Day 1 Linear Systems: Solve with Graphing

<table>
<thead>
<tr>
<th>System of Linear Equations</th>
<th>Solution of a System of Linear Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• Two or more linear equations</td>
</tr>
<tr>
<td></td>
<td>• An ordered pair that makes all of the equations in a system true; the point of intersection</td>
</tr>
</tbody>
</table>

**One solution:** A system of linear equations has one solution when the graphs intersect at a point.

**No solution:** A system of linear equations has no solution when the graphs are parallel.

**Infinite solutions:** A system of linear equations has infinite solutions when the graphs are the exact same lines.

**Example:** Graph to find solution:

a) \( y = 3x + 1 \)

b) \( 2y = -4x - 8 \)

**Step 1:** Graph the lines.

**Methods:**
1. In slope intercept form; graph using the \( y \)-intercept (\( b \)) and slope (\( m \)).
2. Put equations into slope intercept form:
   * Add or subtract the \( x \)-term.
   * Divide all terms by \( y \) in front of \( y \).
   Graph using \( y \)-intercept (\( b \)) and slope (\( m \)).
3. Make a table and find points to plot.
4. Find the \( x \)- and \( y \)-intercepts.

**Step 2:** Identify the solutions.
(Ordered pair where the lines intersect)

**Step 1:**

**Equation a:** \( y = 3x + 1 \)

*is in Slope-Intercept form.
Use method 1

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*is in Slope-Intercept form.
Use method 1
Select the correct system of equations for each graph:

<table>
<thead>
<tr>
<th></th>
<th>1.</th>
<th>2.</th>
<th>3.</th>
<th>4.</th>
<th>5.</th>
<th>6.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A)</td>
<td>( y = x + 2 ) ( y = 3x - 2 )</td>
<td>( x = -2y - 3 ) ( y = 3x + 7 )</td>
<td>( y + x = 3 ) ( x = 2 )</td>
<td>( y = x + \frac{1}{2} ) ( y = 3x + \frac{1}{2} )</td>
<td>( 2x + 3y = 15 ) ( y = \frac{2}{3}x + 5 )</td>
<td>( y = x + 4 ) ( 2y - 4x = 2 )</td>
</tr>
<tr>
<td>B)</td>
<td>( y = -2x - 3 ) ( y = 3x + 7 )</td>
<td>( y = \frac{1}{3}x - 4 ) ( y = \frac{1}{2}x + 2 )</td>
<td>( y = 2 ) ( y = x + 3 )</td>
<td>( y - x = 3 ) ( y + x = 4 )</td>
<td>( y = -\frac{2}{3}x + 5 ) ( y = x - 2 )</td>
<td>( y = -x - 4 ) ( y = -2x + 1 )</td>
</tr>
<tr>
<td>C)</td>
<td>( y = \frac{1}{3}x - 2 ) ( y = \frac{1}{2}x + 2 )</td>
<td>( y = \frac{1}{2}x + 1 ) ( y = \frac{1}{2}x + 2 )</td>
<td>( y + x = -3 ) ( x = 2 )</td>
<td>( y = \frac{1}{2}x + 1 ) ( y = \frac{1}{2}x + 3 )</td>
<td>( y = x + 4 ) ( y = x + 4 )</td>
<td>( y = -x + 4 ) ( y = 2x + 1 )</td>
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</table>
### Example 1:

**A)** \( y = 2x - 1 \)

**B)** \( 3x + 2y = 26 \)

<table>
<thead>
<tr>
<th>Step 1: Isolate one of the variables</th>
<th>Step 1: Equation (A) already has ( y ) isolated.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 2:</strong> Substitute the expression from Step 1 into the OTHER equation.</td>
<td><strong>Step 2:</strong> ( 3x + 2y = 26 ) ( 3x + 2(2x - 1) = 26 )</td>
</tr>
<tr>
<td>- The resulting equation should have only one variable, not both ( x ) and ( y ).</td>
<td><strong>Step 3:</strong> ( 3x + 4x - 2 = 26 ) ( 7x - 2 = 26 ) ( +2 + 2 ) ( 7x = 28 ) ( \frac{7x}{7} = \frac{28}{7} ) ( x = 4 )</td>
</tr>
<tr>
<td><strong>Step 3:</strong> Solve the new equation.</td>
<td><strong>Step 4:</strong> ( y = 2(4) - 1 ) ( y = 7 )</td>
</tr>
<tr>
<td>- This will give you one of the coordinates.</td>
<td><strong>Step 5:</strong> ( y = 7 )</td>
</tr>
<tr>
<td><strong>Step 4:</strong> Substitute the result from Step 3 into either of the original equations.</td>
<td><strong>Step 6:</strong> ((4, 7))</td>
</tr>
<tr>
<td><strong>Step 5:</strong> Solve for the other coordinate.</td>
<td></td>
</tr>
<tr>
<td><strong>Step 6:</strong> Write the solution as an ordered pair. ((x, y))</td>
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### Example 2:

