Math Analysis/Honors Math Analysis Summer Assignment

To be successful in Math Analysis or Honors Math Analysis, a full understanding of the topics listed below is required prior to the school year. To ensure you are proficient in this material, you are strongly advised to complete the attached summer assignment. You should work on it over the course of the few weeks leading up to school to ensure you are prepared for rigor of this class. It is NOT going to be collected by your teacher, however, **you will be assessed in the form of a quiz or test on this material during the first week of school**.

If you have questions on the summer assignment or any of the following topics, it is recommended you

- Ask a teacher for help
- Ask a friend for help (but work should not be identical)
- Search through old notes or textbooks
- Look up review videos by searching the topic name (Recommended sites: patrickjmt.com and mathispower4u.yolasite.com)

Vocabulary

You should be able to define the following terms…

Sum, difference, product, quotient, factor, prime, composite, *x*-intercept, *y*-intercept, reciprocal, exponent, base, equation, inequality, expression, slope, linear, quadratic, parabola, absolute value, undefined, coefficient, term,

Arithmetic Basics

You should be able to…

Add, subtract, multiply, and divide fractions

Apply the order of operations

Find the greatest common factor

Clear fractions by multiplying both sides of an equation by the least common denominator

Convert between improper fractions and mixed numbers

Convert between fractions and decimals

Know that $\frac{0}{real\#}=0$ and $\frac{real\#}{0}=undefined$

Apply properties of fractions $\frac{a\pm b}{c}=\frac{a}{c}\pm\frac{b}{c}$

Understand exponents and have memorized up to $2^6, 3^5, 4^3, 5^3$

Apply properties of exponents $a^{m+n}=a^m a^n$, $\left(a^m\right)^n=a^{mn}$

Understand fractional exponents $\sqrt[n]{a}=a^{1/n}$ $\sqrt[n]{a^m}=a^{m/n}$

Understand negative exponents $a^{-m}=\frac{1}{a^m}$

Algebra Basics

You should be able to…

Explain the concept of a function

Label an *xy*-coordinate plane

Solve a linear system of equations using substitution, elimination, and graphing

Graph a function by creating a table and plotting points

Know the equations for a vertical vs. horizontal line

Find the slope between two points (including zero slope and undefined slope)

Know when two lines are parallel or perpendicular

Give a linear in slope-intercept or point-slope form

Understand graphically when slope is positive (rising from left to right) or negative (falling from left to right)

Recognize and graph a linear function

How to find *x* and *y* intercepts
Factor a GCF from a polynomial, factor quadratics
Understand the difference between factor and solve
Understand the difference between an expression and an equation
State and apply the quadratic formula
Combine rational expressions
Determine if a given value is a solution to an equation or inequality

Geometry Basics
You should be able to…
State the formula for area and perimeter of a circle, rectangle, and triangle
State the formula for volume of a rectangular prism

Calculator Basics*
You should be able to…
Graph a function
Understand the difference between negative and minus on your calculator
Switch between degree and radian mode
Reset your calculator
Graph on the standard viewing window
Adjust the viewing window manually to see a comprehensive graph
**You should have a TI-84 or TI-84 Plus (Silver Edition) calculator ONLY*

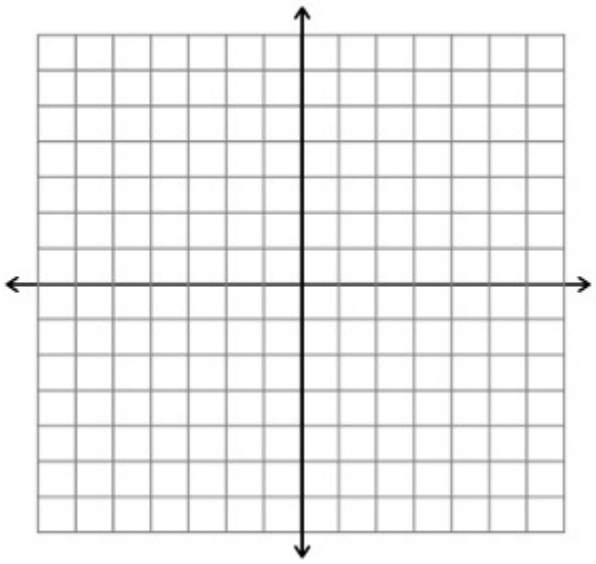
What NOT to do!
The following are examples of "rules" frequently made up by students. In other words, you can NEVER ever perform the following operations:

