AP Calculus AB Syllabus

Course Overview

This course is designed to provide a form foundation in beginning differential and integral Calculus. The topics covered are those listed in the topic outline for AP Calculus AB in AP Calculus course description. Each topic will be approached from a variety of ways, providing students the opportunity to solve problems in four wars- graphically, numerically, analytically, and verbally. An emphasis will be placed on the inter connectedness of each approach and the importance of being able to support and verify an answer in more than one way. Unlike many traditional math classes, this course will require students to explain solutions to problems verbally as well as in written form. A simple numerical answer often is insufficient evidence to fully explain a solution to a problem.

Our study of Calculus is divided into two major topics: differential and integral calculus. Differential Calculus enables us to calculate rates of change, find the slope of a curve, and calculate velocities and acceleration of moving bodies. Integral Calculus is used to find the area of an irregular region in a plane, measure lengths of curves, and calculate centers of mass of arbitrary solids. Our task is to perfect each student's mechanics and to develop his or her understanding of the theory of Calculus and the ability to use these ideas in applied Calculus. Through additional practice of the mechanics and the development of the applications of derivatives and antiderivatives in problem solving, each student may accomplish this task.

Technology requirements

Each student is required to have a graphing calculator. Students will be strongly encouraged to use the TI-83 or the TI-84 models.

I. Functions and Models

A. Functions and Models

1. Definition of a Function
   a. Domain, Range of a Function
   b. Evaluating Functions at specific points
   c. Graphing Functions (with and without graphing calculator)
2. Representations of Functions
   a. Verbal, Numeric, Graphic, Algebraic
3. Vertical Line Test
4. Piecewise Functions
5. Step Functions
6. Symmetry of Even and Odd Functions
7. Increasing and Decreasing Functions

Throughout this section, we will take time to graph functions (including setting up the window) and evaluating functions at specific points using the graph, the table, and the function itself. Also, we will review the graphing calculator's regression capabilities. Finally, we will review how to graph piecewise functions with the graphing calculator.

A common theme will be the relationship between different functional representations. For instance, how can the quadratic functions f(x) x2 be described (decreasing at a decreasing rate then turns at the vertex to increasing at an increasing rate)? What in the table suggests it is quadratic (second differences), what are the features of the graph, what information does the algebraic representation provide. This concept will be applied throughout all the function that will be analyzed.

B. Mathematical Models: A Catalog of Essential Functions

1. Linear Models
2. Polynomials
   b. Including quadratic and cubic functions
3. Power Functions
4. Rational Functions
5. Algebraic Functions
6. Trigonometric Functions
7. Exponential Functions

8. Logarithmic Functions
9. Transcendental Functions

Approx Time: 6 periods

Again, part of the discussion of each type of function will be the comparisons of the different types of representations (graphical, numerical, analytical, verbal) as well as graphing functions (with an appropriate viewing window) in the graphing calculator.

C. New Functions from Old Functions
1. Transformations of Functions
a. translations, stretching, reflecting
2. Combinations of Functions

Approx Time: 4 periods

The graphing calculator is a great exploration tool for transformations. Students will compare functions that have been shifted (using the graphing calculator) and describe the resulting function, making special note of the original and new function's graph, table of values, and algebraic representation. Also, students will learn to graph composite functions in the graphing calculator.

II. Limits
A. The Tangent and Velocity Problems
1. The Tangent Problem
a. Find the equation of a tangent line to a function at a specified point.
2. The Velocity Problem
a. Find the instantaneous velocity of a function

Approx Time: 4 periods

Note: Although students have not heard the term "derivative" yet, they are gaining experience in finding slope by selecting points that are extremely close to one another. Also, students will have opportunities to define the slope as a new function and make a table of values in the graphing calculator.

B. The Limit of a Functions
1. Definition
2. Evaluating Limits with a Table of Values and with a Graph
3. Left-Hand/Right-Hand Limits
4. Limit where there is a Vertical Asymptote

Approx Time: 4 periods

There will be a focus on analytical evaluation of a limit (if possible) as well as using the graphing calculator to evaluate limits numerically and graphically. -

C. Calculating Limits using Limit Laws
1. Limit Laws (ie: The limit of the sum of 2 functions is the sum of the limits of each function, etc...)
2. Theorem (The limit of a function exists only if both the right and left side limits exist and are equal.)
3. Squeeze Theorem

Approx Times 6 periods

The beginning of this section consists of analytically evaluating limits. Students will also be asked to compare analytical solutions to graphical and numerical solutions.

