.\overline{)19500.} \Rightarrow \begin{array}{r} 780. \\ 25\overline{)19500.} \\ \underline{175} \\ 200 \\ \underline{200} \\ 0 \end{array}$

Here are some more to try. Divide these fractions and decimals.

1. $\frac{2}{3} \div \frac{1}{2}$
2. $\frac{5}{6} \div \frac{3}{4}$
3. $14.3 \div 8$
4. $\frac{4}{7} \div \frac{3}{5}$
5. $100.32 \div 24$
6. $1.32 \div 0.032$
7. $1\frac{1}{3} \div \frac{1}{6}$
8. $\frac{4}{5} \div \frac{1}{8}$
9. $25.46 \div 5.05$
10. $2\frac{2}{5} \div 1\frac{7}{9}$
11. $\frac{3}{7} \div \frac{1}{4}$
12. $\frac{9}{20} \div \frac{5}{7}$
13. $\frac{7}{11} \div \frac{3}{4}$
14. $306.4 \div 3.2$
15. $3.24 \div 1.5$
16. $207.3 \div 4.4$
17. $\frac{2}{3} \div \frac{1}{5}$
18. $7\frac{1}{3} \div 3\frac{1}{9}$
19. $53.7 \div 0.023$
20. $\frac{8}{9} \div 3\frac{1}{3}$

Answers:

1. $1\frac{1}{3}$
2. $1\frac{1}{9}$
3. 1.7875
4. $\frac{20}{21}$
5. 4.18
6. 41.25
7. 8
8. $6\frac{2}{5}$
9. ≈ 5.04
10. $1\frac{7}{20}$
11. $1\frac{5}{7}$
12. $\frac{63}{100}$
13. $\frac{28}{33}$
14. 95.75
15. 2.16
16. ≈ 47.11
17. $3\frac{1}{3}$
18. $2\frac{5}{14}$
19. ≈ 2334.78
20. $\frac{4}{15}$

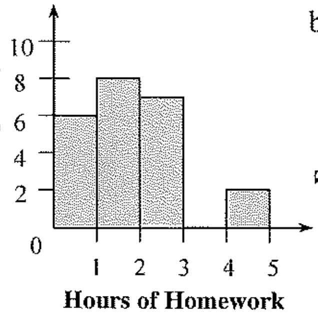
Checkpoint 9A

Problem 9-39

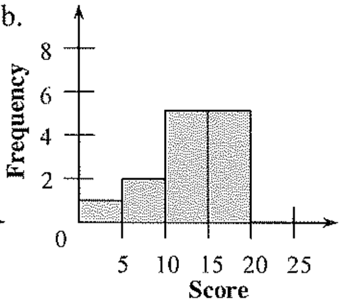
Displays of Data: Histograms and Box Plots

Answers to problem 9-39:

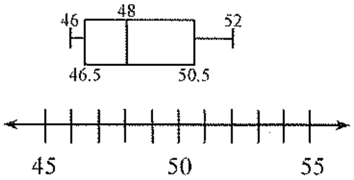
a.



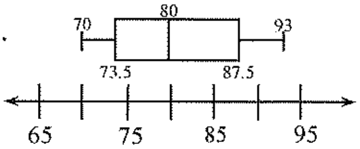
b.



c.



d.



c: IQR = 4 d: IQR = 14

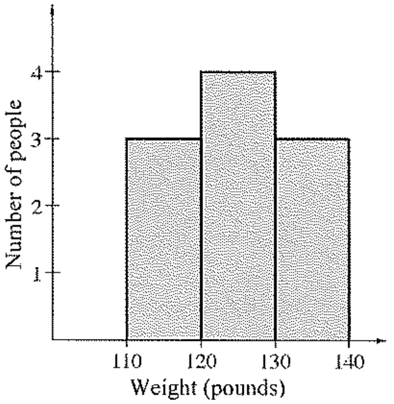
Histograms

A histogram is a method of showing data. It uses a bar to show the frequency (the number of times something occurs). The frequency measures something that changes numerically. (In a bar graph the frequency measures something that changes by category.) The intervals (called bins) for the data are shown on the horizontal axis and the frequency is represented by the height of a rectangle above the interval. The labels on the horizontal axis represent the lower end of each interval or bin.

