System of Equations

In this packet we will be reviewing graphing a line and then solving a system of equations. The topic may seem a little scary, but as you will see if you can graph a line, then you can solve a system of equations. So let’s quickly review how to graph a line. There are several ways that you can graph a line and we’ll look at several.

Slope-Intercept

One of the easiest ways to graph a line is to use the slope intercept method. The hardest part of using the slope intercept is getting the equation in the form $y = mx + b$, if it’s not already in that form. If it’s in that form, you should remember that the $b$ is the y-intercept. It’s where it crosses the y-axis. The $m$ is the slope and that’s what we use to find a few more points so that we can draw the line. Remember it takes two points to make a line.

Graph $y = 2x + 4$.

So on the y-axis, (the y-axis is the one that goes up and down) you will go up to 4 and put a point. Then the slope is 2 which can be written as $\frac{2}{1}$. So from the point, we will go up two and then to the right 1, we will do this again. (We can also go down to and to the left two, because two negatives, down and left, make a positive. Finally we connect the point, and we graphed the line.

YOUR TURN  Graph each of the following linear equations.

1. $y = \frac{2}{3}x - 3$

2. $y = -3x + 1$
We can also graph by finding the **x**- and **y**-intercepts. Depending on what the equation looks like, this may be the easiest way. For example, let’s look at graphing \( 3x + 4y = 12 \). Rather than rewriting this equation in the form \( y = mx + b \). We are just going to find the **x**- and **y**-intercepts. To do this, we let \( x = 0 \) and solve for \( y \), and then we let \( y = 0 \) and solve for \( x \).

\[
3x + 4y = 12
\]

- Let \( x = 0 \) and solve for \( y \).
  
  \[
 3(0) + 4y = 12 \\
 4y = 12 \\
  y = 3
  \]

- Let \( y = 0 \) and solve for \( x \).
  
  \[
 3x + 4(0) = 12 \\
 3x = 12 \\
  x = 4
  \]

Now we just need to graph the point. On the **x**-axis, at 4 you will put a point and then on the **y**-axis, you will put a point at 3, and then just connect the dots.

**Graphing by table**

You also should remember you can graph by creating a table. We will not be reviewing that method.

Now, let’s begin working on solving a system of equations. Systems can involve many equations, but we will focus on solving just two linear equations.

### Example: Graph to find solution:

<table>
<thead>
<tr>
<th>Equation</th>
<th>( a )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 3x + 1 )</td>
<td>( a )</td>
<td>( b )</td>
</tr>
<tr>
<td>( 2y = -4x - 8 )</td>
<td>( a )</td>
<td>( b )</td>
</tr>
</tbody>
</table>

**Step 1:** **Graph the lines.**

**Methods:**

1. In **slope intercept form**; graph using the **y**-intercept (\( b \)) and slope (\( m \)).
2. Put equations into slope intercept form:
   * Add or subtract the **x**-term.
   * Divide all terms by \( # \) in front of **y**.
   Graph using **y**-intercept (\( b \)) and slope (\( m \)).
3. Make a table and find points to plot.
4. Find the **x**- and **y**-intercepts.

**Step 2:** **Identify the solutions.**

(Ordered pair where the lines intersect)

**Equation a:** \( y = 3x + 1 \)

*is in **Slope-Intercept form**.

**Use method 1**

![Slope-Intercept Graph](image)

**Solution:** (-1, -2)

**Equation a:** \( 2y = -4x - 8 \)

*is not in **Slope-Intercept form**.

**Use method 2, 3, or 4.**
Systems can have 1 solution, no solutions or an infinite number of solutions. If the line intersect, then the system has one solution. When the lines are parallel, there is no solution. Lines are parallel when the slopes are equal. Finally, lines can have an infinite number of solutions. This occurs, when the lines are the same. For example, if you have $x + y = 7$, and $2x + 2y = 14$, then you are going to have an infinite number of solutions. These two equations are identical. All of the terms in the first equation have been multiplied by 2 to give you the second equation.

YOUR TURN: Graph each of the following system to equations. Identify your solution.

1. \begin{align*}
y &= x + 2 \\
y &= 3x - 2
\end{align*}

2. \begin{align*}
y &= x - 2 \\
y &= -4
\end{align*}

3. \begin{align*}
y &= -\frac{2}{3}x + 4 \\
y &= -2x + 12
\end{align*}

4. \begin{align*}
y &= -x + 4 \\
y &= 2x + 1
\end{align*}
We can also solve a system using substitution.

**Example:**

A) \(-4x + y = 6\)
B) \(-5x – y = 21\)

**Step 1:** Isolate one of the variables

**Step 2:** Substitute the expression from Step 1 into the OTHER equation.
- The resulting equation should have only one variable, not both \(x\) and \(y\).

**Step 3:** Solve the new equation.
- This will give you one of the coordinates.

**Step 4:** Substitute the result from Step 3 into either of the original equations.

**Step 5:** Solve for the other coordinate.

**Step 6:** Write the solution as an ordered pair. \((x, y)\)

**CHECK:** After you find your solution you should check your work. Put your solutions back into both equations to make sure they work.

We know our solution is correct because in our check, both equations turn out to be true.

**YOUR TURN:** Solve each of the following system of equations by using the substitution method. (Remember to use the substitution method, you must re-write one of the equations as \(x = \) or \(y = \).) Remember you need to check your answers.

1. \[
\begin{align*}
x & = 5 \\
x + y & = 12
\end{align*}
\]

2. \[
\begin{align*}
y & = 2x \\
x + y & = 9
\end{align*}
\]
There is a third method and that’s called the elimination method.

Example: \[\begin{cases} 3x + y = 9 \\ 5x + 4y = 22 \end{cases}\]

**Step 1:** Line up the x’s and y’s

**Step 2:** Look to see if one variable has opposite coefficients
- Yes, move to step 3
- No, multiply one or both equations by a (LCM) in order to make the coefficients of the x or y terms opposites

**Step 3:** Add the equations together to eliminate one of the variables

**Step 4:** Solve for the remaining variable

**Step 5:** Substitute the value you found into one of the original equations to solve for the other variable

**Step 6:** Write your answer as an ordered pair.

**Check:** Put the values back into both equations to make sure that they work.

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\[\begin{cases} x = 3y + 1 \\ 2x + 4y = 12 \end{cases}\]
Since in our checks, the answers were the same we know that our solution, is correct. If it wasn’t, you should go back into your work to find where you made a mistake. Checking allows you to make sure that your answers are correct. (It also should get you a 100% on your assignment.)

**YOUR TURN:** Solve each of the following system of equations by using the elimination method. Remember you need to check your answers.

1. \[
\begin{align*}
2x + 2y &= -2 \\
3x - 2y &= 12
\end{align*}
\]

2. \[
\begin{align*}
6x + 5y &= 4 \\
6x - 7y &= -20
\end{align*}
\]
   **Hint:** Change all the signs on the second row.

CHECK:

CHECK:

3. \[
\begin{align*}
-x + 9y &= -5 \\
x - 5y &= 1
\end{align*}
\]

4. \[
\begin{align*}
3x + 2y &= 0 \\
x - 5y &= 17
\end{align*}
\]

CHECK: