# AP Calculus Summer Assignment

This is the Summer assignment for both AP Calculus AB and AP Calculus BC. The purpose of this assignment is to reinforce what you learned in Calculus Honors so that you retain mastery of that material through the Summer and into the new school year. To be successful in AP Calculus, it is essential that you are familiar with all topics covered in this assignment prior to the start of class. **There will be a test on the content of this assignment during the first full week of classes.**

Please complete all of the following problems. You must write legibly and coherently, and in an organized manner. All work must be shown, all computations must be labelled, and all final answers must be clearly indicated. **This assignment will be graded on completion and presentation and will count as a quiz grade for the first quarter.**

**THIS ASSIGNMENT IS DUE AT THE START OF THE FIRST CLASS OF THE YEAR.**

If you have any questions about this assignment or about AP Calculus, please feel free to contact me via email at **aschultz@oratoryprep.org**

# Limits and Continuity

1. Evaluate the limit $\lim_{x\to 4} \frac{4x^2-64}{2\sqrt{x}-4}$
2. Evaluate the limit $\lim_{x\to -1} \frac{x}{\frac{1}{1+x}-1}$
3. Evaluate the limit $\lim_{x\to \infty} \frac{x^2+1}{3x^3-2x-5}$
4. Evaluate the limit $\lim_{x\to \infty} \frac{3x^3+x}{5x^3+x^2-4}$
5. For what values of $x$ is $f(x) = \frac{2x-2}{x^2+x-2}$ discontinuous?
6. Find a value for $k$ such that the function $f(x) = \begin{cases} kx+1 & x \leq 0 \\ x^2-2k & x > 0 \end{cases}$ is continuous.

# Differentiation

1. Evaluate $\lim_{h\to 0} \frac{\ln(5+h)-\ln(5)}{h}$.
2. Explain, using limits, why $f(x) = |x|$ is not differentiable at $x = 0$.
3. Find the equation for the line tangent to $f(x) = x - e^{\frac{x^2}{2}}$ at the point where $x = 0$.
4. Determine $\frac{dy}{dx}$ for $y = \sqrt{e^x \cos(2x)}$.
5. Determine $\frac{dy}{dx}$ for $y = \frac{\ln(x+1)-1}{(3x^2+2)^4}$.
6. Determine $\frac{dy}{dx}$ for $y$ defined implicitly by $xy^2 + 3xy = x - \sin(2y)$.
7. A 13ft ladder slides down a wall at a rate of 3ft/s. How fast is the base of the ladder sliding away from the wall when the top of the ladder is 5ft above the ground?
8. For $f(x) = \frac{2}{x^2+2}$, determine the intervals where $f$ is increasing or decreasing and where $f$ is concave up or concave down. List any critical points and inflection points.
9. Determine the $x$-coordinates of all local extrema of $f(x) = \frac{2}{3}x^3 + \frac{5}{2}x^2 - 3x + 1$ and say which correspond to local minima and which correspond to local maxima.
10. Determine the absolute extreme values of $f(x) = f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ on the interval $[-1, 1]$.
11. What is the volume of the largest right cylindrical cone that can be inscribed in a sphere of radius 3? Justify your answer by showing that it corresponds to a global maximum value of a volume function.

## Integration

1. Use a right Riemann sum with four subintervals of equal width to approximate $\int_0^{12} e^{x^2} dx$
2. Evaluate the integral $\int \sqrt{2x-1} + x^5 dx$
3. Evaluate the integral $\int \frac{x+5}{x+1} dx$
4. Evaluate the integral $\int \frac{\sin(\ln x)}{x} dx$
5. Evaluate the integral $\int_1^5 x^2 - 3x + 1 dx$
6. Evaluate the integral $\int_0^1 \frac{e^x+1}{e^x+x} dx$
7. Determine the average value of $f(x) = e^x + \cos(\pi x)$ on the interval $[0, 3]$.
8. If $f'(x) = -x^3 + 2x^2$ and $f(1) = 5$, determine $f(x)$.
9. Find the area of the region in the first quadrant bounded by the curves $y = 8x^3$ and $y = \sin(\pi x)$.
10. Let $R$ be the region in the first quadrant bounded by the curves $y = \sqrt{x}$, $y = 6 - x$, and the $x$-axis. Suppose $R$ is the base of a solid where, for each $y$-value, the cross-section of the solid taken taken perpendicular to the $y$-axis is a rectangle whose base lies in $R$ and whose height is $2y$. Find the volume of this solid.
11. Find the volume of the solid obtained when the region bounded by the curves $y = x^2$ and $y = -2x^2 + 6x$ is revolved around the line $x = -1$.

## Differential Equations

1. Draw a slope field corresponding to the solution of the differential equation $y' = \frac{x^2 y}{1+y}$ using all points with integer coordinates $(x, y)$ such that $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$.
2. Find the general solution of the differential equation $y' = y^2 \cos(x + 1)$.
3. Find the general solution of the differential equation $y' = \frac{\ln(x^y)+y}{x}$.
4. Find the particular solution of the differential equation $y' = 3xy$ with initial condition $y(0) = 3$.