CALCULUS HONORS SUMMER ASSIGNMENT

Please complete the following problem numbers from the enclosed packet. You may do the work on the packet itself. If there is not enough space to do your work, please use additional paper and label the problem that you are doing, and label each problem with the section & problem # (for example, A-5.)

A.	**Functions**	2, 3, 5, 7, 9 10
B.	**Domain and Range Assignment**	1, 3, 4, 7, 8, 10, 11, 12, 17
E.	**Transformation of Graphs Assignment**	1, 3, 4, 5, 10, 11, 13
F.	**Special Factorization**	1, 3, 4, 9, 11, 12, 13, 15, 16
G.	**Linear Functions**	1b, 2a, 2c, 3b, 3c, 4, 5
H.	**Solving Quadratic Equations**	1a, 1b, 1d, 1h, 1j, 1k, 11
I.	**Asymptotes**	1, 3, 4, 6, 7, 8, 13
J.	**Negative and Fractional Exponents**	2, 3, 4, 7, 8, 10, 12, 14
K.	**Eliminating Complex Fractions**	1, 3, 4, 5, 9
M.	**Adding Fractions and Solving Fractional Equations**	2a, 2b, 4a, 4b
Q.	**Right Angle Trigonometry**	1a, 1b, 1c, 2, 3, 4, 5a, 5b, 5c
S.	**Trigonometric Identities**	1, 2, 5

A. Functions

The lifeblood of precalculus is functions. A **function** is a set of points (x,y) such that for every x, there is one and only one y. In short, in a function, the x-values cannot repeat while the y-values can. In AB Calculus, all of your graphs will come from functions.

The notation for functions is either "$y=$" or "$f(x)=$". In the $f(x)$ notation, we are stating a rule to find y given a value of x.

1. If $f(x)=x^2-5x+8$, find a) $f(-6)$ b) $f\left(\frac{3}{2}\right)$ c) $\frac{f(x+h)-f(x)}{h}$

a) $f(-6)=(-6)^2-5(-6)+8$
$36+30+8$
74

b) $f\left(\frac{3}{2}\right)=\left(\frac{3}{2}\right)^2-5\left(\frac{3}{2}\right)+8$
$\frac{9}{4}-\frac{15}{2}+8$
$\frac{11}{4}$

c) $\frac{f(x+h)-f(x)}{h}$
$\frac{(x+h)^2-5(x+h)+8-(x^2-5x+8)}{h}$
$\frac{x^2+2xh+h^2-5x-5h+8-x^2+5x-8}{5}$
$\frac{h^2+2xh-5h}{h}=\frac{h(h+2x-5)}{h}=h+2x-5$

Functions do not always use the variable x. In calculus, other variables are used liberally.

2. If $A(r)=\pi r^2$, find a) $A(3)$ b) $A(2s)$ c) $A(r+1)-A(r)$

$A(3)=9\pi$

$A(2s)=\pi(2s)^2=4\pi s^2$

$A(r+1)-A(r)=\pi(r+1)^2-\pi r^2$
$\pi(2r+1)$

One concept that comes up in AP calculus is **composition of functions.** The format of a composition of functions is: plug a value into one function, determine an answer, and plug that answer into a second function.

3. If $f(x)=x^2-x+1$ and $g(x)=2x-1$, a) find $f(g(-1))$ b) find $g(f(-1))$ c) show that $f(g(x))\neq g(f(x))$

$g(-1)=2(-1)-1=-3$
$f(-3)=9+3+1=13$

$f(-1)=1+1+1=3$
$g(3)=2(3)-1=5$

$f(g(x))=f(2x-1)=(2x-1)^2-(2x-1)+1$
$=4x^2-4x+1-2x+1+1=4x^2-6x+3$
$g(f(x))=g(x^2-x+1)=2(x^2-x+1)-1$
$=2x^2-2x+1$

Finally, expect to use **piecewise functions.** A piecewise function gives different rules, based on the value of x.

