ARE YOU READY FOR CALCULUS?

Congratulations! You made it to Calculus AB!

Calculus AB Summer Work

The rationale for this summer homework is to refresh your Algebra 2 and Pre Calculus skills, give you an idea of the prior skills required for this course and help you decide on if AP Calculus AB is for you. You will have a quiz within the first few days of school, and the grade you receive will hopefully give you a good indication on how your math skills are prior to taking this course. If you find that you are struggling with these concepts, you can use this assignment to self evaluate and create a learning plan for yourself.

4 Guiding Questions for Students Inquiring about Calculus AB

1.) Do you personally want to take the class?
2.) Are you okay with initially struggling and perhaps not earning the grade you are hoping for?
3.) Are you willing to work hard on improving and get extra help when needed?
4.) Do you have the time to put in the additional work and get the extra help when needed?

Instructions

1. There are 14 learning targets you need to be familiar with. Please complete five or more learning targets out of the total 14 learning targets, which will be due the day of registration.

Registration & Parent night dates for 2020-2021
Please turn in your summer work on registration day, which are as follows:
10th grade – Class of 2023 Registration – Friday, August 7, 2020 – 12:30 pm – 3:00 pm
11th grade – Class of 2022 Registration – Friday, August 7, 2020 – 7:45 am – 11:00 am
12th grade – Class of 2021 Registration – Thursday, August 6, 2020 – 12:00 pm – 3:00 pm

***Makeup Registration Wednesday, August 12, 2020 – 8 am – 12:30 pm ***

If not turned in by the mentioned dates you will be dropped from the course. Focus on the topics that you need the most work on. If you are good with a topic you can skip that section.

2. Do your work on a separate sheet of paper clearly and neatly and write the answer the actual worksheet.

3. If you need help to review look up the topics on the Internet or in your book. There are 14 learning targets you need to be familiar with. If you get stuck on a problem please refer to the second part of the packet which provides a mini lesson and examples to help refresh your memory.

4. All of these topics should be review and are skills you will be expected to know the first day of class. (OF PARTICULAR IMPORTANCE IS THE UNIT CIRCLE which should be memorized.)

HOW TO TURN IN SUMMER WORK

Summer assignments should be turned in electronically via Mr. Dien’s Google classroom.
Join code: hex2uv2

MISC. Helpful Links

You will need at minimum a TI-84 calculator (any other TI is also good just make sure to know how to use it) which is required for the class. http://mathbits.com is a website where you can find tutorials on how to use the basic functions of the calculator and are also part of the learning packet.

Good Luck!
Mr. Dien
1.) If \( f(x) = 4x - x^2 \), find:

a.) \( f(4) - f(-4) \)  
b.) \( \sqrt{\frac{3}{2}} \)  
c.) \( \frac{f(x+h) - f(x)}{2h} \)

2.) If \( f(x) \) and \( g(x) \) are given in the graph, find:

a.) \( (f - g)(3) \)  
b.) \( f(g(3)) \)

3.) If \( f(x) = \begin{cases} 
-x, & x < 0 \\
x^2 - 1, & 0 \leq x < 2 \\
\sqrt{x+2} - 2, & x \geq 2 
\end{cases} \), find:

a.) \( f(0) - f(2) \)  
b.) \( \sqrt{5 - f(-4)} \)  
c.) \( f(f(3)) \)
Find the domain of the following functions using interval notation:

1.) \( f(x) = 3 \)  
2.) \( y = x^3 - x^2 + x \)  
3.) \( y = \frac{x^3 - x^2 + x}{x} \)

4.) \( y = \frac{x - 4}{x^2 - 16} \)  
5.) \( f(x) = \frac{1}{4x^2 - 4x - 3} \)  
6.) \( y = \sqrt{2x - 9} \)

7.) \( y = \log(x - 10) \)  
8.) \( y = \frac{\sqrt{2x + 14}}{x^2 - 49} \)  
9.) \( y = \frac{\sqrt{5 - x}}{\log(x)} \)

Find the range of the following functions:

10.) \( y = x^4 + x^2 - 1 \)  
11.) \( y = 100^x \)  
12.) \( y = \sqrt{x^2 + 1} + 1 \)

Find the domain and range of the following functions using interval notation:

13.)

14.)

15.)
Topic 3: Graphs of Common Functions

Sketch each of the following as accurately as possible. You will need to be VERY familiar with each of these graphs throughout the year. You may use a graphing calculator for some of them if you have access to one over the summer. Another option is to find a graphing app (www.desmos.com) or generate a table of values. Again, these are VERY important graphs to know. Be very accurate with regards to “open circles” and “closed circles.”

1. \( y = x \)

2. \( y = x^2 \)

3. \( y = x^3 \)

4. \( y = \sqrt{x} \)

5. \( y = |x| \)

6. \( y = \sqrt{4-x^2} \)
7. $y = x^{\frac{1}{3}}$

8. $y = x^{\frac{2}{3}}$

9. $y = \sin x$

10. $y = \cos x$

15. $y = e^x$

16. $y = \ln x$

17. $y = \frac{1}{x}$

19. $y = \frac{1}{x^2}$
Show work to determine if the relation is even, odd, or neither.

1.) \( f(x) = 7 \)

2.) \( f(x) = 2x^2 - 4x \)

3.) \( f(x) = -3x^3 - 2x \)

4.) \( f(x) = \sqrt{x + 1} \)

5.) \( f(x) = |8x| \)

6.) \( f(x) = 8x \)
Topic 5: Function Transformations

If \( f(x) = x^2 - 1 \), describe in words what the following would do to the graph of \( f(x) \):

1.) \( f(x) - 4 \)

2.) \( f(x - 4) \)

3.) \( -f(x + 2) \)

4.) \( 5f(x) + 3 \)

5.) \( f(2x) \)

6.) \( |f(x)| \)

Here is a graph of \( y = f(x) \):

Sketch the following graphs:

7.) \( y = 2f(x) \)

8.) \( y = -f(x) \)

9.) \( y = f(x - 1) \)

10.) \( y = f(x) + 2 \)

11.) \( y = |f(x)| \)

12.) \( y = f(|x|) \)
Factor completely.