**A)** \(-4x + y = 6\)

**B)** \(-5x - y = 21\)

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<thead>
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<th>Step 1: Isolate one of the variables</th>
<th>Step 2: Substitute the expression from Step 1 into the OTHER equation.</th>
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<tr>
<td><strong>Step 2:</strong> Substitute the expression from Step 1 into the OTHER equation.</td>
<td><strong>Step 2:</strong> (-5x - y = 21) (-5x - (6 + 4x) = 21)</td>
</tr>
<tr>
<td>- The resulting equation should have only one variable, not both ( x ) and ( y ).</td>
<td><strong>Step 3:</strong> (-5x - 6 - 4x = 21) (-9x - 6 = 21) (+6 +6) ( -9x = 27) (-9\cdot9) ( x = -3 )</td>
</tr>
<tr>
<td><strong>Step 3:</strong> Solve the new equation.</td>
<td><strong>Step 4:</strong> ( y = 6 + 4(-3) ) ( y = 6 - 6 )</td>
</tr>
<tr>
<td>- This will give you one of the coordinates.</td>
<td><strong>Step 5:</strong> ( y = -6 )</td>
</tr>
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<td><strong>Step 4:</strong> Substitute the result from Step 3 into either of the original equations.</td>
<td><strong>Step 6:</strong> ((-3, -6))</td>
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<td><strong>Step 5:</strong> Solve for the other coordinate.</td>
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<td><strong>Step 6:</strong> Write the solution as an ordered pair. ((x, y))</td>
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**Try It Out:** (Solutions are after the Practice Problems)
Practice Problems:

1. \( \begin{cases} x = 5 \\ x + y = 12 \end{cases} \)
2. \( \begin{cases} y = 5 \\ -3x + 4y = 8 \end{cases} \)
3. \( \begin{cases} y = 2x \\ x + y = 9 \end{cases} \)
4. \( \begin{cases} y = -3x \\ x + y = 4 \end{cases} \)
5. \( \begin{cases} y = 3x - 4 \\ 4x + 3y = 1 \end{cases} \)
6. \( \begin{cases} x = 3y + 1 \\ 2x + 4y = 12 \end{cases} \)
7. \( \begin{cases} -5x + y = -3 \\ 3x - 8y = 24 \end{cases} \)
8. \( \begin{cases} x + 3y = 1 \\ -3x - 3y = -15 \end{cases} \)

Try It Out Answers:

A) \( \begin{cases} x = -2 \\ x + 3y = 4 \end{cases} \) \( \begin{array}{c} x = -2 \\ -2 + 3y = 4 \\ 3y = 6 \\ 3y = 6 \ \\ 3 = 3 \ \\ y = 2 \end{array} \) \( (2, -2) \)

B) \( \begin{cases} x - 3y = 0 \\ x - 3y = 0 \end{cases} \) \( \begin{array}{c} x = 2 \\ y = 4(2) - 10 \\ 8 - 10 \\ y = -2 \end{array} \) \( (2, -2) \)

C) \( \begin{cases} y = -3x + 4 \\ y = 4x - 10 \end{cases} \) Since both equations tell us what \( y \) equals we will set them equal to each other \( -3x + 4 = 4x - 10 \) \( +3x \underline{+3x} \) \( 4 = 7x - 10 \) \( +10 \underline{+10} \) \( 14 = 7x \) \( 7 \ 7 \) \( X = 2 \) \( Y = 4(2) - 10 \) \( 8 - 10 \) \( Y = -2 \) \( (2, -2) \)

D) \( \begin{cases} -5x + y = -2 \\ -3x + 6y = -12 \end{cases} \) \( -5x + y = -2 \) \( +5x \underline{+5x} \) \( y = -2 + 5x \) \( -3x + 6(-2 + 5x) = -12 \) \( -3x - 12 + 30x = -12 \) \( +12 \underline{+12} \) \( -3x + 30x = 0 \) \( 27x = 0 \) \( X = 0 \) \( -5(0) + y = -2 \) \( Y = -2 \) \( (0, -2) \)
### DAY 3 Linear Systems: Elimination Method

**Example:** \[
\begin{align*}
2x + 5y &= 17 \\
6x - 5y &= -9
\end{align*}
\]

**Step 1:** Line up the x’s and y’s

**Step 2:** Look to see if one variable has opposite coefficients
- Yes, move to step 3
- No, multiply one or both equations by a (LCM) in order to make the coefficients of the x or y terms opposites

