

## Welcome to AP Calculus BC!

Calculus is an advanced math that a select few students reach in high school. Congratulations on making it this far and welcome to the beginning of a long but rewarding journey! This class is going to be a lot of work. The expectation is that each student will study AT HOME for a minimum of 8 (eight) hours a week. As an enrolled student in this class, your intention should be to pass the AP exam with a score of 3 or higher. This is not a class for those who just want “AP” next to the class name or the extra grade point for their GPA. Your grade will coincide with your knowledge of calculus at the CONCEPTUAL level; it is not enough to be good at memorizing or computations. Calculus problems are worded using advanced mathematical terminology and often require answers in the form of sentence explanations, not just boxed in numerical values. This Advanced Placement class will be taught at the *college level* because upon passing the AP exam, you will receive college credit. It is expected that students who enter AP Calculus be mature and have a strong work ethic. Being consistently absent or late for any reason will be detrimental to your success in AP Calculus. Students with a lot of extracurricular activities including sports must budget their time accordingly due to the high expectation of time required for this class.

By enrolling in BC you are part of an elite group of students who want to challenge themselves to the fullest extent. Students in BC should have the intention of majoring in a Physical Science or Engineering. What’s the difference between AB and BC? The amount of material! In AB students cover 3 major topics and BC covers 6. The hardest part? Not enough time. If you are coming straight from Honors Math Analysis, you will have the biggest leap to make but IB and AB students also struggle with the pace of the material. Bottom line: you will eat, sleep, and breathe calculus starting now.

This summer, you will begin AP Calculus BC with a “flipped” classroom. The traditional classroom has a teacher that lectures IN CLASS with practice problems done AT HOME. You will be experiencing the opposite. At home, you are required to watch online video lectures ranging from 20-80 minutes and take handwritten notes. Then you will actively work through practice exercises.

Students find AP Calculus difficult because you must have a thorough conceptual and computational understanding of every math concept you have ever been taught. College Board has provided a strict outline of the prior mathematical knowledge necessary to be successful in AP Calculus. The following topics will be considered *assumed* knowledge next year. There is little to no time for in class review on any of the following topics. From Collegeboard.com: *Before studying calculus, all students should complete 4 years of secondary mathematics classes in which they study algebra, geometry, trigonometry, and elementary functions. These functions include linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric and piecewise-defined functions. In particular, before studying calculus, students must be familiar with the properties of functions, the algebra of functions and the graphs of functions. Students must also understand the language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts and so on) and know the values of the trigonometric functions on the unit circle.*

Most students that brave AP Calculus are mathematically capable of succeeding in the class and on the AP exam. The most difficult part of this class is going to be maintaining an academically mature attitude that entails working diligently throughout the entire year. **You must come mentally prepared every day and be willing to get help right away when a concept is not clear. Hard work will earn you the grade that you want. There will be no drastic curve to the grades or extra credit opportunities.**

## 2018-2019 BC Summer Assignment

There are two parts to the summer assignment that are detailed below. You will need a graphing calculator to complete some problems on these summer assignments. It's imperative that you own a TI-84 (or 84 plus) graphing calculator and your own copy of the textbook *Calculus: Early Transcendentals* by Jon Rogawski by the first day of class.

**Part 1: (STRONGLY recommended)** Review topics lists and complete the attached 250+ math review problems.

Since students taking AP Calculus come from different classes, all students are at different places in their mathematical knowledge of the prerequisite material. Regardless of what class you came from, you **MUST** have a complete and thorough understanding of each review topic on the list provided. To help you prepare, you should work through the attached review problems although it will NOT be collected. While working through problems, keep in mind that **on the FIRST DAY OF SCHOOL, you will take a QUIZ on all of the content from the list of topics given.**

Completing the review will help prepare you for the quiz but ultimately the quiz will cover all material from the list given even if there are no practice problems like that on the review. Although this counts as a quiz, you should treat it like a test. It will be entirely free response with the requirement that detailed work is shown. Students are expected to earn an 80% or higher on this assessment as it is an indicator of success in AP Calculus. Should a student not meet this expectation, a meeting will be arranged with the student, parents, teacher, and counselor to discuss an action plan for the course. Should you need help with this assignment or reviewing these topics, there will be links posted to helpful videos on the class website as well as a schedule of availability for summer tutoring.

**Part 2: (MANDATORY)** Watch video lessons, takes notes, and complete exercises from the textbook/worksheets.

The required part of the summer assignment will be to watch and take notes on 14 video lessons (over 8 hours of videos) as well as complete accompanying book problems and/or worksheets also found on the class website. You must watch the videos on EDpuzzle\* which tracks your viewing and includes embedded questions. You are not graded on the embedded questions but you are graded on your completion of watching the videos. Some videos are review content and some are NEW material. Set aside at least 2-3 weeks to watch the lessons and complete the exercises. You will submit notes and completed exercises on the first day of class. **You will have a TEST on the new material on the SECOND DAY OF SCHOOL.**

*\*If you believe that internet access might be a problem for you over summer or during the year, come speak with me individually so I can work out how you will be watching the lessons. Lack of internet access will never be considered a valid excuse for not watching the video lessons over summer or during the regular academic year.*

Success in this class will be difficult if...

1. You plan to skip the recommended summer assignment OR wait until the day before to complete video lessons.
2. You do not fully understand each topic from the review problems before the year begins.
3. You consider yourself a procrastinator or you feel like "senioritis" is something that may strike you.
4. You don't like asking for help. Most students need regular tutoring in calculus, even those "good" at math.
5. You don't do your assignments/homework or you copy assignments/homework.
6. You think "cramming" is an acceptable form of studying.
7. You are excited about online video lectures because now you think you won't have any homework.
8. You "got by" in many of your other math class without really learning.
9. You think you can skip the "word problems." Calculus is mostly word problems.
10. You don't like doing math.

This class will be one of the most challenging classes you will take in high school but it can also be one of the most rewarding! Have a wonderful summer!

Mrs. Kawa

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<https://sites.google.com/a/bishopamat.org/ckawa/> ←Bookmark and check it weekly over summer.

To sign up for REMIND text @mrsckawa to 81010

## REQUIRED BC Calculus Summer Video Lessons

To get started:

1. Visit [edpuzzle.com](http://edpuzzle.com) and create a login. Enter your first AND last name even though it only asks for first name.
2. Enter the BC Class Code: **apcovel** Videos lessons will be available on [EDpuzzle.com](http://EDpuzzle.com) by July 1<sup>st</sup>. You MUST watch all the videos within [edpuzzle.com](http://edpuzzle.com), not on [youtube.com](http://youtube.com). There are 2 different deadlines. See below.
3. Some videos have PDF notes templates you must complete and required assignments. You will find them on the website: <https://sites.google.com/a/bishopamat.org/ckawa>
4. If you have any questions, email [ckawa@bishopamat.org](mailto:ckawa@bishopamat.org)

**Watch video lessons 1-6 by Friday July 20th** (Total time: 273 minutes; approx. 4.5 hours PLUS exercises)

1. *A Crash Course on Functions* [review material] (53 minutes)
  - No required notes or worksheets
2. *Review of Rational Functions* [review material] (42 minutes)
  - No required notes or worksheets
3. *All about trigonometry* [review material] (46 minutes)
  - No required notes or worksheets
4. *Calculator basics* [new material] (38 minutes)
  - No required notes; complete required worksheet
5. *Intro to Rates of Change* [new material] (51 minutes)
  - Take notes (on printed template<sup>^</sup>) and complete required worksheet
6. *Limits: An Introduction* [new material] (43 min)
  - Take notes (on printed template<sup>^</sup>) and complete required book exercises<sup>^</sup>