4. If $f(x)=\begin{cases} x^2-3, & x\geq 0 \\ 2x+1, & x<0 \end{cases}$, find a) $f(5)$ b) $f(2)-f(-1)$ c) $f(f(1))$

$f(5)=25-3=22$

$f(2)-f(-1)=1-(-1)=2$

$f(1)=-2,\ f(-2)=-3$

A. Function Assignment

- If $f(x) = 4x - x^2$, find

1. $f(4) - f(-4)$

2. $\sqrt{f\left(\frac{3}{2}\right)}$

3. $\frac{f(x+h) - f(x-h)}{2h}$

- If $V(r) = \frac{4}{3}\pi r^3$, find

4. $V\left(\frac{3}{4}\right)$

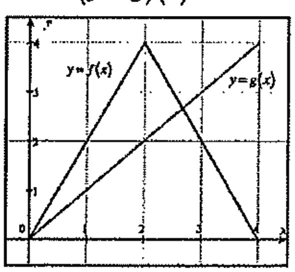
5. $V(r+1) - V(r-1)$

6. $\frac{V(2r)}{V(r)}$

- If $f(x)$ and $g(x)$ are given in the graph below, find

7. $(f-g)(3)$

8. $f(g(3))$



- If $f(x) = x^2 - 5x + 3$ and $g(x) = 1 - 2x$, find

9. $f(g(x))$

- If $f(x) = \begin{cases} \sqrt{x+2} - 2, & x \geq 2 \\ x^2 - 1, & 0 \leq x < 2 \\ -x, & x < 0 \end{cases}$, find

10. $f(0) - f(2)$

11. $\sqrt{5 - f(-4)}$

12. $f(f(3))$

B. Domain and Range

First, since questions in calculus usually ask about behavior of functions in intervals, understand that intervals can be written with a description in terms of $<, \leq, >, \geq$ or by using **interval notation.**

Description	Interval notation	Description	Interval notation	Description	Interval notation
$x>a$	(a,∞)	$x\leq a$	$(-\infty,a]$	$a\leq x<b$	$[a,b)$
$x\geq a$	$[a,\infty)$	$a<x<b$	(a,b) - open interval	$a<x\leq b$	$(a,b]$
$x<a$	$(-\infty,a)$	$a\leq x\leq b$	$[a,b]$ - closed interval	All real numbers	$(-\infty,\infty)$

If a solution is in one interval or the other, interval notation will use the connector $\cup$. So $x\leq 2$ or $x>6$ would be written $(-\infty,2]\cup(6,\infty)$ in interval notation. Solutions in intervals are usually written in the easiest way to define it. For instance, saying that $x<0$ or $x>0$ or $(-\infty,0)\cup(0,\infty)$ is best expressed as $x\neq 0$.

The **domain of a function** is the set of allowable x-values. The domain of a function f is $(-\infty,\infty)$ except for values of x which create a zero in the denominator, an even root of a negative number or a logarithm of a non-positive number. The domain of $a^{p(x)}$ where a is a positive constant and $p(x)$ is a polynomial is $(-\infty,\infty)$.

• Find the domain of the following functions using interval notation:

1. $f(x)=x^2-4x+4$ — $\boxed{(-\infty,\infty)}$

2. $y=\dfrac{6}{x-6}$ — $\boxed{x\neq 6}$

3. $y=\dfrac{2x}{x^2-2x-3}$ — $\boxed{x\neq -1, x\neq 3}$

4. $y=\sqrt{x+5}$ — $\boxed{[-5,\infty)}$

5. $y=\sqrt[3]{x+5}$ — $\boxed{(-\infty,\infty)}$

6. $y=\dfrac{x^2+4x+6}{\sqrt{2x+4}}$ — $\boxed{(-2,\infty)}$

The **range of a function** is the set of allowable y-values. Finding the range of functions algebraically isn't as easy (it really is a calculus problem), but visually, it is the [lowest possible y-value, highest possible y-value]. Finding the range of some functions are fairly simple to find if you realize that the range of $y=x^2$ is $[0,\infty)$ as any positive number squared is positive. Also the range of $y=\sqrt{x}$ is also positive as the domain is $[0,\infty)$ and the square root of any positive number is positive. The range of $y=a^x$ where a is a positive constant is $(0,\infty)$ as constants to powers must be positive.