NO!	**What should you do?**
$(x+2)^2 = x^2+4$	Distribute! $(x+2)^2 = (x+2)(x+2) = x^2+4x+4$
$\sqrt{x^2+4} = x+2$	Nothing! This cannot be simplified.
$\frac{\cancel{x}+1}{\cancel{x}}$ or $\frac{\cancel{\sin x}+\cos x}{\cancel{\sin x}}$	Nothing! These do not cancel.
$x^2 = x$ $\frac{x^2}{x} = \frac{x}{x}$ (cannot divide by x) $x = 1$ $\{1\}$	$x^2 = x$ $x^2 - x = 0$ $x(x-1) = 0$ $x = 0, x = 1$ $\{0,1\}$

GRAPHICAL REPRESENTATION OF DATA

- You should be able to plot points on the coordinate axis.
- You should know that the the midpoint of the line segment joining (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
- You should know that the distance between (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- You should know the equation of a circle: $(x-h)^2 + (y-k)^2 = r^2$.
- You should be able to construct scatter plots, bar graphs and line graphs for a set of data.

Plot the point and label which quadrant each point is located.



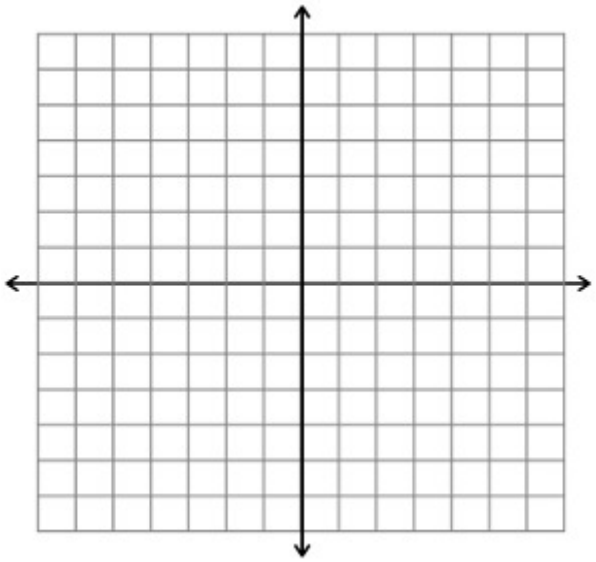
1. A. (4,-3)
2. B. (2,1)
3. C. (-5,-2)
4. D. (-3,5)
5. E. (4,0) (Hint: can't list more than one quadrant)

Determine the quadrant(s) in which the points (x,y) are located so that the conditions are satisfied.

6. $x>0$ and $y = -2$
7. $xy<0$
8. $xy=4$ and $xy>0$

Plot the points and draw the line segment between them. Then find the distance between the points.

9. (-3,5) and (1,7)



10. (-4,0) and (0,5)

Write the equation of the circle in standard form given the following information. You may want to sketch the circle to help determine the equation.