**Watch video lessons 7-14 by August 15<sup>th</sup>** (Total time: 205 minutes; approx. 3.5 hours PLUS exercises)

7. *Limits at Infinity and Horizontal Asymptotes* [new material] (39 min)
  - Take notes (on printed template<sup>^</sup>) and complete required book exercises\*
8. *Reading Limits from a graph* [new material] (12 min)
  - Take notes (on printed template<sup>^</sup>) and complete required worksheet
9. *Limit Laws* [new material] (16 min)
  - Take notes (on printed template<sup>^</sup>) and complete required book exercises\*
10. *Continuity* [new material] (39 min)
  - Take notes (on printed template<sup>^</sup>) and complete required book exercises\*
11. *Substitution Method* [new material] (13 min)
  - Take notes (on printed template<sup>^</sup>) and complete required book exercises\*
12. *Special Trig Limits* [new material] (11 min)
  - Take notes (on printed template<sup>^</sup>) and complete required book exercises\*
13. *Evaluating Limits Algebraically* [new material] (45 min)
  - Take notes (on printed template<sup>^</sup>) and complete required book exercises\*
14. *Intermediate Value Theorem* [new material] (30 min)
  - Take notes (on printed template<sup>^</sup>) and complete required worksheet and book exercises\*

\*If you don't yet have a copy of the textbook, see the class website for scanned images of the needed pages.

<sup>^</sup>If you don't want to use the printed templates, you can handwrite notes but ALL information must be written.

All required notes, exercises from the textbook, and worksheets will be collected on the first day of school at the start of class. Make sure they are binder clipped together and in the order shown above. If you have any questions or need the summer videos put on a flash drive, please see me ASAP.

## AP Calculus BC Prerequisite Topics List

To be successful in AP Calculus BC, a full understanding of the topics listed below is required prior to the school year. The summer assignment covers a majority of these topics but not all of them. All topics on the list may be asked on the first test of the year even if there are no practice problems covering that topic. If you are struggling with a topic, you are encouraged to look up review views of the topic online by searching the key-words in the topic or come for tutoring during summer.

You MUST have an understanding of each problem:  
ANALYTICALLY, GRAPHICALLY, NUMERICALLY, and VERBALLY.

### VOCABULARY

*You should be able to define and be comfortable verbally using the following terms this year:*

Sum, difference, product, quotient, prime, composite, opposite, reciprocal,  $x$ -intercept,  $y$ -intercept, zeroes of a function, coordinate point, exponent, base, equation, inequality, expression, equation, slope, linear, quadratic, parabola, cubic, factor, solve, absolute value, inverse, undefined, conjugate, domain, range, even function, odd function, root function, exponential function, rational function, polynomial, degree, (leading) coefficient, (dominating) term, increasing, decreasing, asymptote, asymptote, holes in the graph, piecewise function, rationalize, numerator, denominator, improper fraction, interval notation, parallel, perpendicular, horizontal, vertical, area, volume, perimeter, surface area, and units of measure.

### ARITHMETIC AND ALGEBRA BASICS

- Order of operations  $PeM \rightarrow D, A \rightarrow S$
- Simplifying expressions and parentheses basics
- Zero vs. undefined vs. indeterminate
- Properties of exponents and radicals (fractional and negative)
- Operations and properties of fractions
- Interval notation; when to use  $()$  vs.  $[]$
- Inequality notation  $< > \geq \leq$

### FUNCTIONS

- Define and evaluate a function
- Compose and decompose functions (extremely important!)
- Graphing functions by plotting points (input/output on axes)
- Find domain and range from a graph or given a function
- Know the graphs and properties (domain/range, even/odd/neither, asymptotes, intercepts, inc/dec) of

$$f(x) = x$$

$$f(x) = |x|$$

$$f(x) = x^2$$

$$f(x) = \sqrt{x}$$

$$f(x) = x^3$$

$$f(x) = \sqrt[3]{x}$$

$$f(x) = \frac{1}{x}$$

$$f(x) = \sin x$$

$$f(x) = \cos x$$

$$f(x) = \tan x$$

$$f(x) = \arctan x = \tan^{-1} x$$

$$f(x) = e^x$$

$$f(x) = \ln(x)$$

$$f(x) = \frac{1}{x^2}$$

- Translations (left/right/up/down), reflections (over  $x$  or  $y$  axis), absolute value of basic functions
- Even (symmetric about  $y$ -axis) and odd (symmetric about origin) functions graphically
- Find  $x$  and  $y$  intercepts of a function graphically and analytically
- Find zeroes of a function

### LINEAR FUNCTIONS AND EQUATIONS

- Slope formula and slope from a graph
- Point slope form
- Slope intercept form
- Vertical vs. horizontal line equation
- Parallel vs. perpendicular line relationships

## **POINTS OF INTERSECTION**

- Find points of intersection by hand and with a graph calculator

## **PIECEWISE FUNCTIONS**

- Read and evaluate piecewise functions
- Graph piecewise functions

## **INVERSE FUNCTIONS**

- Graphical interpretation of inverse functions
- Inverse given a table of values
- Relationship between exponentials and logarithms
- Log properties  $a \log_b x = \log_b x^a$   $\log 10 = 1$   $\ln e = 1$   $\ln e^x = x$   $e^{\ln x} = x$

## **FACTORING EXPRESSIONS VS. SOLVING EQUATIONS AND INEQUALITIES**

- Factoring means to “break down into smaller parts”
- Factor quadratics and certain cubics (should know how to factor by grouping)
- Solving an equation or inequality is find all  $x$  values that satisfy the statement.
- Determine on which  $x$ -intervals  $f(x) = 0$ ,  $f(x) < 0$ ,  $f(x) > 0$  given graph of  $f$  (and  $\leq, \geq$ )
- Solve for  $x$ -intervals  $f(x) = g(x)$ ,  $f(x) < g(x)$ ,  $f(x) > g(x)$  given graph of  $f$  and  $g$  (and  $\leq, \geq$ )

## **RATIONAL FUNCTIONS**

- Define a rational function as a polynomial divided by a polynomial
- Find horizontal asymptotes
- Vertical asymptotes vs. holes in graph
- Find HA and VA from graphs
- Find domain and range of rational functions

## **TRIGONOMETRY AND THE UNIT CIRCLE**

- Memorize all values on the unit circle
- Trig identities (Pythagorean, reciprocal, quotient, double angle)
- Solve trig equations
- Evaluate inverse trig functions

## **GRAPH READING**

- Determine on what  $x$ -interval(s) a function is increasing, decreasing, or constant
- Determine where the local max/min of  $f(x)$  occur and know that they are  $y$  values
- Determine where the absolute max/min of  $f(x)$  occur and know that they are  $y$  values
- Find the real zeroes of a function graphically

## **CALCULATOR BASICS** (You should have a TI-84 or TI-84 Plus (Silver Edition) calculator ONLY)

- Graph a function
- Understand the difference between negative and minus on your calculator
- Switch between degree and radian mode
- Reset your calculator
- Graph on the standard viewing window
- Adjust the viewing window manually to see a comprehensive graph
- Evaluate a function at a point using trace
- Create a table
- Find points of intersection
- Find zeroes
- Store values