Example: Sam and her friends weighed themselves and here is their weight in pounds: 110, 120, 131, 112, 125, 135, 118, 127, 135, and 125. Make a histogram to display the information. Use intervals of 10 pounds.

Solution:

See histogram at right. Note that the person weighing 120 pounds is counted in the next higher bin.

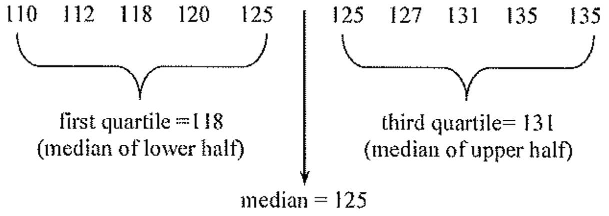


Box Plots

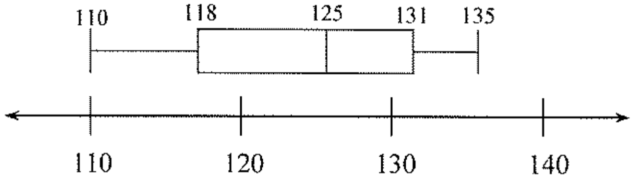
A box plot displays a summary of data using the median, quartiles, and extremes of the data. The box contains the "middle half" of the data. The right segment represents the top 25% of the data and the left segment represent the bottom 25% of the data.

Example: Create a box plot for the set of data given in the previous example.

Solution: Place the data in order to find the median (middle number) and the quartiles (middle numbers of the upper half and the lower half.)



Based on the extremes, first quartile, third quartile, and median, the box plot is drawn.



The interquartile range IQR = 131–118 = 13.

Now we can go back to the original problem.

a. The 0–1 bin contains the six students who do less than one hour of homework. The 1–2 bin contains the 10 students who do at least one hour but less than two hours. The 2–3 bin contains the seven students who do at least two hours but less than three hours. There are no students who do at least three hours and less than four. Two students did four hours and less than five. See the histogram above.

b. The 0–5 bin contains two scores less than 5 points. The 5–10 bin contains the two scores of a least five but less than 10. The 10–15 bin contains the eight scores at least 10 but less than 15. The 15–20 bin contains the seven scores at least 15 but less than 20. See the histogram above.

c. Place the ages in order: 46, 46, 47, 47, 48, 49, 50, 51, 52.
The median is the middle age: 48. The first quartile is the median of the lower half of the ages. Since there are four lower-half ages, the median is the average of the middle two: $\frac{46+47}{2} = 46.5$. The third quartile is the median of the upper half ages. Again, there are four upper-half ages, so average the two middle ages: $\frac{50+51}{2} = 50.5$. The interquartile range is the difference between the third quartile and the first quartile: 50.5–46.5 = 4. See the box plot above.

d. Place the scores in order: 70, 72, 75, 76, 80, 82, 85, 90, 93.
The median is the middle score: 80. The lower quartile is the median of the lower half of the scores. Since there are four lower-half scores, the median is the average of the middle two: $\frac{72+75}{2} = 73.5$. The third quartile is the median of the upper half of the scores. Again, there are four upper-half scores, so average the two middle ages: $\frac{85+90}{2} = 87.5$. The interquartile range is the difference between the third quartile and the first quartile: 87.5–73.5 = 14. See the box plot above.

Here are some more to try. For problems 1 through 6, create a histogram. For problems 7 through 12, create a box plot. State the quartiles and the interquartile range.