11. Center (3,-1) and a point on the circle (-5,1)

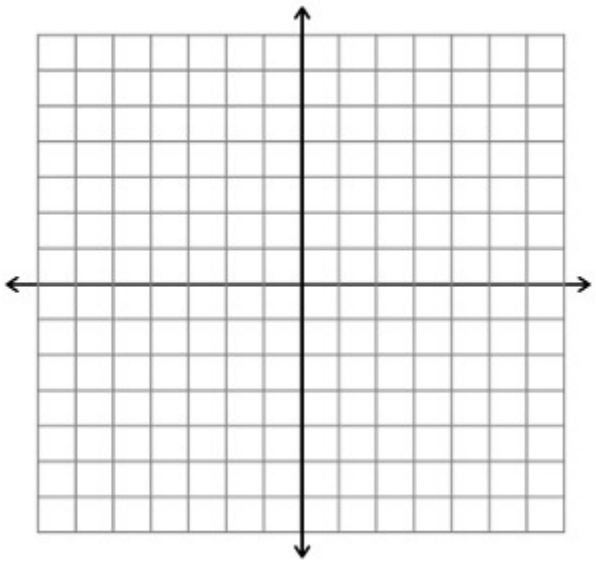
12. End points of the diameter: (-4,6) and (10,-2)

GRAPHING

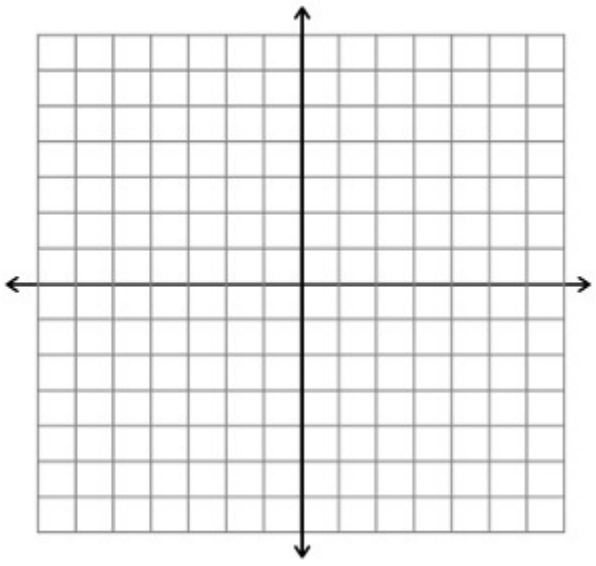
- You should be able to use the point-plotting method of graphing.
- You should be able to find x- and y-intercepts.
  a. To find the x-intercept, let y=0 and solve for x.
  b. To find the y-intercept, let x=0 and solve for y.
- You should know how to graph an equation with a graphing utility. You should be able to determine an appropriate viewing rectangle.
- You should be able to use the zoom and trace features of a graphing utility.

For each of the following, create a table with at least 5 points *(x,y)*. Choose *x* values strategically to avoid fractional answers. Then plot the points and sketch a graph of the equation. Label the tick marks appropriately. You may need to adjust the scale to draw a better graph.