1.) $x^3 + 8$

2.) $x^3 - 8$

3.) $27x^3 - 125y^3$

4.) $x^4 + 11x^2 - 80$

5.) $ac + cd - ab - bd$

6.) $2x^2 + 50y^2 - 20xy$

7.) $x^2 + 12x + 36 - 9y^2$

8.) $x^3 - xy^2 + x^2y - y^3$

9.) $(x - 3)^2 (2x + 1)^3 + (x - 3)^3 (2x + 1)^2$
1.) Find the equation of the line in point-slope form, with the given slope, passing through the given point.
   a.) \( m = -7, \ (-3, -7) \) 
   b.) \( m = -\frac{1}{2}, \ (2, -8) \)

2.) Find the equation of the line in point-slope form, passing through the given points.
   a.) \((-3, 6), \ (-1, 2)\) 
   b.) \((-7, 1), \ (3, -4)\)

3.) Find the equations of the lines through the given point that are a.) parallel and b.) normal to the given line.
   a.) \((-6, 2), \ 5x + 2y = 7\) 
   b.) \((-3, -4), \ y = -2\)

4.) Find the equation of the line in general form, containing the point \((4, -2)\) and parallel to the line containing the points \((-1, 4)\) and \((2, 3)\).

5.) Find \(k\) if the lines \(3x - 5y = 9\) and \(2x + ky = 11\) are a.) parallel and b.) perpendicular.
Topic 8: Asymptotes

For each function, find the equations of both the vertical asymptote(s) and horizontal asymptote (if it exists) and the location of any holes.

1.) \( y = \frac{x-1}{x+5} \)  
2.) \( y = \frac{8}{x^2} \)  
3.) \( y = \frac{2x+16}{x+8} \)

4.) \( y = \frac{2x^2 + 6x}{x^2 + 5x + 6} \)  
5.) \( y = \frac{x}{x^2 - 25} \)  
6.) \( y = \frac{x^2 - 5}{2x^2 - 12} \)

7.) \( y = \frac{4+3x-x^2}{3x^2} \)  
8.) \( y = \frac{5x+1}{x^2 - x - 1} \)  
9.) \( y = \frac{x^3 + 4x}{x^3 - 2x^2 + 4x - 8} \)
Simplify and write with positive exponents.

1.) \(-12^2 x^{-5}\)  
2.) \((-12x^5)^{-2}\)  
3.) \((4x^{-1})^{-1}\) 

4.) \((-4/x^4)^{-3}\)  
5.) \((5x^3/y^2)^{-3}\)  
6.) \((x^3 - 1)^{-2}\) 

7.) \((121x^8)^{\frac{1}{2}}\)  
8.) \((8x^2)^{-\frac{4}{3}}\)  
9.) \((-32x^{-5})^{-\frac{3}{5}}\) 

10.) \((x + y)^2\)  
11.) \(x\left(x^{\frac{1}{2}} - x\right)^{-2}\)
Topic 10: Complex Fractions

Eliminate the complex fractions:

1.) \( \frac{x - \frac{1}{x}}{x + \frac{1}{x}} \)

2.) \( \frac{1 + x^{-1}}{1 - x^{-2}} \)

3.) \( \frac{x^{-1} + y^{-1}}{x + y} \)
Topic 11: Absolute Value Equations

Solve the following equations:

1.) \(4|x + 8| = 20\) 

2.) \(|1 - 7x| = 13\)

3.) \(|8 + 2x| + 2x = 40\)

4.) \(|4x - 5| + 5x + 2 = 0\)
Topic 12: Exponential Functions and Logarithms

Simplify the following: (remember ln of a number (not variable) is just a number!)

1.) \( \log_2 \frac{1}{4} \)  
2.) \( \log_8 4 \)  
3.) \( \ln \frac{1}{\sqrt[3]{e^2}} \)  

4.) \( 5^{\log_5 40} \)  
5.) \( e^{\ln 12} \)  
6.) \( \log_{12} 2 + \log_{12} 9 + \log_{12} 8 \)

7.) \( \log_2 \frac{2}{3} + \log_2 \frac{3}{32} \)  
8.) \( \log_{\frac{4}{3}} 3 - \log_{\frac{4}{3}} 12 \)  
9.) \( \log_3 (\sqrt{3})^5 \)

Solve the following:

10.) \( \log_5 (3x - 8) = 2 \)  
11.) \( \log_9 (x^2 - x + 3) = \frac{1}{2} \)  
12.) \( \log (x - 3) + \log 5 = 2 \)

13.) \( \log_2 (x - 1) + \log_2 (x + 3) = 5 \)  
14.) \( \log_5 (x + 3) - \log_5 x = 2 \)  
15.) \( \ln x^3 - \ln x^2 = \frac{1}{2} \)

16.) \( 3^{x-2} = 18 \)  
17.) \( e^{3x+1} = 10 \)  
18.) \( 8^x = 5^{2x-1} \)
Topic 13: Basic Right Angle Trigonometry

Solve the following:

If point \( P \) is on the terminal side of \( \theta \), find all 6 trigonometric functions of \( \theta \). (Answers need not be rationalized.)

1.) \( P(-2, 4) \)

2.) \( P(\sqrt{5}, -2) \)

Topic 14: Special Angles

1.) \( \left( \cos \frac{2\pi}{3} - \tan \frac{3\pi}{4} \right)^3 \)

2.) \( \left( \sin \frac{11\pi}{6} - \tan \frac{5\pi}{6} \right) \left( \sin \frac{11\pi}{6} + \tan \frac{5\pi}{6} \right) \)

Determine whether each of the following statements is true or false.

3.) \( \sin \frac{\pi}{6} + \sin \frac{\pi}{3} = \sin \left( \frac{\pi}{6} + \frac{\pi}{3} \right) \)

4.) \( \frac{\cos \frac{5\pi}{3} + 1}{\tan \frac{5\pi}{3}} = \frac{\cos \frac{5\pi}{3}}{\sec \frac{5\pi}{3} - 1} \)

Good job now go to [http://mathbits.com/MathBits/TISection/CachingPage.html](http://mathbits.com/MathBits/TISection/CachingPage.html) and go through how to use basic functions of the graphing calculator. [Http://mathbits.com](http://mathbits.com) also has lessons to teach you how to use the more modern TI calculators.
Topic 1: Functions

The lifeblood of precalculus is functions. A function is a set of points \((x, y)\) such that for every \(x\), there is one and only one \(y\). In short, in a function, the \(x\)-values cannot repeat while the \(y\)-values can. In AB Calculus, all of your graphs will come from functions.

The notation for functions is either "\(y = f(x)\)" or "\(f(x) = \)". In the \(f(x)\) notation, we are stating a rule to find \(y\) given a value of \(x\).

1. If \(f(x) = x^2 - 5x + 8\), find a) \(f(-6)\), b) \(f\left(\frac{3}{2}\right)\), c) \(\frac{f(x+h)-f(x)}{h}\)

   \[a) f(-6) = (-6)^2 - 5(-6) + 8 = 36 + 30 + 8 = 74\]
   \[b) f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2 - 5\left(\frac{3}{2}\right) + 8 = \frac{9}{4} - \frac{15}{2} + 8 = \frac{11}{4}\]
   \[c) \frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2 - 5(x+h) + 8 - (x^2 - 5x + 8)}{h} = \frac{x^2 + 2xh + h^2 - 5x - 5h + 8 - x^2 + 5x - 8}{h} = \frac{h^2 + 2xh - 5h}{h} = h + 2x - 5\]

Functions do not always use the variable \(x\). In calculus, other variables are used liberally.