1. Number of heads showing in 20 tosses of three coins:
   2, 2, 1, 3, 1, 0, 2, 1, 2, 1, 1, 2, 0, 1, 3, 2, 1, 3, 1, 2

2. Number of even numbers in 5 rolls of a dice done 14 times:
   4, 2, 2, 3, 1, 2, 1, 1, 3, 3, 2, 2, 4, 5

3. Number of fish caught by 7 fishermen:
   2, 3, 0, 3, 3, 1, 5

4. Number of girls in grades K-8 at local schools:
   12, 13, 15, 10, 11, 12, 15, 11, 12

5. Number of birthdays in each March in various 2nd grade classes:
   5, 1, 0, 0, 2, 4, 4, 1, 3, 1, 0, 4

6. Laps jogged by 15 students:
   10, 15, 10, 13, 20, 14, 17, 10, 15, 20, 8, 7, 13, 15, 12

7. Number of days of rain:
   6, 8, 10, 9, 7, 7, 11, 12, 6, 12, 14, 10

8. Number of times a frog croaked per minute:
   38, 23, 40, 12, 35, 27, 51, 26, 24, 14, 38, 41, 23, 17

9. Speed in mph of 15 different cars:
   30, 35, 40, 23, 33, 32, 28, 37, 30, 31, 29, 33, 39, 22, 30

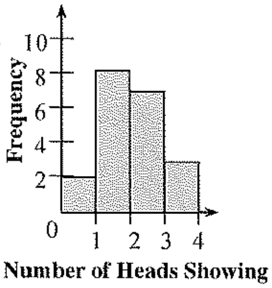
10. Typing speed of 12 students in words per minute:
    28, 30, 60, 26, 47, 53, 39, 42, 48, 27, 23, 86

11. Number of face cards pulled when 13 cards are drawn 15 times:
    1, 4, 2, 1, 1, 0, 0, 2, 1, 3, 3, 0, 0, 2, 1

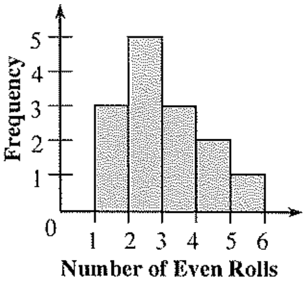
12. Height of 15 students in inches:
    48, 55, 56, 65, 67, 60, 60, 57, 50, 59, 62, 65, 58, 70, 68

Answers:

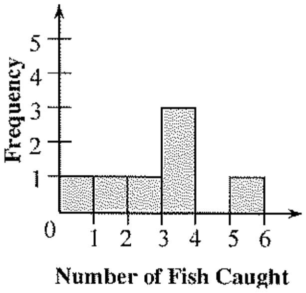
1.



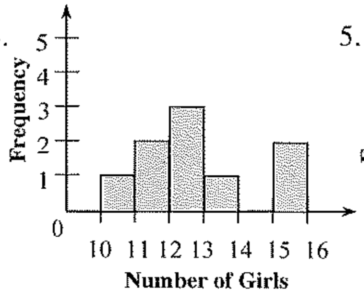
2.



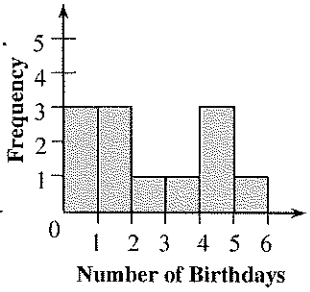
3.



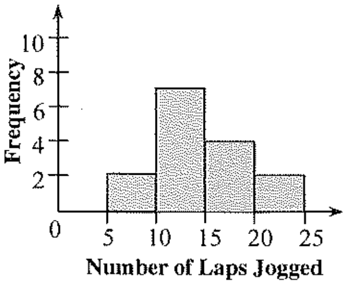
4.



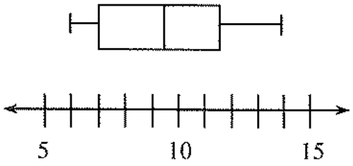
5.



6.

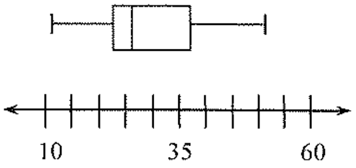


7.



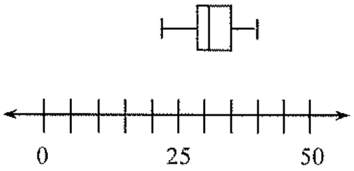
Q1 = 7; Med = 9.5; Q3 = 11.5;
IQR = 4.5

8.