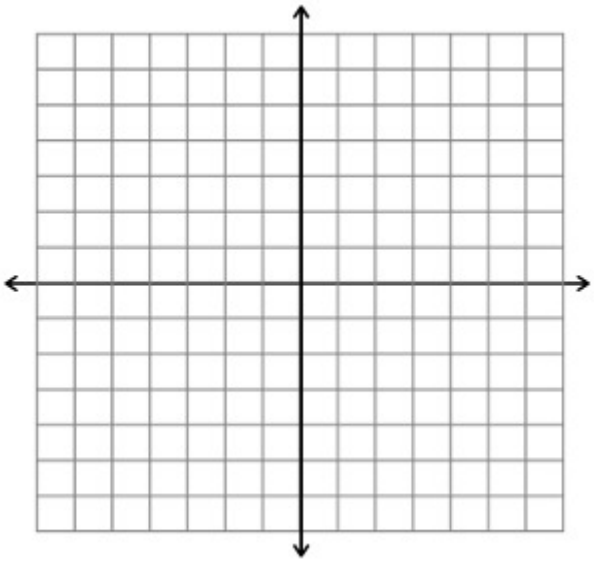
13. $y = \frac{-1}{2}x + 2$



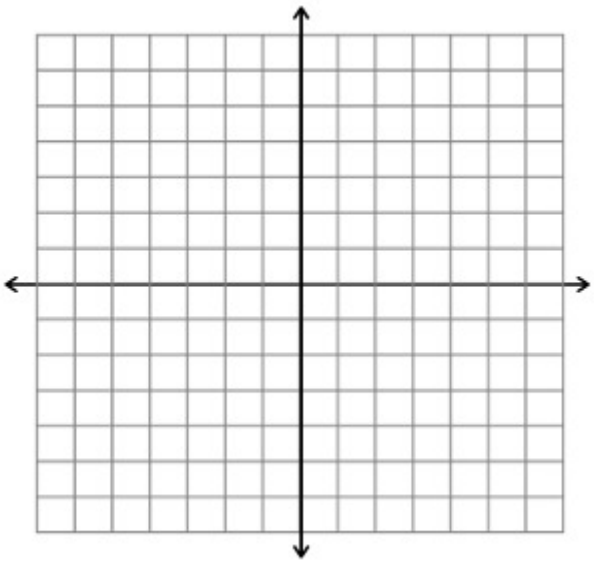
14. $2x - 3y = 6$



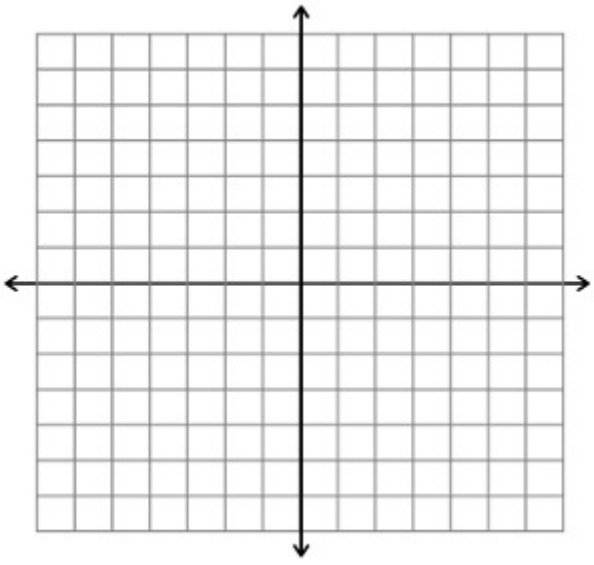
15. $y = x^2 - 3x$



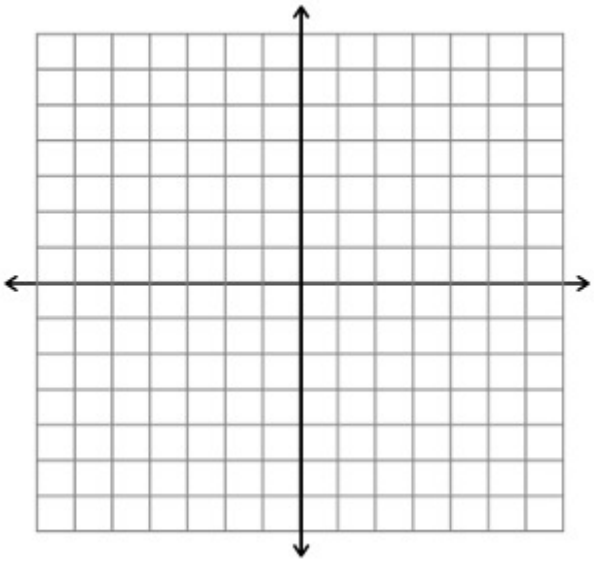
16. $y = x^3$



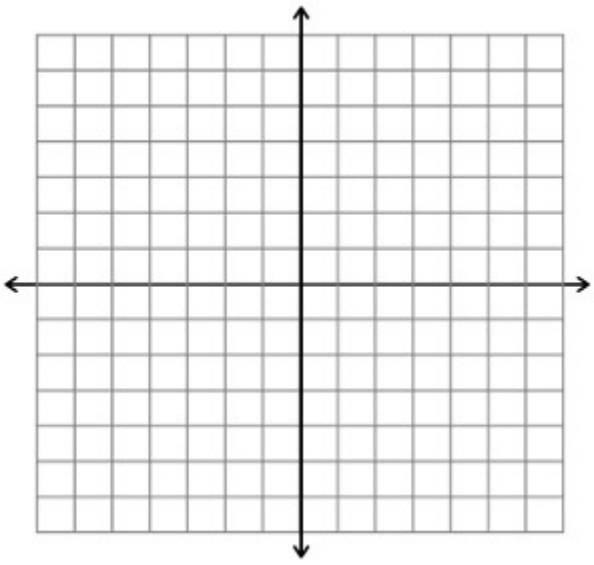
17. $y = 3 - |x|$



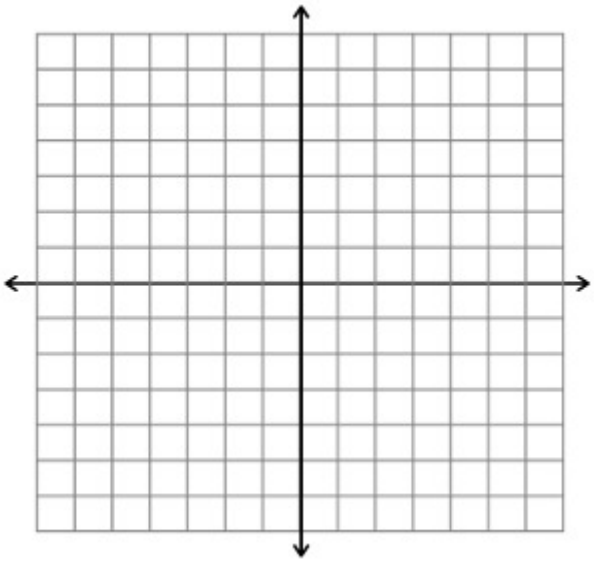
18. $y = \sqrt{5 - x}$



19. $y = -x^2 + 4$



20. $y = (x+1)^2$



LINEAR EQUATIONS

You should know the following important facts about lines.

- The graph of $y = mx + b$ is a straight line. It is called a linear equation.
- The slope of the line through $(x_1, y_1) and (x_2, y_2)$ is $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- If $m > 0$, the line rises from left to right.
- If $m < 0$, the line falls from left to right.
- If $m = 0$, the line is horizontal.
- If m is undefined, the line is vertical.
- Equations of Lines
  - Slope-Intercept: $y = mx + b$
  - Point-Slope: $y - y_1 = m(x - x_1)$
  - Two-Point: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
  - General: $Ax + By + C = 0$
  - Vertical: $x = a$