2. If \(A(r) = \pi r^3\), find a) \(A(3)\), b) \(A(2x)\), c) \(A(r+1) - A(r)\)

   \[A(3) = 9\pi\]
   \[A(2x) = \pi (2x)^3 = 8\pi x^3\]
   \[A(r+1) - A(r) = \pi (r+1)^3 - \pi r^3 = \pi (r^3 + 3r^2 + 3r + 1 - r^3) = 3\pi r^2 + 3\pi r + \pi\]

One concept that comes up in AP calculus is composition of functions. The format of a composition of functions is: plug a value into one function, determine an answer, and plug that answer into a second function.

3. If \(f(x) = x^2 - x + 1\) and \(g(x) = 2x - 1\), a) find \(f(-1)\) b) find \(g(f(-1))\) c) show that \(f(g(x)) \neq g(f(x))\)

   \[g(-1) = 2(-1) - 1 = -3\]
   \[f(-1) = (-1)^2 - (-1) + 1 = 3\]
   \[g(3) = 2(3) - 1 = 5\]

Finally, expect to use piecewise functions. A piecewise function gives different rules, based on the value of \(x\).

4. If \(f(x) = \begin{cases} 
  x^2 - 3, & x \geq 0 \\
  2x + 1, & x < 0 
\end{cases}\), find a) \(f(5)\) b) \(f(2) - f(-1)\) c) \(f(f(1))\)

   \[f(5) = 5^2 - 3 = 22\]
   \[f(2) - f(-1) = 2(2) + 1 - (2(-1) - 1) = 5 - 1 = 2\]
   \[f(1) = -2, \quad f(-2) = -3\]
Topic 2: Domain and Range

First, since questions in calculus usually ask about behavior of functions in intervals, understand that intervals can be written with a description in terms of $<$, $\le$, $>$, $\ge$ or by using interval notation.

<table>
<thead>
<tr>
<th>Description</th>
<th>Interval notation</th>
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<th>Interval notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x &gt; a$</td>
<td>$(a, \infty)$</td>
<td>$x \le a$</td>
<td>$(-\infty, a]$</td>
<td>$a \le x &lt; b$</td>
<td>$[a, b)$</td>
</tr>
<tr>
<td>$x \ge a$</td>
<td>$[a, \infty)$</td>
<td>$a &lt; x &lt; b$</td>
<td>$(a, b)$ · open interval</td>
<td>$a &lt; x \le b$</td>
<td>$(a, b]$</td>
</tr>
<tr>
<td>$x &lt; a$</td>
<td>$(-\infty, a)$</td>
<td>$a \le x \le b$</td>
<td>$[a, b]$ · closed interval</td>
<td>All real numbers</td>
<td>$(\infty, \infty)$</td>
</tr>
</tbody>
</table>

If a solution is in one interval or the other, interval notation will use the connector $\cup$. So $x \le 2$ or $x > 6$ would be written $(-\infty, 2] \cup (6, \infty)$ in interval notation. Solutions in intervals are usually written in the easiest way to define it. For instance, saying that $x < 0$ or $x > 0$ or $(-\infty, 0) \cup (0, \infty)$ is best expressed as $x \neq 0$.

The domain of a function is the set of allowable $x$-values. The domain of a function $f$ is $(-\infty, \infty)$ except for values of $x$ which create a zero in the denominator, an even root of a negative number or a logarithm of a non-positive number. The domain of $a^{p(x)}$ where $a$ is a positive constant and $p(x)$ is a polynomial is $(-\infty, \infty)$.

- Find the domain of the following functions using interval notation.

1. $f(x) = x^2 - 4x + 4$
   
   $(-\infty, \infty)$

2. $y = \frac{6}{x - 6}$
   
   $x \neq 6$

3. $y = \frac{2x}{x^2 - 2x - 3}$
   
   $x \neq -1, x \neq 3$

4. $y = \sqrt{x + 5}$
   
   $[-5, \infty)$

5. $y = \sqrt[3]{x + 5}$
   
   $(-\infty, \infty)$

6. $y = \sqrt{x^3 + 4x + 6}$
   
   $(-2, \infty)$

The range of a function is the set of allowable $y$-values. Finding the range of functions algebraically isn't as easy (it really is a calculus problem), but visually, it is the [lowest possible $y$-value, highest possible $y$-value]. Finding the range of some functions are fairly simple to find if you realize that the range of $y = x^2$ is $[0, \infty)$ as any positive number squared is positive. Also the range of $y = \sqrt{x}$ is also positive as the domain is $[0, \infty)$ and the square root of any positive number is positive. The range of $y = a^x$ where $a$ is a positive constant is $(0, \infty)$ as constants to powers must be positive.

- Find the range of the following functions using interval notation.

7. $y = 1 - x^2$
   
   $(-\infty, 1]$

8. $y = \frac{1}{x^2}$
   
   $(0, \infty)$

9. $y = \sqrt{x - 8} + 2$
   
   $[2, \infty)$

- Find the domain and range of the following functions using interval notation.

10. [Graph]
    Domain: $(-\infty, \infty)$

    Range: $[-0.5, 2.5]$

11. [Graph]
    Domain: $(0, 4)$

    Range: $[0, 4)$
Topic 3: Graphs of Common Functions

There are certain graphs that occur all the time in calculus and students should know the general shape of them, where they hit the x-axis (zeros) and y-axis (y-intercept), as well as the domain and range. There are no assignment problems for this section other than students memorizing the shape of all of these functions. In section 5, we will talk about transforming these graphs.

Function: $y = a$
Domain: $(-\infty, \infty)$
Range: $[a, a]$

Function: $y = x$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

Function: $y = x^2$
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

Function: $y = x^3$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

Function: $y = \sqrt{x}$
Domain: $[0, \infty)$
Range: $[0, \infty)$

Function: $y = |x|$
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

Function: $y = \frac{1}{x}$
Domain: $x \neq 0$
Range: $(-\infty, 0) \cup (0, \infty)$

Function: $y = \ln x$
Domain: $(0, \infty)$
Range: $(-\infty, \infty)$

Function: $y = e^x$
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$

Function: $y = e^{-x}$
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$

Function: $y = \sin x$
Domain: $(-\infty, \infty)$
Range: $[-1, 1]$

Function: $y = \cos x$
Domain: $(-\infty, \infty)$
Range: $[-1, 1]$
Topic 4: Even/Odd Functions and Symmetry

Functions that are even have the characteristic that for all \(a\), \(f(-a) = f(a)\). What this says is that plugging in a positive number \(a\) into the function or a negative number \(-a\) into the function makes no difference; you will get the same result. Even functions are symmetric to the \(y\)-axis.