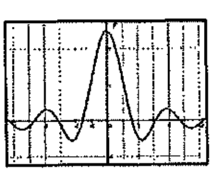
• Find the range of the following functions using interval notation:

7. $y=1-x^2$ — $\boxed{(-\infty,1]}$

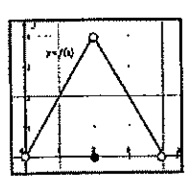
8. $y=\dfrac{1}{x^2}$ — $\boxed{(0,\infty)}$

9. $y=\sqrt{x-8}+2$ — $\boxed{[2,\infty)}$

• Find the domain and range of the following functions using interval notation.

10. 

Domain: $(-\infty,\infty)$
Range: $[-0.5,2.5]$

11. 

Domain: $(0,4)$
Range: $[0,4)$

B. Domain and Range Assignment

• Find the domain of the following functions using interval notation:

1. $f(x)=3$

2. $y=x^3-x^2+x$

3. $y=\dfrac{x^3-x^2+x}{x}$

4. $y=\dfrac{x-4}{x^2-16}$

5. $f(x)=\dfrac{1}{4x^2-4x-3}$

6. $y=\sqrt{2x-9}$

7. $f(t)=\sqrt{t^3+1}$

8. $f(x)=\sqrt[5]{x^2-x-2}$

9. $y=5^{x^2-4x-2}$

10. $y=\log(x-10)$

11. $y=\dfrac{\sqrt{2x+14}}{x^2-49}$

12. $y=\dfrac{\sqrt{5-x}}{\log x}$

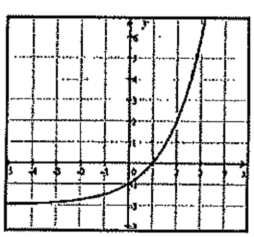
Find the range of the following functions.

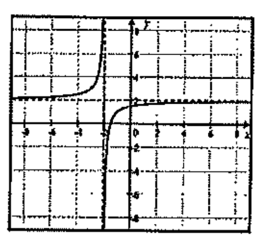
13. $y=x^4+x^2-1$

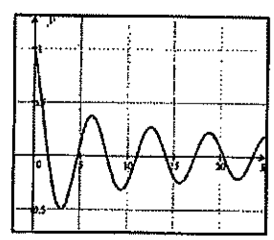
14. $y=100^x$

15. $y=\sqrt{x^2+1}+1$

Find the domain and range of the following functions using interval notation.

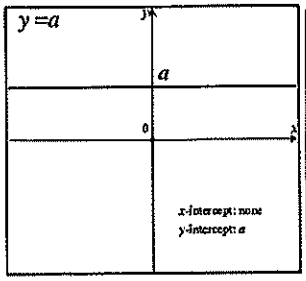
16. 

17. 

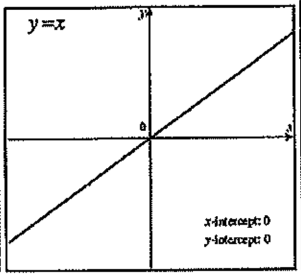
18. 

C. Graphs of Common Functions

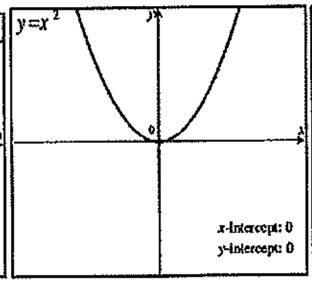
There are certain graphs that occur all the time in calculus and students should know the general shape of them, where they hit the x-axis (zeros) and y-axis (y-intercept), as well as the domain and range. There are no assignment problems for this section other than students memorizing the shape of all of these functions. In section 5, we will talk about transforming these graphs.



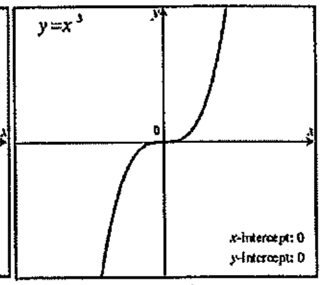
Function: $y = a$
Domain: $(-\infty,\infty)$
Range: $[a,a]$