  - Horizontal: $y = b$
- Given two distinct non-vertical lines:
  - $L_1 = m_1 x + b_1$
  - $L_2: y = m_2 x + b_2$
  - L_1 is parallel to L_2 if and only if $m_1 = m_2$ and $b_1 \neq b_2$.
  - L_2 is perpendicular to L_2 if and only if $m_1 = \frac{-1}{m_2}$.

Find the slope of the line that passes through the given points.

21. (6,-1), (7,12)

22. $\left(\frac{3}{2}, 1\right), \left(5, \frac{5}{2}\right)$

23. $\left(\frac{-3}{4}, \frac{5}{6}\right), \left(\frac{1}{2}, \frac{-5}{2}\right)$

24. (-3,2), (8,2)

25. (4,1), (4, -2)

Find an equation of the line that passes through the given point and has the given slope. Write your answer in **slope-intercept form.**

26. Point (2,-1) , slope $m = \frac{1}{4}$

27. Point (0,3) , slope $m = \frac{-2}{3}$

28. Point (-2,6) , slope $m = 0$

29. Point (5,4) , slope m is undefined

Find an equation of the line that passes through the given points. Write your answer in **point-slope form.**

30. (2,-1), (4,-2)

31. (-1,0), (6,2)

32. (1,6), (4,2)

Write the equation of the line that satisfies the following conditions. You can leave your answer in any form.

