CONGRUENT AND SIMILAR TRIANGLES

CONGRUENT (≡) TRIANGLES
Triangles are congruent when they have exactly the same three sides and exactly the same three angles.

What is “Congruent”…?
It means that one shape can become another using rotations (turns), reflections (flips) and/or translations (slides).

**Congruent Triangles**
**Example 1:**

Notice all of the sides have the same measure, the only difference is the placement of the triangles.

**Not Congruent Triangles**

The two triangles at right are not congruent because two sides do not have the exact same measure.

**What if two triangles have the exact same angle measures, will they be congruent?** Not always. In this case the angles are the same but one triangle is larger than the other. However, if two triangles are congruent, their angles will all be the same size.

When two triangles are congruent, we mark corresponding sides and angles like this:

Looking at the triangles, we can tell which sides and angles are congruent by looking at the markings. For Example $ED$ has one mark on it and $AB$ has one mark on it. That means $ED \cong AB$. The symbol $\cong$ means congruent.
Let’s see if the triangles are congruent.
\[ \angle A \cong \angle D \quad BE \cong CE \]
\[ \angle B \cong \angle C \quad AB \cong DC \]
\[ \angle AEB \cong \angle DEC \quad AE \cong DE \]

Since all of the sides and angles are congruity, the two triangles are congruent. \( \triangle ABE \cong \triangle DCE \)

When stating the triangles are congruent, notice that we have matched up the congruent sides and angles. \( \triangle ABE \cong \triangle DCE \)

YOU TRY: For each pair of triangles below, state the parts that are congruent and then state a congruency statement for the two triangles.

\[
\begin{align*}
\triangle JIK & \cong \triangle \_ \_ \_ \\
\triangle \_ \_ \_ & \cong \triangle \_ \_ \_
\end{align*}
\]

\[
\begin{align*}
\triangle \_ \_ \_ & \cong \triangle \_ \_ \_
\end{align*}
\]
There are several ways of proving triangles.

<table>
<thead>
<tr>
<th>Method</th>
<th>Diagram</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSS</td>
<td><img src="image1" alt="SSS Diagram" /></td>
<td>Each triangle has a corresponding side that is the same length.</td>
</tr>
<tr>
<td>SAS</td>
<td><img src="image2" alt="SAS Diagram" /></td>
<td>The congruent angles are located between the two marked congruent sides. The sides make up the sides of the angle.</td>
</tr>
<tr>
<td>ASA</td>
<td><img src="image3" alt="ASA Diagram" /></td>
<td>The congruent sides are located between the two congruent angles.</td>
</tr>
<tr>
<td>AAS</td>
<td><img src="image4" alt="AAS Diagram" /></td>
<td>In this case the congruent side is <strong>NOT</strong> between the two angles that are marked congruent.</td>
</tr>
<tr>
<td>HL</td>
<td><img src="image5" alt="HL Diagram" /></td>
<td>These triangles <strong>MUST</strong> be right triangles.</td>
</tr>
</tbody>
</table>
YOUR TURN: For each set of triangles shown below, determine if the triangles are congruent. If the triangles are congruent, state how you know they are congruent.

1. 
2. 
3. 
4. 

SIMILAR TRIANGLES
Two shapes are Similar when one can become the other after a dilation (resize), reflection (flip), translation (slide) or a rotation (turn).

In each case, the triangles are similar.

<table>
<thead>
<tr>
<th>Dilation</th>
<th>Rotation</th>
<th>Reflection</th>
<th>Translation</th>
</tr>
</thead>
</table>

When two shapes are similar, then:
- **Corresponding angles are equal, and**
- **Corresponding sides are proportional**

Are Congruent Shapes also Similar?
Yes, because corresponding angles are congruent and the sides are in proportion. The sides are actually in a 1 to 1 ratio.

Two triangles are similar if the only difference is size, rotation, or reflection.
Corresponding Sides
In similar triangles, corresponding sides are always in the same ratio. For example:

Triangles R and S are similar. The equal angles are marked with the same number of arcs.
What are the corresponding lengths?
- The lengths 7 and a are corresponding
- The lengths 8 and 6.4 are corresponding
- The lengths 6 and b are corresponding

Calculating the Lengths of Corresponding Sides
Step 1: Find the ratio of the corresponding sides
Step 2: Use that ratio to find the unknown length(s)

Example:

YOUR TURN: Each pair of triangles are similar. Find the missing side.

1. 

2. 

**FINDING SIMILAR (\(~\)) TRIANGLES**

We do **not need** to all three sides and all three angles to show that the triangles are similar. Usually if we know two or three of the six pieces, we have enough information to determine if the triangles are similar.

There are three way to show that two triangles are similar:

<table>
<thead>
<tr>
<th>Method</th>
<th>Condition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>AA</strong></td>
<td>If two triangles have two of their angles equal, then the triangles will be similar. If two of their angles are equal, then the third angle must be equal because angles of a triangle always to make 180°.</td>
<td><img src="image" alt="AA Example" /></td>
</tr>
<tr>
<td><strong>SAS</strong></td>
<td>If two triangles have two pairs of sides in the same ration and included angles are equal, the triangles are similar. In the triangles at right, we can see: - One pair of sides is in the ratio of 21:14 = 3:2 - Another pair of sides is in the ratio of 15:10 = 3:3 - There is a matching angle of 75° in between them.</td>
<td><img src="image" alt="SAS Example" /></td>
</tr>
<tr>
<td><strong>SSS</strong></td>
<td>If two triangle have three pairs of sides in the same ratio, then the triangles are similar. In the triangles at right, the ratio of the sides are: (\frac{8}{10} = \frac{4}{5} = \frac{6}{7.5}). (\frac{8}{10}) simplifies to (\frac{4}{5}) and (\frac{6}{7.5}) which simplifies to (\frac{4}{5}). All of the sides are in the same ratio, so the triangles are similar.</td>
<td><img src="image" alt="SSS Example" /></td>
</tr>
</tbody>
</table>

Name the \(\Delta\) walk around it \(\Delta ABC\) Name the \(\Delta\) the same way \(\Delta XYZ\)

To write a similarity statement: \(\Delta ABC \sim \Delta XYZ\)
YOUR TURN: State if each triangle pair is similar. If so, name the similar triangles, and how they are similar.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. <img src="image1.png" alt="Diagram of triangle pair 1" /></td>
<td></td>
</tr>
<tr>
<td>2. <img src="image2.png" alt="Diagram of triangle pair 2" /></td>
<td></td>
</tr>
<tr>
<td>3. <img src="image3.png" alt="Diagram of triangle pair 3" /></td>
<td></td>
</tr>
<tr>
<td>4. <img src="image4.png" alt="Diagram of triangle pair 4" /></td>
<td></td>
</tr>
</tbody>
</table>