## GEOMETRY

- SohCahToa
- Pythagorean Theorem
- Area formulas for a square, rectangle, triangle, equilateral triangle, isosceles right triangle, trapezoid, circle
- Volume formulas for a cube, rectangular prism
- Find perimeter
- Similar triangles (understand ratios)

## BC ONLY REVIEW TOPICS

### SERIES

- Define a sequence
- Determine if a sequence converges or diverges
- Define a series (finite vs. infinite)
- Define partial sum
- Understand summation (sigma) notation
- Understand convergence vs. divergence of an infinite series
- Recognize a geometric series
- Find the sum of a geometric series if possible

### POLAR/VECTORS/PARAMETRIC EQUATIONS

- Plot polar coordinates
- Graph polar equations on a calculator
- Find polar zeroes (by hand)
- Know the horizontal and vertical component of a vector
- Define a parameter in a set of parametric equations

### WARNINGS! These are common student mistakes.

Cannot distribute an exponent over addition or subtraction  $(x+2)^2 \neq x^2 + 4$

Cannot distribute a root over addition or subtraction  $\sqrt{x^2 + 4} \neq x + 2$

Cannot cancel when addition or subtraction is involved like this  $\frac{\cancel{x} + 1}{\cancel{x}}$  or  $\frac{\cancel{\sin} x + \cos x}{\cancel{\sin} x}$

$$x^2 = x$$

$$\frac{x^2}{x} = \frac{x}{x}$$

Cannot divide or multiply an equation by an  $x$  or you lose/gain solutions.

$$x = 1$$

$$\{1\}$$

Cannot “cancel”  $e$  and  $\ln$  if they are not right next to each other (composed.)  $e^{2\ln x} \neq 2x$

**Part 1: STRONGLY RECOMMENDED Practice Problems**

You should begin the habit of showing all work. Mental math is great for double checking but NOT acceptable in this class so get used to writing every single step. Work VERTICALLY whenever possible.

**ARITHMETIC AND ALGEBRA BASICS**

Simplify by applying the properties of exponents. Do NOT leave negative exponents.

1.  $-8x^{-2}$

2.  $(-5x^3)^2$

3.  $\left(\frac{-3}{x^4}\right)^{-2}$

4.  $(36x^{10})^{1/2}$

5.  $(27x^3)^{-2/3}$

6.  $(16x^{-2})^{3/4}$

$$7. \frac{-2}{3}(8x)^{-5/3} (8)$$

$$8. \frac{(x+4)^2}{(x+4)^4(x-4)^{-1/2}}$$

Simplify by factoring out the greatest common factor. Then combine like terms when appropriate.

$$9. (x-1)^3(2x-3) - (2x+12)(x-1)^2$$

$$10. (2x^2)(x^2-1)^2 - 8x^3(x^2-1)$$

$$11. \frac{(x-1)^3(2x-3) - 2(x-1)(3)}{(x-1)^4}$$

Divide by multiplying by the reciprocal. Then simplify.

$$12. \frac{\frac{\sqrt{x}}{3}}{\frac{1}{x^2}}$$

$$13. \frac{\frac{2}{x}}{x-1}$$

$$14. \frac{\frac{x^2-1}{x}}{\frac{x+1}{x^3}}$$

Combine by finding common denominators

$$15. \frac{1}{x} - x$$

$$16. \frac{2}{x-3} + \frac{3}{x+3}$$

$$17. \frac{4}{x} - \frac{2}{x+1}$$

## FUNCTIONS

Given  $f(x) = x^3 + 1$ ,  $g(x) = \sqrt{x}$ , and  $h(x) = 2x - 3$ , find each of the following,

1.  $f(-2) + g(4)$

2.  $f(h(2))$

3.  $f(g(9))$

4.  $h(x+1)$

5.  $g(x + \Delta x)$

6.  $h(g(x))$

7.  $g(f(x))$

8.  $f(h(g(4)))$

9.  $h(x) - h(2)$

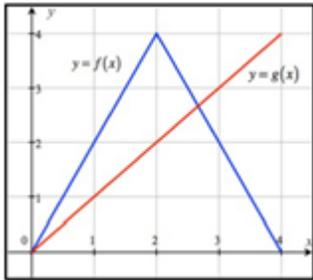
10. Write the domain of each in interval notation

a.  $f(x)$

b.  $g(x)$

c.  $g(h(x))$  [This one is tricky]

The graphs of  $f(x)$  and  $g(x)$  are given below. Find each of the following based on the graphs.



11.  $g(f(1))$

12.  $(f - g)(3)$

13.  $f(g(3))$

14. For what value of  $k$  does the graph of  $y = kx^3$  pass through the point?

a) (1,4)

b) (-2,1)

The following is a table of values for the function  $f(x)$ .

$x$	-1	0	2	3	5	6
$f(x)$	5	3	4	1	0	7

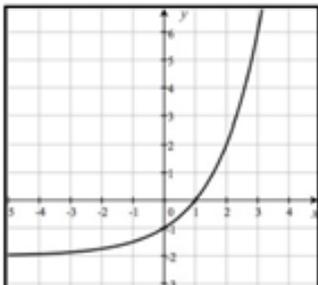
15. Find  $f(2)$

16. Find  $f(0)+f(6)$

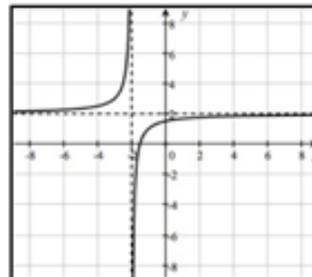
17. Find  $f(f(0))$

Write the domain and range of each of the following functions using interval notation.

18. (This function has a horizontal asymptote at  $y = -2$ )



19. (This function has a horizontal asymptote at  $y = 1$  and vertical asymptote at  $x = -1$ )



The following are examples of a compositions of function. Describe which function in “inside” the other.

Ex. Given  $\sqrt{x+2}$   $x+2$  is “inside”  $\sqrt{x}$

20.  $\ln(x+3)$

23.  $(x^2-1)^3$

21.  $\sin(x^2+x)$

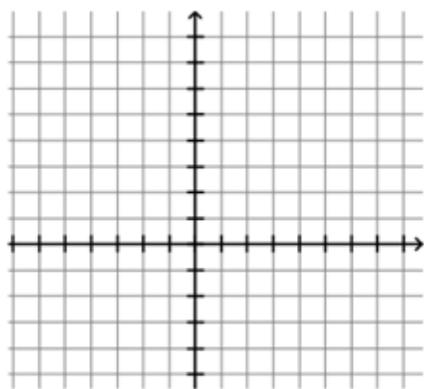
24.  $\sqrt{\ln x}$

22.  $\cos^2 x$

25.  $e^{\cos x}$

Sketch each function and label the axes and tick marks appropriately. Describe, in words, the translations and reflections. State the domain and range in interval notation.