Functions that are odd have the characteristic that for all \(a\), \(f(-a) = -f(a)\). What this says is that plugging in a negative number \(-a\) into the function will give you the same result as plugging in the positive number and taking the negative of that. So, odd functions are symmetric to the origin. If a graph is symmetric to the \(x\)-axis, it is not a function because it fails the vertical-line test.

1. Of the common functions in section 3, which are even, which are odd, and which are neither?

| Even: \(y = a, y = x^2, y = |x|, y = \cos x\) | Odd: \(y = x, y = x^3, y = \frac{1}{x}, y = \sin x\) |
|---------------------------------------------|-----------------------------------------------|
| Neither: \(y = \sqrt{x}, y = \ln x, y = e^x, y = e^{-x}\) |

2. Show that the following functions are even.

a) \(f(x) = x^4 - x^2 + 1\)

\[
f(-x) = (-x)^4 - (-x)^2 + 1 = x^4 - x^2 + 1 = f(x)
\]

b) \(f(x) = \frac{1}{x}\)

\[
f(-x) = \frac{1}{-x} = \frac{1}{x} = f(x)
\]

c) \(f(x) = x^{2g}\)

\[
f(-x) = (-x)^{2g} = (\sqrt{x})^2 = f(x)
\]

3. Show that the following functions are odd.

a) \(f(x) = x^3 - x\)

\[
f(-x) = (-x)^3 + x = -x^3 + x = -f(x)
\]

b) \(f(x) = \sqrt[3]{x}\)

\[
f(-x) = \sqrt[3]{-x} = -\sqrt[3]{x} = -f(x)
\]

c) \(f(x) = e^x - e^{-x}\)

\[
f(-x) = e^{-x} - e^x = -(e^x - e^{-x}) = -f(x)
\]

4. Determine if \(f(x) = x^3 - x^2 + x - 1\) is even, odd, or neither. Justify your answer.

\[
f(-x) = -x^3 - x^2 - x - 1 \neq f(x) \text{ so } f \text{ is not even.} \quad -f(x) = -x^3 + x^2 - x - 1 \neq f(-x) \text{ so } f \text{ is not odd.}
\]

Graphs may not be functions and yet have \(x\)-axis or \(y\)-axis or both. Equations for these graphs are usually expressed in “implicit form” where it is not expressed as “\(y = \)” or “\(f(x) = \)”. If the equation does not change after making the following replacements, the graph has these symmetries:

- \(x\)-axis. \(y \) with \(-y\)
- \(y\)-axis. \(x \) with \(-x\)
- origin. both \(x \) with \(-x\) and \(y \) with \(-y\)

5. Determine the symmetry for \(x^2 + xy + y^2 = 0\)

\[
x \text{ - axis. } x^2 + x(-y) + (-y)^2 = 0 \Rightarrow x^2 - xy + y^2 = 0 \text{ so not symmetric to } x \text{ - axis}
\]

\[
y \text{ - axis. } (-x)^2 + (-x)(y) + y^2 = 0 \Rightarrow x^2 - xy + y^2 = 0 \text{ so not symmetric to } y \text{ - axis}
\]

origin. \((-x)^2 + (-x)(-y) + y^3 = 0 \Rightarrow x^2 + xy + y^2 = 0 \text{ so symmetric to origin}
\]
**Topic 5: Function Transformations**

A curve in the form \( y = f(x) \), which is one of the basic common functions from section C can be transformed in a variety of ways. The shape of the resulting curve stays the same but zeros and y-intercepts might change and the graph could be reversed. The table below describes transformations to a general function \( y = f(x) \) with the parabolic function \( f(x) = x^2 \) as an example.

<table>
<thead>
<tr>
<th>Notation</th>
<th>How ( f(x) ) changes</th>
<th>Example with ( f(x) = x^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) + a )</td>
<td>Moves graph up ( a ) units</td>
<td><img src="image1" alt="Graph" /></td>
</tr>
<tr>
<td>( f(x) - a )</td>
<td>Moves graph down ( a ) units</td>
<td><img src="image2" alt="Graph" /></td>
</tr>
<tr>
<td>( f(x + a) )</td>
<td>Moves graph ( a ) units left</td>
<td><img src="image3" alt="Graph" /></td>
</tr>
<tr>
<td>( f(x - a) )</td>
<td>Moves graph ( a ) units right</td>
<td><img src="image4" alt="Graph" /></td>
</tr>
<tr>
<td>( af(x) )</td>
<td>( a &gt; 1 ) Vertical Stretch</td>
<td><img src="image5" alt="Graph" /></td>
</tr>
<tr>
<td>( af(x) )</td>
<td>( 0 &lt; a &lt; 1 ) Vertical shrink</td>
<td><img src="image6" alt="Graph" /></td>
</tr>
<tr>
<td>( f(ax) )</td>
<td>( a &gt; 1 ) Horizontal compress (same effect as vertical stretch)</td>
<td><img src="image7" alt="Graph" /></td>
</tr>
<tr>
<td>( f(ax) )</td>
<td>( 0 &lt; a &lt; 1 ) Horizontal elongated (same effect as vertical shrink)</td>
<td><img src="image8" alt="Graph" /></td>
</tr>
<tr>
<td>( -f(x) )</td>
<td>Reflection across x-axis</td>
<td><img src="image9" alt="Graph" /></td>
</tr>
<tr>
<td>( f(-x) )</td>
<td>Reflection across y-axis</td>
<td><img src="image10" alt="Graph" /></td>
</tr>
</tbody>
</table>
Topic 6: Special Factorization

While factoring skills were more important in the days when A topics were specifically tested, students still must know how to factor. The special forms that occur most regularly are:

| Common factor: $x^3 + x^2 + x = x(x^2 + x + 1)$ |
| Difference of squares: $x^2 - y^2 = (x + y)(x - y)$ or $x^{2n} - y^{2n} = (x^n + y^n)(x^n - y^n)$ |
| Perfect squares: $x^2 + 2xy + y^2 = (x + y)^2$ |
| Perfect squares: $x^2 - 2xy + y^2 = (x - y)^2$ |
| Sum of cubes: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ - Trinomial unfactorable |
| Difference of cubes: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ - Trinomial unfactorable |
| Grouping: $xy + xb + ay + ab = x(y + b) + a(y + b) = (x + a)(y + b)$ |

The term “factoring” usually means that coefficients are rational numbers. For instance, $x^2 - 2$ could technically be factored as $(x + \sqrt{2})(x - \sqrt{2})$ but since $\sqrt{2}$ is not rational, we say that $x^2 - 2$ is not factorable.