33. Passes through (-2,1) and is parallel to $y = 3x - 2$

34. Passes through (-2,1) and is perpendicular to $y = 3x - 2$

35. Passes through (3,-2) and is parallel to $5x - 4y = 8$

36. Passes through (3,-2) and is perpendicular to $5x - 4y = 8$

37. Passes through (3,-4) and is parallel to $x = 4$

38. Passes through (3,-4) and is perpendicular to $x = 4$

FACTORING POLYNOMIALS

Factoring polynomials is the inverse process of multiplying polynomials. We are try to find "smaller" polynomials that when multiplied, get us back to the original polynomial. First try to find a GCF or recognize a special form like $a^2 - b^2 = (a-b)(a+b)$.

If given a general quadratic trinomial $ax^2 + bx + c$, use the following method:

1. Find the product ac.
2. Find two integers h and k such that $hk = ac$ AND $h + k = b$
3. Rewrite the quadratic as $ax^2 + hx + kx + c$
4. Group the two pairs of terms that have common factors and simplify.

$(ax^2 + hx) + (kx + c)$

$x(ax + h) + (kx + c)$

(Note: because of the way you chose h and k, you will be able to factor a constant out of the second parentheses, leaving you with two identical expressions in parentheses as in the examples).

Example:

Given:	$4x^2 + 7x - 15$
Find the product ac:	$(4)(-15) = -60$
Think of two factors of -60 that add up to 7:	-5 and 12
Write the $7x$ as the sum of $-5x$ and $12x$:	$4x^2 - 5x + 12x - 15$
Group the two pairs of terms:	$(4x^2 - 5x) + (12x - 15)$
Remove common factors from each group:	$x(4x - 5) + 3(4x - 5)$
Notice that the two quantities in parentheses are now identical. That means we can factor out a common factor of $(4x - 5)$:	$(4x - 5)(x + 3)$

For help on this topic, visit patrickjmt.com and look up "Factor by Grouping"

Factor the following polynomials completely, if possible. Do NOT solve. You should know the difference.

39. $x^2 - 9x$

40. $8m^3 + 4m$

41. $x^2 - 36$

42. $36 - x^2$

43. $16x^2 - 81$

44. $4x^3 - 64x^2$

45. $m^2 - 25$

46. $2y^2 - 98y$

47. $4x^2 + 5x + 1$

48. $9x^2 + 15x + 4$

49. $x^3 - 2x^2 - 4x + 8$

50. $3x^3 + 6x^2 + x + 2$

51. $6x^2 + 21x + 9$

52. $7x^2 + 28x + 21$

53. $8x^3 - 48x^2 + 72x$

54. $3x^2 + 3x - 6$

55. $3x^2 - 11x - 20$

56. $2x^2 - 5x + 2$

57. $6x^2 + 2x - 2$

58. $8x^2 + 10x - 3$

59. $10x^2 + 21x - 10$

60. $10x^2 - 17x - 6$

61. $15 - 2x - x^2$

SIMPLIFYING RATIONAL EXPRESSIONS

Simplify the following expressions. You may need to factor.