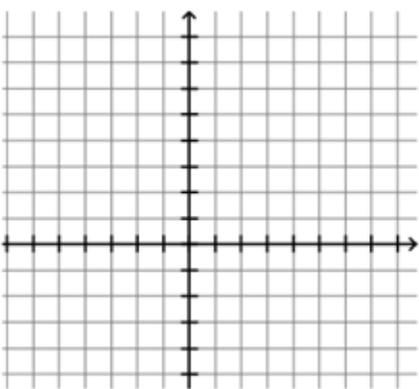
1.  $y = e^x - 3$



D: \_\_\_\_\_

R: \_\_\_\_\_

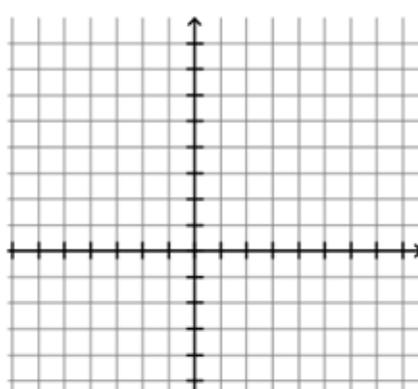
2.  $y = \ln(x+4)$



D: \_\_\_\_\_

R: \_\_\_\_\_

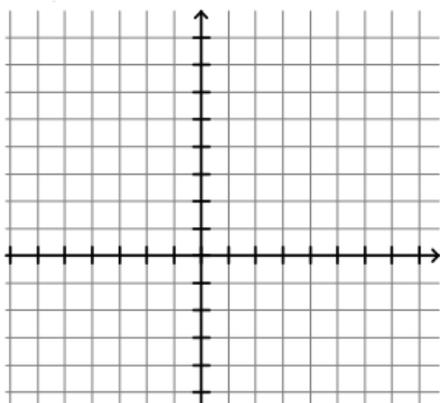
3.  $y = \sqrt{x-4} + 3$



D: \_\_\_\_\_

R: \_\_\_\_\_

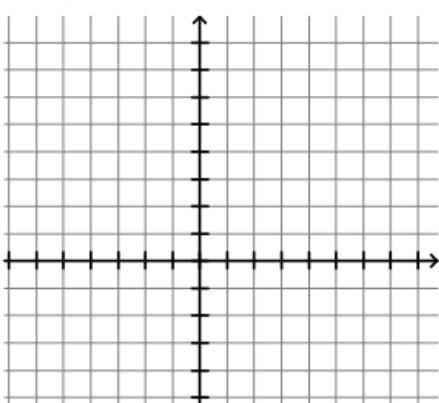
4.  $y = -(x+1)^2 + 2$



D: \_\_\_\_\_

R: \_\_\_\_\_

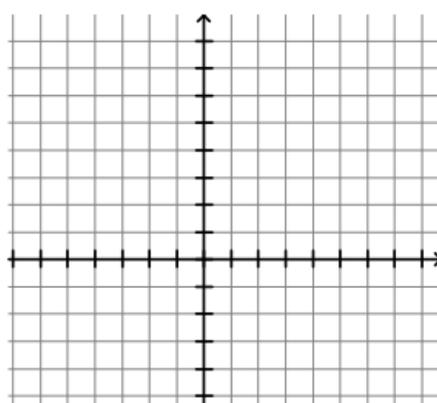
5.  $y = \tan^{-1} x$



D: \_\_\_\_\_

R: \_\_\_\_\_

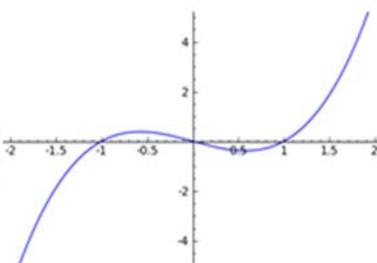
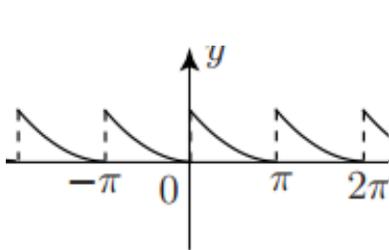
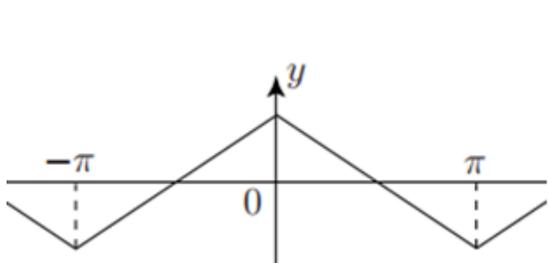
6.  $y = |x-2| + 3$



D: \_\_\_\_\_

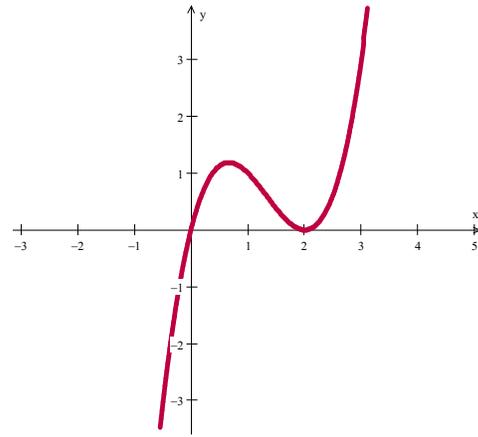
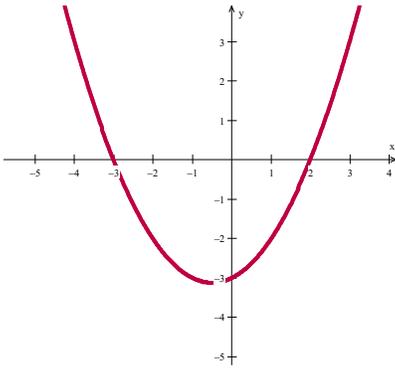
R: \_\_\_\_\_

Determine whether the given functions are even, odd, or neither. If even or odd, explain why based on the graph.



Sketch  $y = |f(x)|$  on the same graph as  $f(x)$  below.

Use a colored pen to draw entire graph.

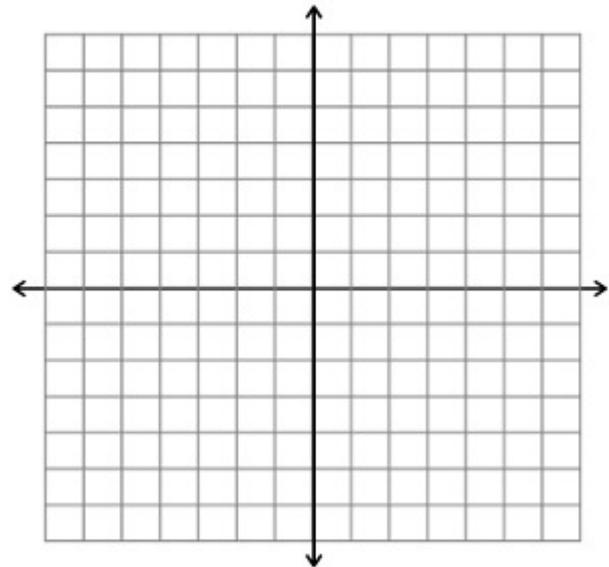
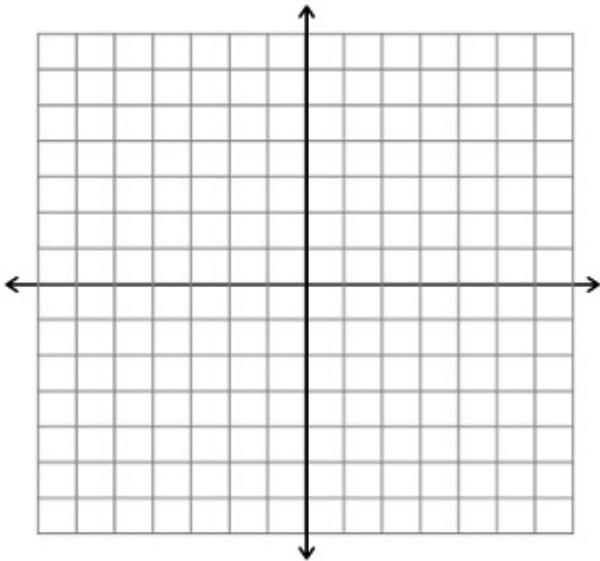


Sketch  $y = |f(x)|$  on the same graph as  $f(x)$  below.