**Step 3:** Add the equations together to eliminate one of the variables

**Step 4:** Solve for the remaining variable

**Step 5:** Substitute the value you found into one of the original equations to solve for the other variable

**Step 6:** Write your answer as an ordered pair.

| Step 1: | \[
\begin{align*}
2x + 5y &= 17 \\
6x - 5y &= -9
\end{align*}
\] |
<table>
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<tr>
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</tr>
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<tbody>
<tr>
<td>Step 2:</td>
<td>Yes</td>
</tr>
</tbody>
</table>
| Step 3: | \[
\begin{align*}
8x &= 8 \\
8 &= 8
\end{align*}
\] |
| Step 4: | \[
x = 1
\] |
| Step 5: | \[
2(1) + 5y = 17 \\
2 + 5y = 17 \\
-2 - 2
\] |
| Step 6: | (1,3) |

**Example:** \[
\begin{align*}
3x + y &= 9 \\
5x + 4y &= 22
\end{align*}
\]

**Step 1:** Line up the x’s and y’s

**Step 2:** Look to see if one variable has opposite coefficients
- Yes, move to step 3
- No, multiply one or both equations by a (LCM) in order to make the coefficients of the x or y terms opposites

**Step 3:** Add the equations together to eliminate one of the variables

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**Step 6:** Write your answer as an ordered pair.

| Step 1: | \[
\begin{align*}
3x + y &= 9 \\
5x + 4y &= 22
\end{align*}
\] |
<table>
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<tbody>
<tr>
<td>Step 2:</td>
<td>No, multiply one or both equations by a constant (LCM) in order to make the coefficients of the x or y terms opposites.</td>
</tr>
</tbody>
</table>
| Step 3: | \[
\begin{align*}
-12x - 4y &= -36 \\
5x + 4y &= 22
\end{align*}
\] |
| Step 4: | \[
x = 2
\] |
| Step 5: | \[
5(2) + 4y = 22 \\
10 + 4y = 22 \\
-10 -10
\] |
| Step 6: | (2, 3) |

**Try It Out:** (Solutions are after the Practice Problems)

A) \[
\begin{align*}
3y + 2x &= 6 \\
5y - 2x &= 10
\end{align*}
\]

B) \[
\begin{align*}
x + 3y &= 18 \\
x - 4y &= -25
\end{align*}
\]

C) \[
\begin{align*}
4x + 2y &= 8 \\
16x - y &= 14
\end{align*}
\]
Practice Problems

1. \[
\begin{align*}
2x + 2y &= -2 \\
3x - 2y &= 12
\end{align*}
\]

2. \[
\begin{align*}
4x - 2y &= -1 \\
-4x + 4y &= -2
\end{align*}
\]

3. \[
\begin{align*}
6x + 5y &= 4 \\
6x - 7y &= -20
\end{align*}
\]

4. \[
\begin{align*}
3x + y &= -21 \\
-x - y &= 5
\end{align*}
\]

5. \[
\begin{align*}
-x + 9y &= -5 \\
x - 5y &= 1
\end{align*}
\]

6. \[
\begin{align*}
-2x + y &= 10 \\
4x - y &= -14
\end{align*}
\]

7. \[
\begin{align*}
3x + 2y &= 0 \\
x - 5y &= 17
\end{align*}
\]

8. \[
\begin{align*}
-x - 15y &= -17 \\
x + 5y &= -13
\end{align*}
\]

Try It Out Answers:

A) \[
\begin{align*}
3y + 2x &= 6 \\
5y - 2x &= 10
\end{align*}
\]

\[
\begin{align*}
8y &= 16 \\
y &= 2
\end{align*}
\]

\[
\begin{align*}
3(2) + 2x &= 6 \\
6 + 2x &= 6 \\
-6 - 6 &= 0 \\
2x &= 0 \\
x &= 0
\end{align*}
\]

(0, 2)

B) \[
\begin{align*}
x + 3y &= 18 \\
-x - 4y &= -25
\end{align*}
\]

\[
\begin{align*}
x + 3(7) &= 18 \\
x + 21 &= 18 \\
-21 - 21 &= -3
\end{align*}
\]

(-3, 7)

C) Since neither variable has opposite coefficients, we must multiply the top equation by -4

\[
\begin{align*}
-4(4x + 2y) &= 8 \\
16x - y &= 14
\end{align*}
\]

\[
\begin{align*}
-16x - 8y &= -32 \\
16x - y &= 14 \\
-9y &= -18 \\
-9 - 9 &= 2 \\
x &= -3 \\
4x + 2(2) &= 8 \\
4x + 4 &= 8 \\
-4 &= -4 \\
4x &= 4 \\
x &= 1
\end{align*}
\]

(1, 2)