It is important to know that $x^2 + y^2$ is unfactorable.

- Completely factor the following expressions.

1. $\frac{4a^2 + 2a}{2a(a + 2)}$  
2. $\frac{x^2 + 16x + 64}{(x + 8)^2}$  
3. $\frac{4x^2 - 64}{4(x + 4)(x - 4)}$

4. $\frac{5x^4 - 5y^4}{5(x^2 + 1)(x + 1)(x - 1)}$  
5. $\frac{16x^2 - 8x + 1}{(4x - 1)^2}$  
6. $\frac{9a^4 - a^2b^2}{a^2(3a + b)(3a - b)}$

7. $\frac{2x^2 - 40x + 200}{2(x - 10)^2}$  
8. $\frac{x^3 - 8}{(x - 2)(x^2 + 2x + 4)}$  
9. $\frac{8x^3 + 27y^3}{(2x + 3y)(4x^2 - 6xy + 9y^2)}$

10. $\frac{x^4 + 11x^2 - 80}{(x + 4)(x - 4)(x^2 + 5)}$  
11. $\frac{x^4 - 10x^2 + 9}{(x + 1)(x - 1)(x + 3)(x - 3)}$  
12. $\frac{36x^2 - 64}{4(3x + 4)(3x - 4)}$

13. $\frac{x^3 - x^2 + 3x - 3}{x^2(x - 1) + 3(x - 1)}(x - 1)(x^2 + 3)$  
14. $\frac{x^3 + 5x^2 - 4x - 20}{x^2(x + 5) - 4(x + 5)}(x + 5)(x - 2)(x + 2)$  
15. $\frac{9 - (x^2 + 2xy + y^2)}{(3 + x + y)(3 - x - y)}$
Topic 7: Linear Functions

Probably the most important concept from precalculus that is required for differential calculus is that of linear functions. The formulas you need to know backwards and forwards are:

Slope: Given two points \((x_1, y_1)\) and \((x_2, y_2)\), the slope of the line passing through the points can be written as:

\[
m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}
\]

Slope-intercept form: the equation of a line with slope \(m\) and y-intercept \(b\) is given by \(y = mx + b\)

Point-slope form: the equation of a line passing through the points \((x_1, y_1)\) and slope \(m\) is given by \(y - y_1 = m(x - x_1)\). While you might have preferred the simplicity of the \(y = mx + b\) form in your algebra course, the \(y - y_1 = m(x - x_1)\) form is far more useful in calculus.

Intercept form: the equation of a line with x-intercept \(a\) and y-intercept \(b\) is given by \(\frac{x}{a} + \frac{y}{b} = 1\)

General form: \(Ax + By + C = 0\) where \(A\), \(B\) and \(C\) are integers. While your algebra teacher might have required you changing the equation \(y - 1 = 2(x - 5)\) to general form \(2x - y - 9 = 0\), you will find that on the AP calculus test, it is sufficient to leave equations for a lines in point-slope form and it is recommended not to waste time changing it unless you are specifically told to do so.

Parallel lines: Two distinct lines are parallel if they have the same slope: \(m_1 = m_2\).

Normal lines: Two lines are normal (perpendicular) if their slopes are negative reciprocals: \(m_1 m_2 = -1\).

Horizontal lines have slope zero. Vertical lines have no slope (slope is undefined).

1. Find the equation of the line in slope-intercept form, with the given slope, passing through the given point.
   
   a. \(m = -4\), \((1, 2)\)
   
   \(y - 2 = -4(x - 1) \Rightarrow y = -4x + 6\)

   b. \(m = \frac{2}{3}\), \((-5, 1)\)

   \(y - 1 = \frac{2}{3}(x + 5) \Rightarrow y = \frac{2x}{3} - 7\)

   c. \(m = 0\), \((-\frac{1}{2}, \frac{3}{4})\)

   \(y = -\frac{3}{4}\)

2. Find the equation of the line in slope-intercept form, passing through the following points.

   a. \((4, 5)\) and \((-2, -1)\)

   \[m = \frac{5 + 1}{4 + 2} = 1\]

   \[y - 5 = x - 4 \Rightarrow y = x + 1\]

   b. \((0, -3)\) and \((-5, 3)\)

   \[m = \frac{-3 + 6 - 6}{-5 - 0} = \frac{-3}{5}\]

   \[y + 3 = \frac{-6}{5} x \Rightarrow y = \frac{-6}{5} x - 3\]

   c. \((\frac{3}{4}, -1)\) and \((1, \frac{1}{2})\)

   \[m = \frac{\frac{1}{2} + 1 - \frac{3}{4}}{1 - \frac{3}{4}} = \frac{2 + 4}{4 - 3} = 6\]

   \[y - \frac{1}{2} = 6(x - 1) \Rightarrow y = 6x - \frac{11}{2}\]

3. Write equations of the line through the given point a) parallel and b) normal to the given line.

   a. \((4, 7)\), \(4x - 2y = 1\)

   \[y = 2x - \frac{1}{2} \Rightarrow m = 2\]

   a) \(y - 7 = 2(x - 4)\)

   b) \(y = \frac{-1}{2}(x - 4)\)

   b. \((-\frac{2}{3}, 1)\), \(x + 5y = 2\)

   \[y = -\frac{1}{5} x + 2 \Rightarrow m = -\frac{1}{5}\]

   a) \(y - 1 = -\frac{1}{5} \left(x - \frac{2}{3}\right)\)

   b) \(y = 5 \left(x - \frac{2}{3}\right)\)
Topic 8: Asymptotes

Rational functions in the form of \( y = \frac{p(x)}{q(x)} \) possibly have vertical asymptotes, lines that the graph of the curve approach but never cross. To find the **vertical asymptotes**, factor out any common factors of numerator and denominator, reduce if possible, and then set the denominator equal to zero and solve.

**Horizontal asymptotes** are lines that the graph of the function approaches when \( x \) gets very large or very small. While you learn how to find these in calculus, a rule of thumb is that if the highest power of \( x \) is in the denominator, the horizontal asymptote is the line \( y = 0 \). If the highest power of \( x \) is both in numerator and denominator, the horizontal asymptote will be the line \( y = \frac{\text{highest degree coefficient in numerator}}{\text{highest degree coefficient in denominator}} \). If the highest power of \( x \) is in the numerator, there is no horizontal asymptote, but a slant asymptote which is not used in calculus.

1) Find any vertical and horizontal asymptotes for the graph of \( y = \frac{-x^2}{x^2 - x - 6} \)

\[
y = \frac{-x^2}{x^2 - x - 6} = \frac{-x^2}{(x-3)(x+2)}
\]

**Vertical asymptotes**: \( x - 3 = 0 \Rightarrow x = 3 \) and \( x + 2 = 0 \Rightarrow x = -2 \)

**Horizontal asymptotes**: Since the highest power of \( x \) is 2 in both numerator and denominator, there is a horizontal asymptote at \( y = -1 \)

This is confirmed by the graph to the right. Note that the curve actually crosses its horizontal asymptote on the left side of the graph.