Use a colored pen to draw entire graph.

Sketch  $y = |x^2 - 4|$

Sketch  $y = |1 - x^2|$



Describe, in a complete sentence, what putting absolute values bars around a function does to the graph of the function.

Find the  $x$ -intercept(s) and  $y$ -intercept of each function. Be sure to write your answers as **coordinate points**.

1.  $y = 2x - 5$

2.  $2x - 3y = 6$

3.  $y = x^2 - x - 2$

4.  $y = x^2 - 5x$

5. Explain why the graph of a function can have more than one  $x$ -intercept but can have only one  $y$ -intercept.

6. Given the equation  $f(x) = x^3 - x$

a) Is the point (3,2) on the graph of  $f(x)$ ? Why or why not?

b) Is the point (2,6) on the graph of  $f(x)$ ? Why or why not?

c) Find the  $y$ -intercept.

d) Find the  $x$ -intercept(s).

### **LINEAR FUNCTIONS AND EQUATIONS**

Find the equation of the line in **point-slope form** with the given slope passing through the given point.

1.  $m = -2, (0, 4)$

2.  $m = 7, (-3, 5)$

3.  $m = 0, (2, 4)$

Find the equation of the line in **point-slope form** passing through the following points

4.  $(-3, -6)$  and  $(-1, 2)$

5.  $(-1, 2)$  and  $(-1, -2)$

Write an equation of the line in **slope-intercept form** that passes

6. through  $(-5, 3)$  and parallel to  $x + y = 4$

7. through  $(0, 4)$  and perpendicular to  $2x - y = 4$

8. through  $(-3, -4)$  and parallel to  $y = 2$

9. through  $(-3, -4)$  and normal to  $y = 2$

10. Find  $k$  if the lines  $3x - 5y = 9$  and  $2x + ky = 11$  are

a. Parallel

b. Perpendicular

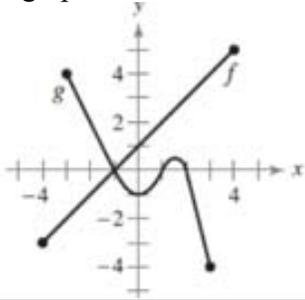
**POINTS OF INTERSECTION (Non-calculator)**

Find the point(s) of intersection *analytically* given the following equations. Then graph the pair of equations to confirm your solutions.

1.  $x - y = 2$   
 $x + y = 6$

2.  $y = 2x + 8$   
 $y = x^2$

3. Use the graphs below to answer the following questions



a) Identify the domains and range of  $f$  and  $g$

b) Identify  $f(-2)$  and  $g(3)$

c) For what  $x$  value(s) is  $f(x) = g(x)$  ?

d) Estimate the solution(s) of  $f(x) = 2$

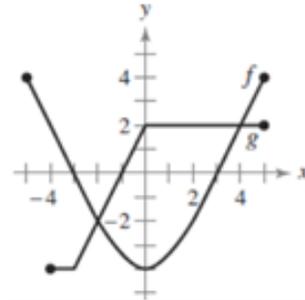
e) Estimate the solution(s) of  $g(x) = 0$

4. In general, explain what you are looking for graphically when solving each of the following

a.  $f(x) = g(x)$

b.  $f(x) > 0$

c.  $f(x) > g(x)$



a) Identify the domains and range of  $f$  and  $g$

b) Identify  $f(-2)$  and  $g(3)$

c) For what  $x$  value(s) is  $f(x) = g(x)$  ?

d) Estimate the solution(s) of  $f(x) = 2$

e) Estimate the solution(s) of  $g(x) = 0$

## PIECEWISE FUNCTIONS

1. If  $f(x) = \begin{cases} x^2 - 3 & x \geq 0 \\ 2x + 1 & x < 0 \end{cases}$  Find

a)  $f(5) =$

b)  $f(2) - f(-1) =$

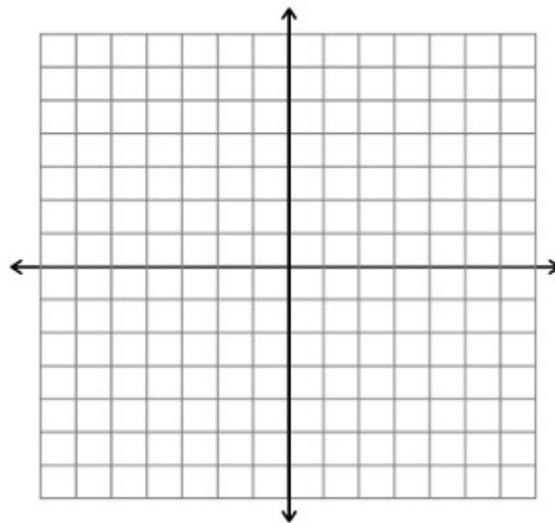
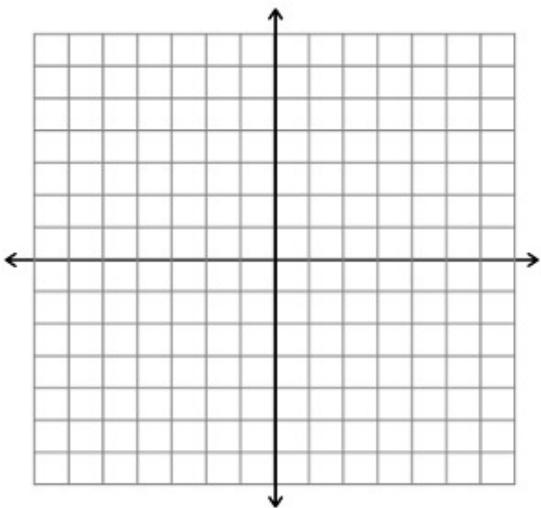
c)  $f(f(1)) =$

d) Can you draw the graph of  $f(x)$  without lifting your pencil? What is this formally called?

2. Graph each of the following piecewise functions

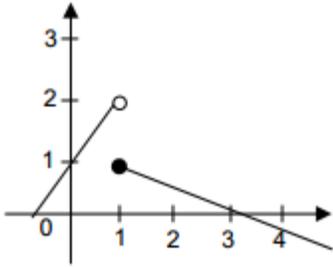
$$f(x) = \begin{cases} x^2 - 1 & x < 1 \\ 4 - x & x \geq 1 \end{cases}$$

$$f(x) = \begin{cases} -x^2 & x < 1 \\ 2 & x = 1 \\ x - 2 & x > 1 \end{cases}$$

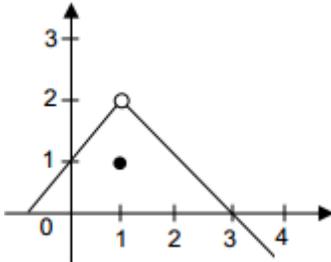


Write the piecewise equation for the following graphs of  $f(x)$ .