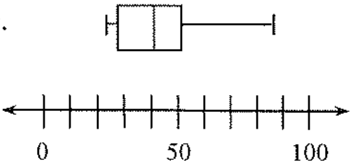
Q1 = 23; Med = 26.5; Q3 = 38;
IQR = 15

9.



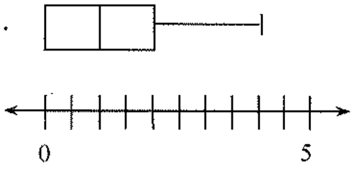
Q1 = 29; Med = 31; Q3 = 35;
IQR = 6

10.



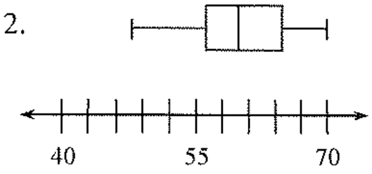
Q1 = 27.5; Med = 40.5Q3 = 50.5;
IQR = 23

11.



Q1 = 0; Med = 1; Q3 = 2;
IQR = 2

12.



Q1 = 56; Med = 60; Q3 = 65;
IQR = 9

Checkpoint 9B

Problem 9-79

Solving One-Step Equations

Answers to problem 4-129: a. 62, b. 17, c. 59, d. 216, e. 72, f. $3\frac{2}{5}$

To solve an equation (find the value of the variable which makes the equation true) we want the variable by itself. To undo something that has been done to the variable, do the opposite arithmetical operation.

Example 1: Solve $x-17=49$

Solution: 17 is subtracted from the variable. To undo subtraction of 17, add 17.
$x=49+17 \Rightarrow x=66$

Example 2: Solve $\frac{y}{3}=17$

Solution: The variable is divided by 3. To undo division by 3, multiply by 3.
$y=17\cdot 3=51$

Now we can go back and solve the original problem.

a. $x-13=49$; To undo subtraction of 13, add 13; $x=49+13=62$.

b. $4m=68$; To undo multiplication by 4, divide by 4; $m=\frac{68}{4}=17$.

c. $78=y+19$; To undo addition of 19, subtract 19; $y=78-19=59$.

d. $\frac{x}{6}=36$; To undo division by 6, multiply by 6; $x=36\cdot 6=216$.

e. $\frac{1}{3}x=24$; Multiplying by $\frac{1}{3}$ is the same as dividing by 3, undo it by multiplying by 3;
$x=24\cdot 3=72$. $x=24\div\frac{1}{3}=72$ is also correct.

f. $5y=17$; To undo multiplication by 5, divide by 5; $y=\frac{17}{5}=3\frac{2}{5}$.

Here are some more to try. Solve each equation.

1. $7+y=37$
2. $a-6=18$
3. $3z=9$
4. $\frac{x}{17}=3$
5. $20=6c$
6. $437=f+219$
7. $17=\frac{s}{4}$
8. $4=h-8$
9. $207+l=911$
10. $50b=150$
11. $\frac{k}{12}=12$
12. $t-489=195$
13. $1=3u$
14. $\frac{v}{11}=8$
15. $n-\frac{1}{4}=5$
16. $d+195=2004$
17. $e-503=0$
18. $146r=877$
19. $17+m=92$
20. $\frac{g}{56}=5$
21. $4q=15$
22. $\frac{j}{12}=8$
23. $0.9+w=3.86$
24. $p-\frac{1}{3}=\frac{3}{4}$

Answers:

1. $y=30$
2. $a=24$
3. $z=3$
4. $x=51$
5. $c=\frac{20}{6}=3\frac{2}{6}=3\frac{1}{3}$
6. $f=218$
7. $s=68$
8. $h=12$
9. $l=704$
10. $b=3$
11. $k=144$
12. $t=684$
13. $u=\frac{1}{3}$
14. $v=88$
15. $n=5\frac{1}{4}$
16. $d=1809$
17. $e=503$
18. $r=\frac{877}{146}=6\frac{1}{146}$
19. $m=75$
20. $g=280$
21. $q=\frac{15}{4}=3\frac{3}{4}$
22. $j=96$
23. $w=2.96$
24. $p=\frac{13}{12}=1\frac{1}{12}$