D. The Precise Definition of a Limit
1. Definition
2. Definition of Left-Hand and Right-Hand Limits

3. Infinite Limits

Approx Time: 4 periods

This is students' first exposure and utilization of such intricate definitions. As a result, students will have difficulties with this content; therefore I am planning on the possibility of taking extra time with this section.

E. Continuity
1. Definition of Continuity
2. Discontinuities
   a. Jump
   b. Removable
3. Right-Hand/Left-Hand Continuity
4. Continuous on an Interval
5. Continuity Theorems
6. Continuous Functions
7. Composite Functions and Continuity
8. Intermediate Value Theorem

Approx Time: 6 periods

There will be a special focus on the Intermediate Value Theorem. Students will use the graphing calculator to provide graphical evidence of continuity. We will discuss why graphs and tables are not sufficient to prove continuity but may be sufficient to prove discontinuity.

III. Derivatives
A. Derivatives and Rates of Change
1. Tangents
   a. Equations of Tangent Line to a Function
2. Definition of Slope using Limits
3. Position Function
4. Instantaneous Velocity
5. Definition of Derivative
6. Rates of Change (average and instantaneous)

Approx Time: 6 periods

Students will begin evaluating slope (to find the equation of the tangent line to a function) by computing the slope of various secant lines on the function and making the points on the secant line come closer and closer together. The graphing calculator will allow students to quickly find the slope of a secant line by making a table of values that show the points getting closer to each other.

Also, students will use the graphing calculator to find the slope of tangent line through local linearization, in which students zoom in on a function until it appears to be a line. Students then find two points to determine the slope of the line. This also allows students the opportunity to use a linear regression (with two points) to find the equation of the tangent line to a function. Finally, students will be shown how to get the equation of a tangent line to a function using the Tangent(function on the DRAW menu of the TI83/84 graphing calculator.

B. The Derivative as a Function
1. Limit definition of a Derivative
2. Other Derivative Notations
3. Differentiability of a Function
   a. Differentiability and Continuity
4. How functions fail to be differentiable
5. Higher Derivatives

Approx Time: 5 periods

Students will use their graphing calculator to provide evidence that their analytical solutions are correct.

C. Differentiation Formulas

1. Derivative of Constant and Power Functions
2. Constant Multiple Rule
3. Sum Rule
4. Difference Rule
5. Product Rule
6. Quotient Rule
7. Power Rule

Approx Time: 6 periods

Students will be introduced to the nDeriv (function on the TI-83/84 (or a similar function on a different calculator)) to graph derivative functions and to evaluate the derivative of a function at a particular point. For evaluating derivative at various points, students will also learn the dy/dx function under the [Calc] menu on the TI-83/84 (or similar function for a different calculator). The use of the graphing calculator also allows students opportunities to compare the analytical rules of derivatives to the derivative graph to the numerical value (where applicable).

D. Derivatives of Trigonometric Functions

1. lim
2. lim
3. Derivative of $\sin\theta$, $\cos\theta$, $\tan\theta$, $\csc\theta$, $\sec\theta$, $\cot\theta$

Approx Time: 4 periods

Students will be asked to graph the trigonometric functions and then sketch the graph of the derivative. Students will then be asked to graph the same derivatives on the graphing calculator and compare the calculator's graph with their sketch and discuss with other students the two graphs and make conjectures about the derivatives. Most students should see the sin x and cos x derivatives but will struggle with the others. As this point
we will discuss the analytic rules for trigonometric derivatives.

E. The Chain Rule

1. Chain Rule
2. Power Rule combined with Chain Rule

Approx Time: 3 periods

Again, students will be encouraged to check any analytical work against the nDeriv(function of the graphing calculator to provide evidence for accuracy of the results.

F. Implicit Differentiation

Approx Time: 4 periods

G. Rates of Change in the Natural and Social Sciences

1. Average, instantaneous rate of change
2. Physics
    a. Velocity, Acceleration
3. Chemistry
    a. Rate of Reaction
4. Biology
    a. Rate of Growth
5. Economics
    a. Marginal Cost

Approx Time: 4 periods

Due to the nature of rates of change in the natural and social sciences there will be a rather large emphasis on working with multiple representations of functions in context and then communicating results in such a way that is mathematically reasonable and reasonable in the context of the problem.