3.



4.



### INVERSE FUNCTIONS

- If the graph of  $f(x)$  has the point  $(2,7)$  then what is one point that will be on the graph of  $f^{-1}(x)$ ? Why?
- Explain how the **graphs** of  $f(x)$  and  $f^{-1}(x)$  compare.

Simplify the following expressions, if possible.

3.  $\ln e^3$

4.  $\ln e$

5.  $\ln e^x$

6.  $e^{\ln 5}$

7.  $e^{2\ln x}$

8.  $\ln 1 - \ln e$

9.  $\ln x + \ln x$

10.  $\ln(6) + \ln(x) - \ln(2)$

13.  $f^{-1}(3)$

Given  $f(x) = x^3 - 1$  Find

14.  $f^{-1}(0)$

15.  $f^{-1}(7)$

$x$	-1	0	2	3	4	5
$f(x)$	5	3	4	1	0	7

11.  $f^{-1}(5)$

12.  $f^{-1}(0)$

## FACTORING

Factor each of the following expressions. Do NOT solve them. *You must understand the difference.*

1.  $x^2 - 8x$

9.  $5x^2 - 6x + 1$

2.  $-3x^2 - 6x$

10.  $x^2 - 81$

3.  $x^2 - 8x + 15$

11.  $2x^2 - x - 3$

4.  $x^2 + 8x + 16$

12.  $4x^2 - 7x - 2$

5.  $x^2 - 9x - 10$

13.  $x^3 - 4x$

6.  $x^2 - 5x + 6$

14.  $x^4 - 81$

7.  $x^2 + 5x - 6$

15.  $x^2 - 14x + 49$

8.  $6x^2 + x - 40$

16.  $9x^2 - 30x + 25$

## SOLVING EQUATIONS AND INEQUALITIES

Solve the following equations analytically.

1.  $4 - 2x = 0$

2.  $x^2 - x - 2 = 0$

3.  $(x - 3)(x + 3) = 0$

4.  $6x^2 - 2x = 0$

5.  $-x^2 - 2x + 8 = 0$

6.  $2x^2 + 5x - 3 = 0$

Solve the following equations GRAPHICALLY. Your answers should be intervals.

7.  $x^2 - x - 2 > 0$

8.  $(x - 3)(x + 3) \leq 0$

9.  $x^2 - 16 \geq 0$

10.  $2x - 4 < 0$

11.  $-x^2 - 2x + 8 < 0$

12. Explain the difference between factoring an expression and solving an equation.

### RATIONAL FUNCTIONS

Determine the following statements are true or false. If false, explain why it is false or provide a counterexample.

1. \_\_\_\_\_ The function  $y = \frac{\sin x}{x^2 - 3x - 4}$  is a rational function
2. \_\_\_\_\_ The graph of a rational function always has a vertical asymptote
3. \_\_\_\_\_ If a rational function has a vertical asymptote at  $x=3$  then the factor  $(x-3)$  must appear in its numerator
4. \_\_\_\_\_ The graph of a rational function sometimes has a hole.
5. \_\_\_\_\_ The graph of a rational function never intersects a vertical asymptote
6. \_\_\_\_\_ The graph of a rational function never intersects a horizontal asymptote

Find all vertical and horizontal asymptotes and the coordinates of any holes in the graph.

7.  $y = \frac{x-1}{x+5}$

8.  $y = \frac{8}{x^2}$

9.  $y = \frac{2x+16}{x+8}$

10.  $y = \frac{2x^2+6x}{x^2+5x+6}$

11.  $y = \frac{x}{x^2-25}$

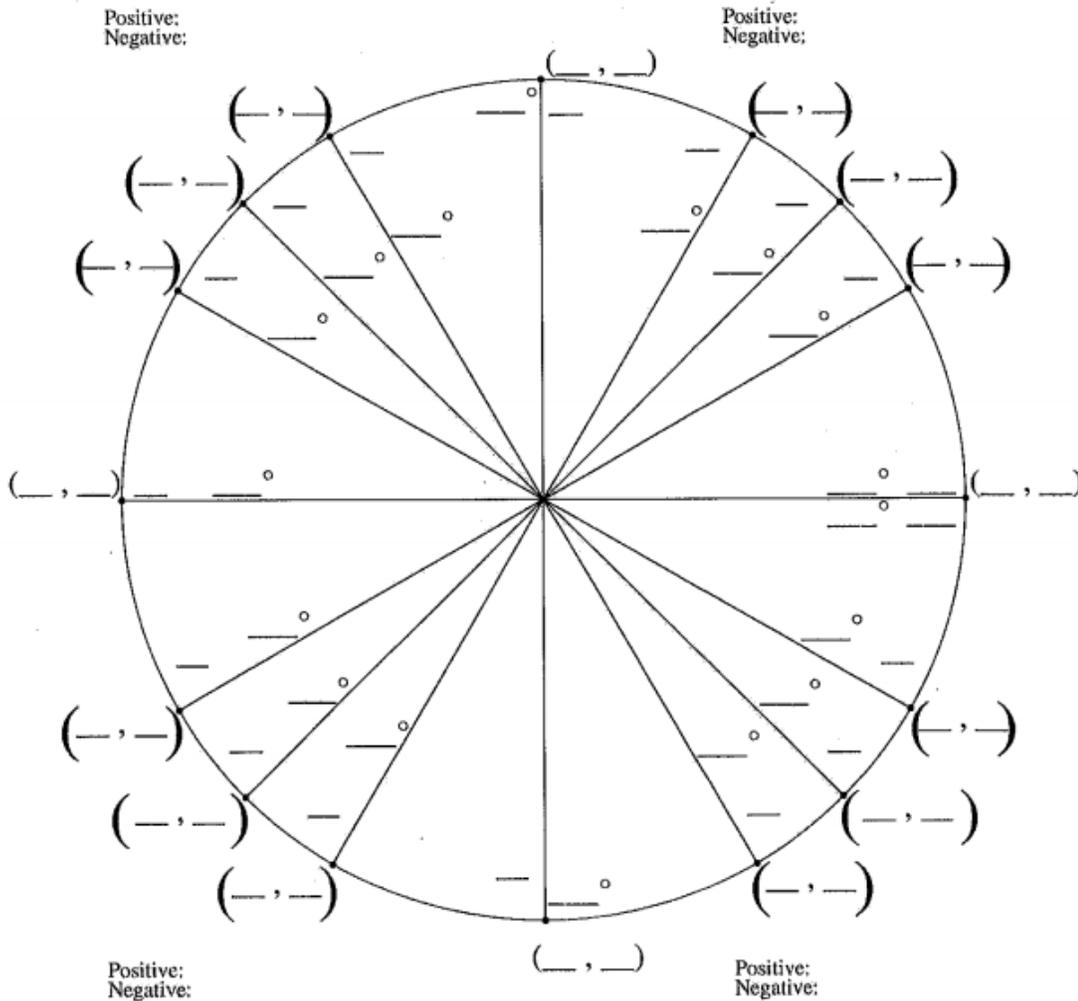
12.  $y = \frac{5x+1}{x^2-x-6}$

13.  $y = \frac{x^3+4x}{x^3-2x^2+4x-8}$  Factor the denominator by grouping.