2) Find any vertical and horizontal asymptotes for the graph of \( y = \frac{3x + 3}{x^2 - 2x - 3} \)

\[
y = \frac{3x + 3}{x^2 - 2x - 3} = \frac{3(x+1)}{(x-3)(x+1)} = \frac{3}{x-3}
\]

**Vertical asymptotes**: \( x - 3 = 0 \Rightarrow x = 3 \) Note that since the \( (x+1) \) cancels, there is no vertical asymptote at \( x = 1 \), but a hole (sometimes called a removable discontinuity) in the graph.

**Horizontal asymptotes**: Since there the highest power of \( x \) is in the denominator, there is a horizontal asymptote at \( y = 0 \) (the \( x \)-axis). This is confirmed by the graph to the right.

3) Find any vertical and horizontal asymptotes for the graph of \( y = \frac{2x^2 - 4x}{x^2 + 4} \)

\[
y = \frac{2x^2 - 4x}{x^2 + 4} = \frac{2x(x-2)}{x^2 + 4}
\]

**Vertical asymptotes**: None. The denominator doesn’t factor and setting it equal to zero has no solutions.

**Horizontal asymptotes**: Since the highest power of \( x \) is 2 in both numerator and denominator, there is a horizontal asymptote at \( y = 2 \). This is confirmed by the graph to the right.
Topic 9: Negative and Fractional Exponents

In calculus, you will be required to perform algebraic manipulations with negative exponents as well as fractional exponents. You should know the definition of a negative exponent: \( x^{-n} = \frac{1}{x^n}, x \neq 0 \). Note that negative powers do not make expressions negative; they create fractions. Typically expressions in multiple-choice answers are written with positive exponents and students are required to eliminate negative exponents. Fractional exponents create roots. The definition of \( x^{1/2} = \sqrt{x} \) and \( x^{a/b} = \sqrt[b]{x^a} = (\sqrt[b]{x})^a \)

As a reminder when we multiply, we add exponents. \((x^a)(x^b) = x^{a+b}\)

When we divide, we subtract exponents: \(\frac{x^a}{x^b} = x^{a-b}, x \neq 0\)

When we raise powers, we multiply exponents: \((x^a)^b = x^{ab}\)

In your algebra course, leaving an answer with a radical in the denominator was probably not allowed. You had to rationalize the denominator \(\frac{1}{\sqrt{x}}\) changed to \((\frac{1}{\sqrt{x}})(\frac{\sqrt{x}}{\sqrt{x}}) = \frac{\sqrt{x}}{x}\) in calculus, you will find that it is not necessary to rationalize and it is recommended that you not take the time to do so.

### *Simplify and write with positive exponents. Note: #12 involves complex fractions, covered in section K.*

1. \(-8x^{-2}\)
   \[-\frac{8}{x^2}\]

2. \((-5x^3)^2\)
   \[(-5)^2 x^{6} = \frac{1}{(-5)^3} x^{6} = \frac{1}{25} x^{6}\]

3. \(\left(\frac{-3}{x^4}\right)^{-2}\)
   \[\left((-3)^{-2}\right) \left(\frac{1}{x^{8}}\right) = \frac{1}{25} x^{8}\]

4. \((36x^{10})^{1/2}\)
   \[6x^5\]

5. \((27x^3)^{-2/3}\)
   \[\left(\frac{1}{(27x^3)^{2/3}}\right) = \frac{1}{9x^2}\]

6. \((16x^{-2})^{3/4}\)
   \[16^{3/4} x^{-3/4} = \frac{8}{x^{3/4}}\]

7. \((x^{1/2} - x)^2\)
   \[\frac{1}{(x^{1/2} - x)^2} = \frac{1}{x^2 - 2x^{3/2} + x}\]

8. \((4x^2 - 12x + 9)^{1/2}\)
   \[\frac{1}{(2x - 3)^2} = \frac{1}{2x - 3}\]

9. \((x^{1/2})\left(\frac{1}{2}x^{-1/2}\right) + (x^{1/2} + 1)\left(\frac{1}{3}x^{-1/2}\right)\)
   \[\frac{x^{1/2} + x^{1/2} + 1}{3x^{1/2}} = \frac{1}{2x^{1/2} + \frac{x^{1/2} + 1}{3x^{1/2}}}\]

10. \(-\frac{2}{3}(8x)^{-5/3}(8)\)
    \[\frac{-16}{3(32)x^{5/3}} = \frac{-16}{6x^{5/3}}\]

11. \((x + 4)^{1/2} - (x - 4)^{-1/2}\)
    \[(x + 4)^{1/2} (x - 4)^{-1/2} = (x^2 - 16)^{1/2}\]

12. \((x^{-1} + y^{-1})^{-1}\)
    \[\frac{1}{\frac{1}{x} + \frac{1}{y}} \left(\frac{xy}{y + x}\right) = \frac{xy}{y + x}\]
**Topic 10: Complex Fractions**

Calculus frequently uses **complex fractions**, which are fractions within fractions. Answers are never left with complex fractions and they must be eliminated. There are two methods to eliminate complex fractions.

When the problem is in the form of $\frac{\frac{a}{b}}{\frac{c}{d}}$, we can "flip the denominator" and write it as $\frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$

However, this does not work when the numerator and denominator are not single fractions. The best way is to eliminate the complex fractions in all cases is to find the LCD (lowest common denominator) of all the fractions in the complex fraction. Multiply all terms by this LCD and you are left with a fraction that is magically no longer complex. **Important:** Note that $\frac{x^{-1}}{y^{-1}}$ can be written as $\frac{y}{x}$ but $\frac{1+x^{-1}}{y^{-1}}$ must be written as $\frac{1}{y} + \frac{1}{x}$

- **Eliminate the complex fractions.**

1. \[ \frac{2}{\frac{3}{5} \frac{6}{6}} = \frac{4}{5} \]

2. \[ \frac{1 + \frac{2}{3} \frac{6}{6}}{1 + \frac{5}{6}} = \frac{10}{11} \]

3. \[ \frac{\frac{3 + 5}{4 + 3}}{\frac{3 + 5}{2 - \frac{1}{6}}} = \frac{29}{22} \]

4. \[ \frac{1 + \frac{1}{2} x^{-1}}{1 + \frac{1}{3} x^{-1}} \]

5. \[ \frac{x - \frac{1}{x}}{x^2 + \frac{1}{4x^2}} \]

6. \[ \frac{2 x^{3x}}{\frac{5}{3}} = \frac{6x^{3x}}{25} \]

7. \[ \frac{x^{-3} + x}{x^{-2} + 1} \]

8. \[ \frac{\frac{1}{2} (2x + 5)^{3/3}}{-\frac{2}{3}} \]

9. \[ \frac{(x-1)^{1/2} - x(x-1)^{-1/2}}{x-1} \]

10. \[ \frac{\frac{1}{2} \frac{-3}{2} (2x+5)^{3/3}}{\frac{3}{4} (2x+5)^{3/3}} \]
Topic 11: Absolute Value Equations

Absolute value equations crop up in calculus, especially in BC calculus. The definition of the absolute value function is a piecewise function. \( f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases} \)

So, to solve an absolute value equation, split the absolute value equation into two equations, one with a positive parentheses and the other with a negative parentheses and solve each equation. It is possible that this procedure can lead to incorrect solutions so solutions must be checked.