H. Related Rates

Approx Time: 6 periods

This section will include a great deal of emphasis on strategies to solve related rates problems. Specifically, we will focus on setting up problems visually and analytically and then use calculus to solve the problem.

I. Linear Approximations and Differentials

1. Linear Approximation or Tangent Line Approximation
2. Differentials

Approx Time: 3 periods

Again, given the content, there will be an emphasis between the analytical, graphical, and numeric representations of problems.

IV. Applications of Differentiation

A. Maximum and Minimum Values

1. Absolute Max/Mm Values
2. Local Max/Mm Values
3. Extreme Value Theorem
4. Fermat's Theorem
5. Critical Numbers
6. Closed Interval Method

Approx Time: 6 periods

We will use the graphing calculator to graph various functions and determine absolute and local max/mm values. Students will be asked to change the window and determine if that changes any of the max/mm values. This will lead to a discussion about the role of the domain of a function when determining an absolute max/mm. Also, students will use the [Calci menu functions "minimum" and "maximum" to find local max/mm values. We will then discuss why the graphing calculator finds local extreme and not absolute extreme.

B. The Mean Value Theorem

1. Rolle's Theorem
2. The Mean Value Theorem

Approx Time: 4 periods

We will focus on the relationship between the theorems and their graphical interpretations.

C. How Derivatives Affect the Shape of a Graph

1. Increasing/Decreasing Test
2. The First Derivative Test
3. Concave Upward/Downward
4. Concavity Test
5. Inflection Point
6. Second Derivative Test

Approx Time: 8 periods

This section begins with the main components of curve sketching. These ideas will be solidified in later sections.

D. Limits at Infinity, Horizontal Asymptotes
1. Definition
2. Horizontal Asymptotes
3. Infinite Limits at Limits

Approx Time: 2 periods

E. Summary of Curve Sketching
1. Guidelines for Sketching a Curve
2. Slant Asymptotes

Approx Time: 2 periods

F. Graphing with Calculus and Calculators

Approx Time: 4 periods

The purpose of this section is to start with a graph produced by a graphing calculator and to refine it using calculus to reveal the important aspects of the curve.

G. Optimization Problems
1. General Procedures in Solving Optimization Problems
2. First Derivative Test for Absolute Extreme Values
3. Applications to Business and Economics

Approx Time: 4 periods

H. Newton's Method
1. Newton's Method

Approx Time: 3 periods

I. Anti-Derivatives
1. Definition
2. Theorem for General Anti-derivative
3. Introduction to Differential Equations
4. Rectilinear Motion

Approx Time: 4 periods

Of particular interest in this section is rectilinear motion using the graphing calculator. Specifically, the class will explore this motion using parametric equations and the parametric mode on the TI-83184 (or other graphing calculator.) Also, given the multitude of viewing options for graphs on graphing calculators, we will further model situations more authentically using various viewing modes. This allows students to accurately model particles moving horizontally or vertically as well as providing evidence about turning points, velocities, etc...

V. Integrals
A. Areas and Distances
1. The Area Problem (flow do we find the area under a curved surface?)
2. The Distance Problem (What is the distance traveled by an object during a period of time if the velocity is known at all times?)

Approx Time: 4 periods

Students will be this section by taking a function (for instance, f(x) x') and attempt to find the area under the curve (between 2 points) using methods they are familiar with. As a class, we will discuss these methods and attempt to arrive at a systematic method of creating rectangles to approximate the area. We will then discuss making rectangles with

the right endpoint on the function, the left endpoint on the function, and the midpoint on the function.

Note: I will have to review sigma notation with students,

B. The Definite Integral
1. Definition of a Definite Integral
a. Explanations of terms and symbols
b. Comparison with Riemann Sum
2. Evaluating Integrals (using limit notation)
3. Midpoint Rule
4. Properties of the Definite Integral
a. Properties of the Integral
b. Comparison Properties of the Integral

Approx Time: 8 periods

Students will need time to develop the ideas of the definite integral. It is important for students to have a conceptual framework of an integral for later units, thus we will spend a great deal of time working through problems and the conceptual ideas of this section.