14.  $y = \frac{1}{x} - \frac{x}{x+2}$  (Hint: Express with a common denominator)

TRIGONOMETRY AND THE UNIT CIRCLE

# Fill in The Unit Circle



Fill in the following chart for quadrant 1 of the unit circle.

$\theta$ Radians/Degree	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
$0 = 0^\circ$			
$\frac{\pi}{6} = 30^\circ$			
$\frac{\pi}{4} = 45^\circ$			
$\frac{\pi}{3} = 60^\circ$			
$\frac{\pi}{2} = 90^\circ$			

1-24 Use your knowledge of the unit circle to evaluate the following trigonometric functions. Leave answers in radical form. You will need to be able to evaluate these *without* using the unit circle. In other words, memorize them.

$$\sin \pi$$

$$\cos \frac{3\pi}{2}$$

$$\sin \left( -\frac{\pi}{2} \right)$$

$$\sin \left( \frac{5\pi}{4} \right)$$

$$\cos \frac{\pi}{4}$$

$$\cos(-\pi)$$

$$\cos \frac{\pi}{3}$$

$$\sin \frac{5\pi}{6}$$

$$\cos \frac{2\pi}{3}$$

$$\tan \frac{\pi}{4}$$

$$\tan \pi$$

$$\tan \frac{\pi}{3}$$

$$\cos \frac{4\pi}{3}$$

$$\sin \frac{11\pi}{6}$$

$$\tan \frac{7\pi}{4}$$

$$\sin \left( -\frac{\pi}{6} \right)$$

$$\sin(30^\circ)$$

$$\cos \frac{\pi}{2}$$

$$\tan 45^\circ$$

$$\tan \frac{\pi}{6}$$

$$\cos \pi$$

$$\tan 0$$

$$\csc \frac{5\pi}{6}$$

$$\cos \frac{3\pi}{4}$$

Evaluate each of the following inverse trig functions. Write your answer in radians. (There should be only one answer.)

25.  $\cos^{-1}\left(\frac{1}{2}\right)$

26.  $\arcsin\left(\frac{\sqrt{2}}{2}\right)$

27.  $\tan^{-1}(1)$

Solve each trigonometric equation for  $0 \leq x \leq 2\pi$  (These are important.)

28.  $\sin x = \frac{-1}{2}$

29.  $2 \cos x = \sqrt{3}$

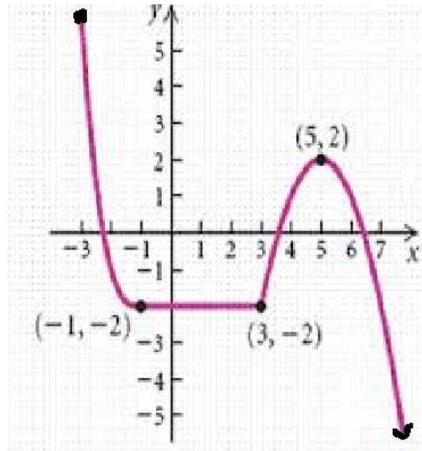
30.  $4 \sin^2 x = 3$  Hint:  $\sin^2 x = (\sin x)^2$  and  $x^2 = 9 \Rightarrow x = \pm 3$

31. Which of the following expressions are identical?

$(\sin x)^{-1}$        $\arcsin(x)$        $\sin x^{-1}$        $\frac{1}{\sin x}$

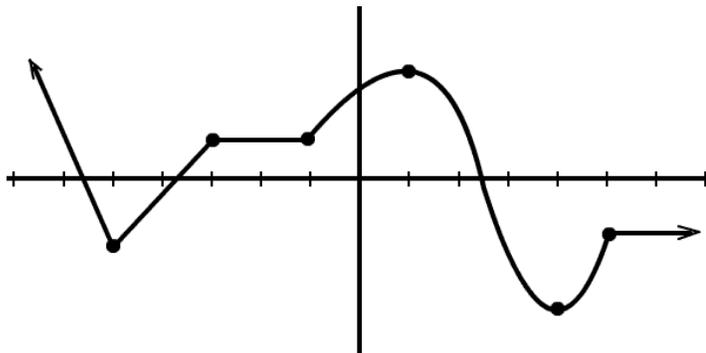
**GRAPH READING**

The following is the graph of  $f$ . Identify the  $x$  intervals where the  $f$  is



1. Increasing
2. Decreasing
3. Constant

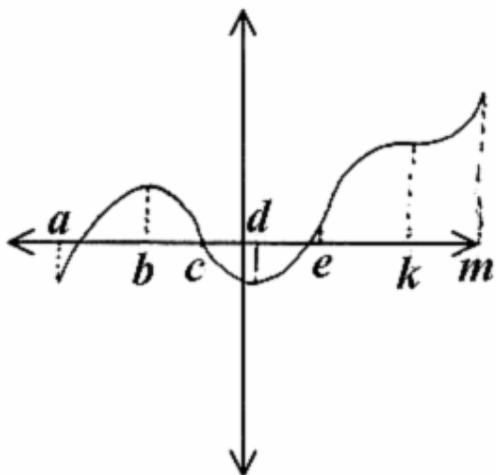
Identify the  $x$  intervals where the function  $f$  shown below is



4. Increasing
5. Decreasing
6. Constant

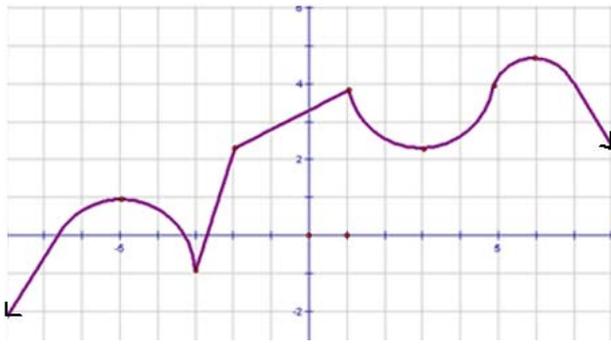
Each tick mark is one unit.

Identify the  $x$  intervals where the function  $f$  shown below is



7. Increasing
8. Decreasing
9. Constant

The following is the graph of  $f$ .



Each tick mark is one unit.

10. Find all local maximum(s) of  $f$

11. Approximate all local minimum(s) of  $f$

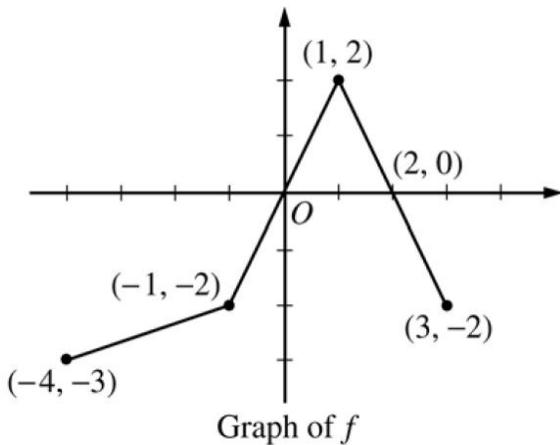
12. Find the absolute maximum of  $f$  (if any)

13. Find the absolute minimum of  $f$  (if any)

14. Find the  $x$ -interval(s) where  $f$  is increasing

15. Find the  $x$ -interval(s) where  $f$  is decreasing

The following is the graph of  $f$ .