• Solve the following equations.

1. \(|x - 1| = 3\)

\[
\begin{align*}
  x - 1 &= 3 \\
  x &= 4 \\
  x + 1 &= 3 \\
  x &= -2 
\end{align*}
\]

2. \(|3x + 2| = 9\)

\[
\begin{align*}
  3x + 2 &= 9 \\
  3x &= 7 \\
  x &= \frac{7}{3} \\
  -3x - 2 &= 9 \\
  -3x &= -11 \\
  x &= \frac{-11}{3} 
\end{align*}
\]

• Solve the following equations.

3. \(|2x - 1| - x = 5\)

\[
\begin{align*}
  2x - 1 - x &= 5 \\
  x &= 6 \\
  -3x &= 4 \\
  x &= \frac{4}{3} 
\end{align*}
\]

4. \(|x + 5| + 5 = 0\)

\[
\begin{align*}
  x + 5 + 5 &= 0 \\
  x &= -10 \\
  -x - 5 + 5 &= 0 \\
  x &= 0 
\end{align*}
\]

Both answers are invalid. It is impossible to add 5 to an absolute value and get 0.

5. \(|x^2 - x| = 2\)

\[
\begin{align*}
  (x^2 - x) &= 2 \\
  x^2 - x - 2 &= 0 \\
  (x - 1)(x + 2) &= 0 \\
  x &= 2, x = -1 \\
  No \ real \ solution \\
  Both \ solutions \ check 
\end{align*}
\]

6. \(|x - 10| = x^2 - 10x\)

\[
\begin{align*}
  x - 10 &= x^2 - 10x \\
  x^2 - 9x - 10 &= 0 \\
  (x - 10)(x + 1) &= 0 \\
  x &= 10, x = -1 \\
  Of \ the \ three \ solutions, \ only \ x = -1 \ and \ x = 10 \ are \ valid. 
\end{align*}
\]

7. \(|x| + |2x - 2| = 8\)

\[
\begin{align*}
  x + 2x - 2 &= 8 \\
  3x &= 10 \\
  x &= \frac{10}{3} \\
  -x + 2x - 2 &= 8 \\
  x &= 6 \\
  -x - (2x - 2) &= 8 \\
  -3x &= 6 \\
  x &= -6 \\
  x &= -2 
\end{align*}
\]

Of the four solutions, only \(x = \frac{10}{3}\) and \(x = -2\) are valid.
Topic 12: Exponential Functions and Logarithms

Calculus spends a great deal of time on exponential functions in the form of \( b^x \). Don’t expect that when you start working with them in calculus, your teacher will review them. So learn them now! Students must know that the definition of a logarithm is based on exponential equations. If \( y = b^x \) then \( x = \log_b y \). So when you are trying to find the value of \( \log_2 32 \), state that \( \log_2 32 = x \) and \( 2^x = 32 \) and therefore \( x = 5 \).

If the base of a log statement is not specified, it is defined to be 10. When we asked for \( \log 100 \), we are solving the equation \( 10^x = 100 \) and \( x = 2 \). The function \( y = \log x \) has domain \((0, \infty)\) and range \((\infty, \infty)\). In calculus, we primarily use logs with base \( e \), which are called natural logs (ln). So finding \( \ln 5 \) is the same as solving the equation \( e^x = 5 \). Students should know that the value of \( e = 2.71828 \).

There are three rules that students must keep in mind that will simplify problems involving logs and natural logs. These rules work with logs of any base including natural logs.

- i. \( \log a + \log b = \log(ab) \)
- ii. \( \log a - \log b = \log\left(\frac{a}{b}\right) \)
- iii. \( \log a^b = b \log a \)

1. Find a. \( \log_4 8 \)
   
   \[
   \log_4 8 = \frac{3}{2} \quad 4^{\frac{3}{2}} = 2^3
   \]

2. Solve
   a. \( \log_9(x^2 - x + 3) = \frac{1}{2} \)
   
   \[
   x^2 - x + 3 = 9^{\frac{1}{2}}
   \]
   \[
   x(x - 1) = 0
   \]
   \[
   x = 0, x = 1
   \]

   d. \( 5^x = 20 \)
   
   \[
   \log 5^x = \log 20 \quad \frac{x \log 5}{\log 5} = \log 20
   \]
   \[
   x = \frac{\log 20}{\log 5} \quad \text{or} \quad x = \frac{\ln 20}{\ln 5}
   \]
Topic 13: Basic Right Angle Trigonometry

Trigonometry is an integral part of AP calculus. Students must know the basic trig function definitions in terms of opposite, adjacent and hypotenuse as well as the definitions if the angle is in standard position.

Given a right triangle with one of the angles named \( \theta \), and the sides of the triangle relative to \( \theta \) named opposite (\( y \)), adjacent (\( x \)), and hypotenuse (\( r \)) we define the 6 trig functions to be:

\[
\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{r} \quad \csc \theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{r}{y} \\
\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{r} \quad \sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{r}{x} \\
\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{x} \quad \cot \theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{x}{y}
\]

The Pythagorean theorem ties these variables together: \( x^2 + y^2 = r^2 \). Students should recognize right triangles with integer sides: 3-4-5, 5-12-13, 8-15-17, 7-24-25. Also any multiples of these sides are also sides of a right triangle. Since \( r \) is the largest side of a right triangle, it can be shown that the range of \( \sin \theta \) and \( \cos \theta \) is \([-1, 1]\), the range of \( \csc \theta \) and \( \sec \theta \) is \((-\infty, -1] \cup [1, \infty)\) and the range of \( \tan \theta \) and \( \cot \theta \) is \((-\infty, \infty)\).

Also vital to master is the signs of the trig functions in the four quadrants. A good way to remember this is A - S - T - C where All trig functions are positive in the 1st quadrant, Sin is positive in the 2nd quadrant, Tan is positive in the 3rd quadrant and Cos is positive in the 4th quadrant.