C. The Fundamental Theorem of Calculus
1. Fundamental Theorem of Calculus, part 1
2. Fundamental Theorem of Calculus, part 2
3. Differentiation and Integration as Inverse Processes

Approx Time: 6 periods

Again, this is another section that is conceptually very rich and students need time to develop ideas. At this point, students will be introduced to the fnInt(function and the ∫f(x)dx function on the TI-83/84 (or other graphing calculator.)

D. Indefinite Integral and the Net Change Theorem
1. Indefinite Integrals
a. Table of Indefinite Integrals (including Integrals of Some Trigonometric Functions)
2. Applications
a. The Net Change Theorem

Approx Time: 4 periods

E. The Substitution Rule
1. The Substitution Rule
2. The Substitution Rule for Definite Integrals
3. Integrals of Symmetric Functions

Approx Time: 7 periods

Due to the difficulty of this section, we will spend some time working through a variety of problems.

VI. Applications of Integration
A. Areas Between Curves
1. Definition
2. Examples

Approx Time: 6 periods

Students will be exposed to methods of finding areas using the integral functions of the graphing calculator. For instance, to find the area given between the curves y =x' and y 2x — x2, from 0 to 1, one would take the upper function and subtract it from the

lower function and then evaluate the integral in the graphing calculator. Thus, in this case, and then the integral is found by using $\int f(x)dx$ in the [Calc] menu from 0 to 1.

B. Volumes
1. Calculus Definition
2. Disk Method
3. Washer Method

Approx Time: 8 periods

C. Volumes by Cylindrical Shells
1. Method (including formulas)

Approx Time: 6 periods

D. Work

Approx Time: 2 periods

E. Average Value of a Function
1. Average Value of a Function
2. Mean Value Theorem for Integrals

Approx Time: 4 periods

F. Approximate Integration
1. Left/Right Endpoint Approximation
2. Midpoint Rule
3. Trapezoidal Rule

Approx Time: 4 periods

Note: Sections B, D, and E require quite a bit of analytical work that students must practice, hence the additional time for each section.

VII. Inverse Functions: Exponential, Logarithmic, and Inverse Trigonometric Functions
A. Inverse Functions
1. Definition of 1-to-i Function
2. Horizontal Line Test
3. Definition of Inverse Function
4. Cancellation Equations
5. How to find Inverse of a 1-to-1 Function
6. The Calculus of Inverse Functions

Approx Time: 6 periods

We will spend time discussing and comparing inverse and 1-to-1 functions analytically, graphically, and numerically. Specifically, how can one note if a function is 1-to-1 (and thus has an inverse) by using its graph and a table of values.

B. Exponential Functions and Their Derivatives
1. Exponential Functions
2. Laws of Exponents
3. Limits Involving Exponents
4. Applications of Exponential Functions
5. Derivatives of Exponential Functions
   a. Definition of the Number e
   b. Derivative of the Natural Exponential Function
6. Exponential Graphs
7. Integration of Natural Exponential Function

Approx Time: 6 periods

C. Logarithmic Functions
1. Logarithmic Function with Base a
2. Laws of Logs
3. Limits Involving Logs
4. Natural Logs
a. Definition
b. Change of Base Formula

Approx Time: 4 periods

D. Derivatives of Logarithmic Functions
1. Derivative of ln x
2. Integral of 1/x
3. General Logarithmic and Exponential Functions
a.

b.

c.

4. Logarithmic Differentiation

Approx Time: 6 periods

E. Exponential Growth and Decay
1. Differential Equation
a. Law of Natural Growth/Decay
2. Population Growth
3. Radioactive Decay
4. Newton's Law of Cooling
5. Continuously Compounded Interest

Approx Time: 6 periods

F. Inverse Trig Functions
1. Inverse Sine Function
2. Inverse Cosine Function
3. Inverse Tangent Function
4. Derivatives of Inverse Trig Functions (including corresponding integration formulas)

Approx Time: 6 periods

VIII. Differential Equations
A. Modeling with Differential Equations
1. Models of Population Growth
2. A Model for the Motion of a Spring
3. General Differential Equations

Approx Time: 6 periods

B. Direction Fields and Euler's Method
1. Direction Fields (Slope Fields)
2. Euler's Method

Approx Time: 6 periods

A wonderful activity for slope fields is to have students create slope fields (at various points) and try to go back and discuss as many features of the original function. This is also a great opportunity for students to create slope fields with a TI-83184 specific
program.