16. Find all local maximum(s) of  $f$

17. Find all local minimum(s) of  $f$

18. Find the absolute maximum of  $f$  (if any)

19. Find the absolute minimum of  $f$  (if any)

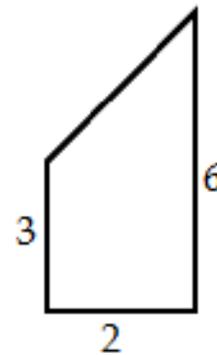
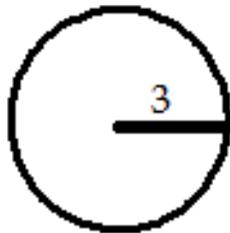
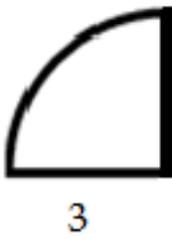
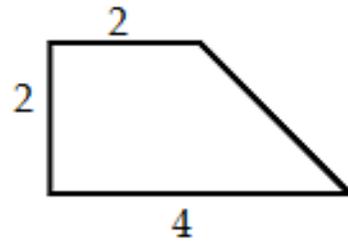
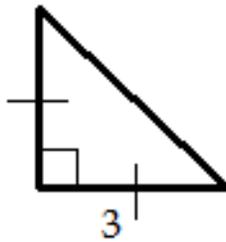
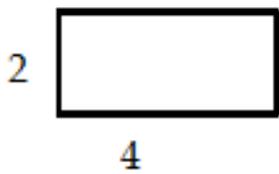
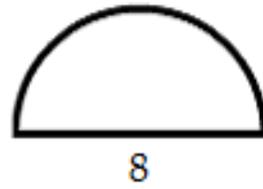
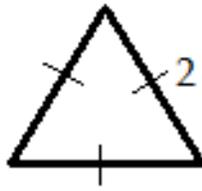
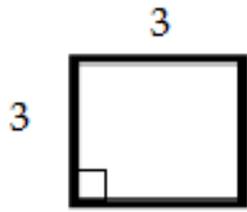
20. Find the  $x$ -interval(s) where  $f$  is increasing

21. Find the  $x$ -interval(s) where  $f$  is decreasing

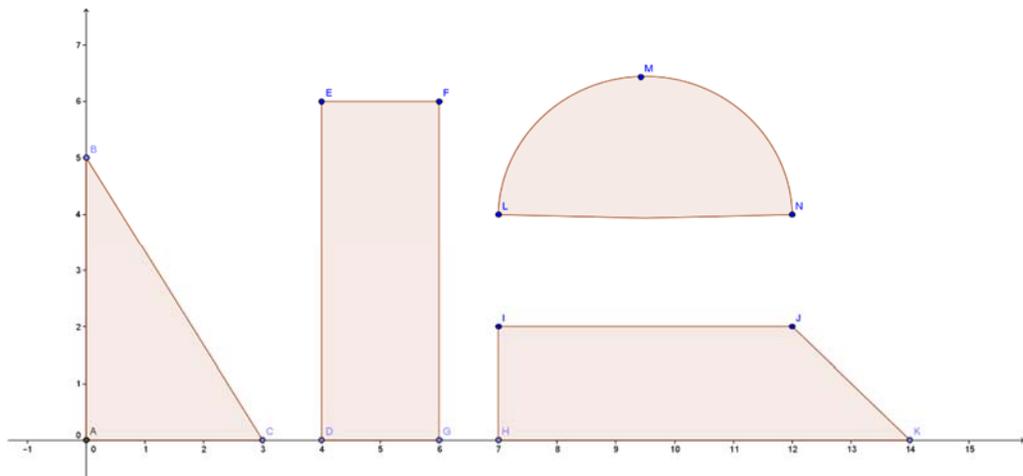
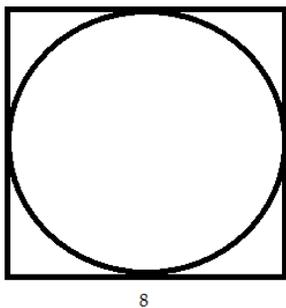
22. Find the zeroes of  $f$

**GEOMETRY-AREA/ /VOLUME/SIMILAR FIGURES**

1-9. Find the area of the given shape. All linear units are in inches. Include appropriate units for answers. Leave your answers in term of pi where appropriate; do not write decimal answers.



10-14. Find the area inside the square & outside the circle and then find the area of each shape on the  $xy$ -plane and write answer in the shape.



Find the volume of each of the following. Include appropriate units. Leave answers exact; do not leave decimal answer.

15. Cube with side length 3 ft

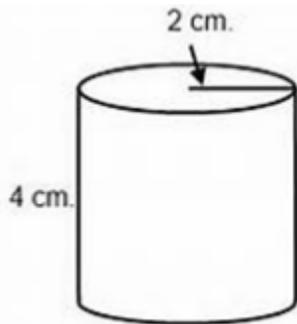
16. Rectangular prism with square base of side 10 in and height 12 in

17. Sphere with diameter 8 in

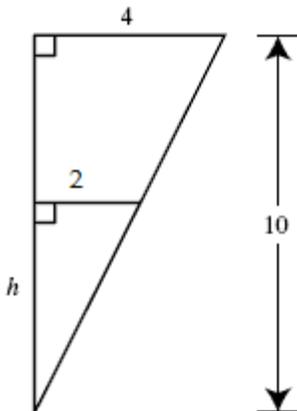
18. Cylinder with radius 2 m and height that is double the diameter.

19. Circular cone with radius 4 cm and height 6 cm

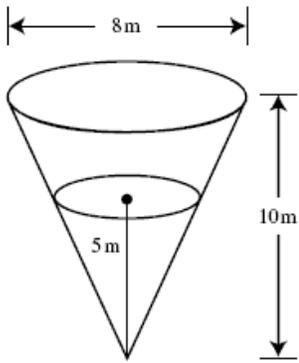
20. Find the volume of the cylinder shown.



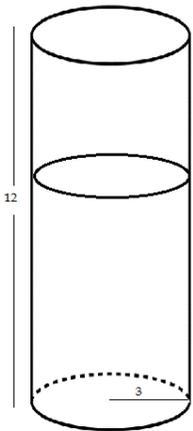
15. Find the area of the smaller triangle



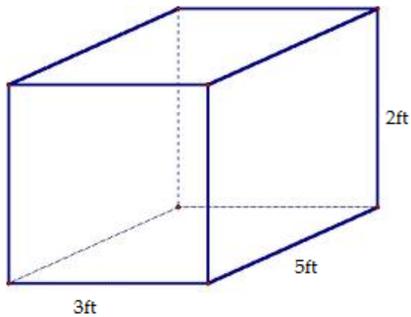
16. The following cone is filled with water 5m high. Find the volume of water in the cone.



17. The following cylinder is two-thirds full with liquid. Find the volume of the liquid.



18. The following bathtub is being filled with water. How much water is in it when it is half full?



19. Explain the difference between units for area and volume. Provide a few examples of each.