1. Let be a point on the terminal side of \( \theta \). Find the 6 trig functions of \( \theta \). (Answers need not be rationalized).

   a) \( P(-8, 6) \)
   
   \[
   x = -8, y = 6, \quad r = 10
   
   \sin \theta = \frac{3}{5}, \quad \csc \theta = \frac{5}{3} \\
   \cos \theta = -\frac{4}{5}, \quad \sec \theta = -\frac{5}{4} \\
   \tan \theta = -\frac{3}{4}, \quad \cot \theta = -\frac{4}{3}
   
   2. If \( \cos \theta = \frac{2}{3} \), \( \theta \) in quadrant IV, find \( \sin \theta \) and \( \tan \theta \)
   
   \[
   x = 2, \ r = 3, \ y = -\sqrt{5}
   
   \sin \theta = -\frac{\sqrt{5}}{3}, \ \tan \theta = -\frac{\sqrt{5}}{2}
   
   3. If \( \sec \theta = \sqrt{3} \), find \( \sin \theta \) and \( \tan \theta \)
   
   \[
   x = 1, \ y = \pm \sqrt{2}, \ r = \sqrt{3}
   
   \sin \theta = \pm \frac{2}{\sqrt{3}}, \ \tan \theta = \pm \sqrt{2}
   
   \theta \ is \ in \ quadrant \ I \ or \ IV
   
   4. Is \( 3 \cos \theta + 4 = 2 \) possible?
   
   \[
   x = -\sqrt{10}, \ y = -\sqrt{6}, \ r = 4
   
   \sin \theta = \frac{\sqrt{6}}{4}, \ \csc \theta = -\frac{4}{\sqrt{6}} \\
   \cos \theta = -\frac{\sqrt{10}}{4}, \ \sec \theta = -\frac{4}{\sqrt{10}} \\
   \tan \theta = \frac{1}{3}, \ \cot \theta = \frac{\sqrt{10}}{3}
   
   4. \cos \theta = -\frac{2}{3} \ which \ is \ possible.
Topic 14: Special Angles

Students must be able to find trig functions of quadrant angles (0, 90°, 180°, 270°) and special angles, those based on the 30° – 60° – 90° and 45° – 45° – 90° triangles.

First, for most calculus problems, angles are given and found in radians. Students must know how to convert degrees to radians and vice-versa. The relationship is \(2\pi\) radians = 360° or \(\pi\) radians = 180°. Angles are assumed to be in radians so when an angle of \(\frac{\pi}{3}\) is given, it is in radians. However, a student should be able to picture this angle as \(\frac{180°}{3} = 60°\). It may be easier to think of angles in degrees than radians, but realize that unless specified, angle measurement must be written in radians. For instance, \(\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}\).

The trig functions of quadrant angles \(0, 90°, 180°, 270°\) or \(0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\) can quickly be found. Choose a point along the angle and realize that \(r\) is the distance from the origin to that point and always positive. Then use the definitions of the trig functions.

<table>
<thead>
<tr>
<th>(\theta)</th>
<th>point</th>
<th>(x)</th>
<th>(y)</th>
<th>(r)</th>
<th>(\sin\theta)</th>
<th>(\cos\theta)</th>
<th>(\tan\theta)</th>
<th>(\csc\theta)</th>
<th>(\sec\theta)</th>
<th>(\cot\theta)</th>
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<tr>
<td>0</td>
<td>(1,0)</td>
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<td>0</td>
<td>1</td>
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<td>(\frac{\pi}{2}) or 90°</td>
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<td>1</td>
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<td>does not exist</td>
<td>1</td>
<td>does not exist</td>
<td>0</td>
</tr>
<tr>
<td>(\pi) or 180°</td>
<td>(-1,0)</td>
<td>-1</td>
<td>0</td>
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<td>does not exist</td>
</tr>
<tr>
<td>(\frac{3\pi}{2}) or 270°</td>
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<td>0</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>Does not exist</td>
<td>-1</td>
<td>does not exist</td>
<td>0</td>
</tr>
</tbody>
</table>
Because over half of the AP exam does not use a calculator, you must be able to determine trig functions of special angles. You must know the relationship of sides in both $30^\circ - 60^\circ - 90^\circ \left(\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}\right)$ and $45^\circ - 45^\circ - 90^\circ \left(\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}\right)$ triangles.

In a $30^\circ - 60^\circ - 90^\circ \left(\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}\right)$ triangle, the ratio of sides is $1 - \sqrt{3} - 2$.

In a $45^\circ - 45^\circ - 90^\circ \left(\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}\right)$ triangle, the ratio of sides is $1 - 1 - \sqrt{2}$

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\sin \theta$</th>
<th>$\cos \theta$</th>
<th>$\tan \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$30^\circ$ (or $\frac{\pi}{6}$)</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$\frac{\sqrt{3}}{3}$</td>
</tr>
<tr>
<td>$45^\circ$ (or $\frac{\pi}{4}$)</td>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>$1$</td>
</tr>
<tr>
<td>$60^\circ$ (or $\frac{\pi}{3}$)</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\sqrt{3}$</td>
</tr>
</tbody>
</table>

Special angles are any multiple of $30^\circ \left(\frac{\pi}{6}\right)$ or $45^\circ \left(\frac{\pi}{2}\right)$. To find trig functions of any of these angles, draw them and find the reference angle (the angle created with the x-axis). Although most problems in calculus will use radians, you might think easier using degrees. This will create one of the triangles above and trig functions can be found, remembering to include the sign based on the quadrant of the angle. Finally, if an angle is outside the range of $0^\circ$ to $360^\circ$ ($0$ to $2\pi$), you can always add or subtract $360^\circ$ ($2\pi$) to find trig functions of that angle. These angles are called co-terminal angles. It should be pointed out that $390^\circ \neq 30^\circ$ but $\sin 390^\circ = \sin 30^\circ$

- Find the exact value of the following

a. $4 \sin 120^\circ - 8 \cos 570^\circ$

Subtract $360^\circ$ from $570^\circ$

$4 \sin 120^\circ - 8 \cos 210^\circ$

$120^\circ$ is in quadrant II with reference angle $60^\circ$

$210^\circ$ is in quadrant III with reference angle $30^\circ$

$4 \left(\frac{\sqrt{3}}{2}\right) - 8 \left(-\frac{\sqrt{3}}{2}\right) = 6\sqrt{3}$

b. $\left[2 \cos \pi - 5 \tan \frac{7\pi}{4}\right]^2$

$2 \cos 180^\circ - 5 \tan 315^\circ$

$180^\circ$ is a quadrant angle

$315^\circ$ is in quadrant III with reference angle $45^\circ$

$\left[2(-1) - 5(-1)\right]^2 = 9$