62. $\dfrac{28x^3}{35x^5}$

63. $\dfrac{5x+40}{4x+32}$

64. $\dfrac{x^2+12x+20}{3x+6}$

65. $\dfrac{6x+30}{x^2+8x+15}$

66. $\dfrac{5x-15}{x^2-3x}$

67. $\dfrac{16k^2\sqrt{mn}n^{-2}}{4k^4mn^3}$

68. $\dfrac{7x-14}{x^2-2x}$

69. $\dfrac{\left(4xyz^3\right)^2}{8x\sqrt[3]{y^2z^5}}$

70. $\dfrac{x^2-6x+8}{x^2+2x-24}$

71. $\dfrac{9x+9}{x^2+8x+7}$

SOLVING EQUATIONS and SYSTEMS OF EQUATIONS ALGEBRAICALLY AND GRAPHICALLY

- You should know how to solve linear equations. $ax + b = 0$
- An identity is an equation whose solution consists of every real number in its domain.
- To solve an equation you can: a) Add or Subtract the same quantity from both sides. b) Multiply or divide both sides by the same nonzero quantity.
- To solve an equation that can be simplified to a linear equation: a) Remove all symbols of grouping and all fractions. b) Combine like terms. c) Solve by algebra. d) Check the answer.
- A "solution" that does not satisfy the original equation is called an extraneous solution.
- You should be able to solve equations graphically.
- You should be able to solve a quadratic equation by factoring, if possible.
- You should be able to solve a quadratic equation of the form $u^2 = d$ by extracting square roots.
- You should be able to solve a quadratic equation by completing the square.
- You should know and be able to use the Quadratic Formula: For $ax^2 + bx + c = 0, a \neq 0, \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
- You should be able to solve polynomials of higher degree by factoring.
- For equations involving radicals or fractional powers, raise both sides to the same power.
- For equations with fractions, multiply both sides by the least common denominator to clear the fractions.
- For equations involving absolute value, remember that the expression inside the absolute value can be positive or negative.
- Always check for extraneous solutions.

Solve the system of equations using substitution or elimination. Write your answer as a coordinate point.

72. $y = -x + 7$
$y = -x + 9$

73. $3x + 5y = -7$
$-x - 2y = 3$

74. $2x - 3y = 6$
$3y = 2x - 10$

75. Why does your answer need to be written as a coordinate point? Explain what we are finding on the graph when solving a system of equations.

Find the x and y intercepts of the graph of the equation algebraically.

76. $y = 3 + x$

77. $2x - 4y = 12$

78. $x = 2$

Solve each equation. Check your work by substituting your solution back into the equation.

79. $3x - 4 = -1$

80. $\frac{1}{2}x - 5 = \frac{2}{3}x$

81. $6x = 3x^2$

82. $16x^2 = 25$

83. $-2x^2 - 14x = 0$

84. $x^2 - x - 6 = 0$

85. $2x^2 - x - 15 = 0$

86. $x^2 - 11x = -30$ (Hint: You can't factor out x on the left side. You have to set the equation to 0 before factoring.)

87. $\sqrt{x+4}=3$

88. $\sqrt[3]{x-1}-4=0$

89. $\frac{x}{2}=\frac{3}{4}$

90. $\frac{1}{x-2}=3$

91. $|x-5|=10$

92. $|x+3|=-2$ (This one is tricky.)

93. $\frac{1}{x}+\frac{2}{x}=3$

94. $\frac{5}{x-5}+\frac{1}{x+5}=\frac{2}{x^2-25}$

95. $\frac{9x}{3x-1}+\frac{4}{3x+1}=3$

SOLVING INEQUALITIES

- You should know the properties of inequalities.
  - Transitive: a < b and b < c implies a < c.
  - Addition: a < b and c < d implies a+c < b+d
  - Adding or Subtracting a Constant: $a \pm c < b \pm c$ a if a < b.
  - Multiplying or Dividing by a constant: For a < b,
    - If c > 0, then ac < bc and $\frac{a}{c} < \frac{b}{c}$.
    - If c < 0, then ac > bc and $\frac{a}{c} > \frac{b}{c}$.
- You should know that $|x| = \{ x \text{ if } x \geq 0 , -x \text{ if } x < 0.$
- You should be able to solve absolute value inequalitites.
  - $|x| < a$ if and only if –a < x < a.
  - $|x| > a$ if and only if x < -a or x > a.
- You should be able to solve polynomial inequalities.
  - Find the critical numbers.
    - Values that make the expression zero.
    - Values that make the expression undefined.
  - Test one value in each interval on the real number line resulting from the critical numbers.
  - Determine the solution intervals.
- You should be able to solve rational and other types of inequalities.

Solving the inequality and graph the solution on the real number line.

96. $8x - 3 < 6x + 15$

97. $\frac{1}{2}x - 4 \geq 2 - x$

98. $|x - 2| < 1$

99. $|x - 3| > 4$

100. $|x + 9| + 7 > 19$