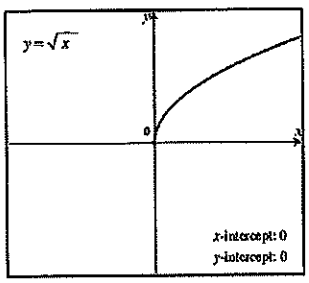
Function: $y = x$
Domain: $(-\infty,\infty)$
Range: $(-\infty,\infty)$



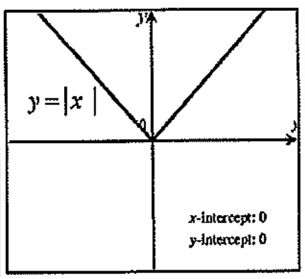
Function: $y = x^2$
Domain: $(-\infty,\infty)$
Range: $[0,\infty)$



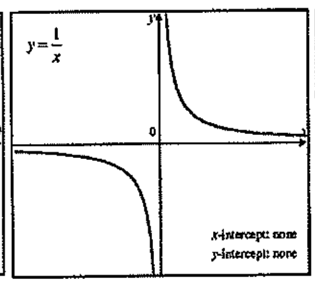
Function: $y = x^3$
Domain: $(-\infty,\infty)$
Range: $(-\infty,\infty)$



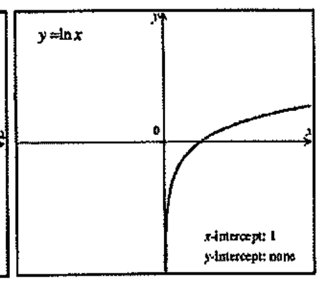
Function: $y = \sqrt{x}$
Domain: $[0,\infty)$
Range: $[0,\infty)$



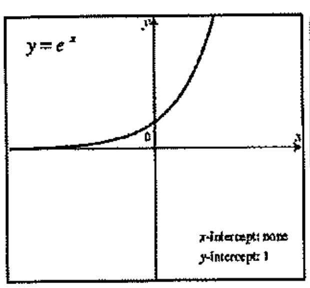
Function: $y = |x|$
Domain: $(-\infty,\infty)$
Range: $[0,\infty)$



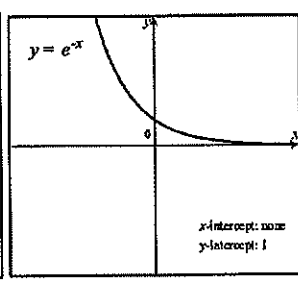
Function: $y = \dfrac{1}{x}$
Domain: $x \neq 0$
Range: $y \neq 0$



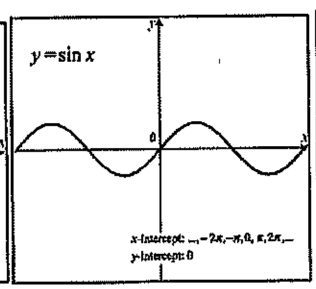
Function: $y = \ln x$
Domain: $(0,\infty)$
Range: $(-\infty,\infty)$



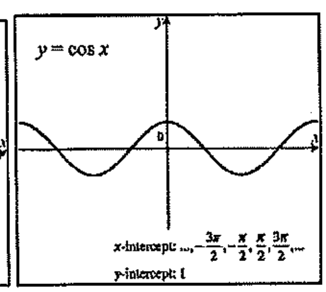
Function: $y = e^x$
Domain: $(-\infty,\infty)$
Range: $(0,\infty)$



Function: $y = e^{-x}$
Domain: $(-\infty,\infty)$
Range: $(0,\infty)$



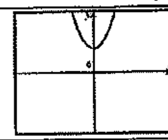
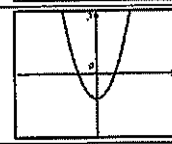
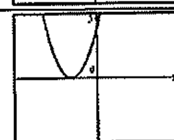
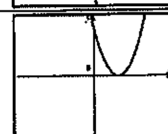
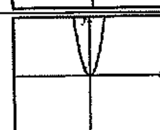
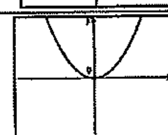
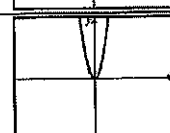
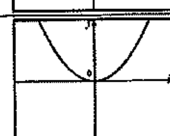
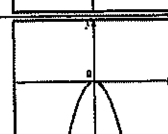
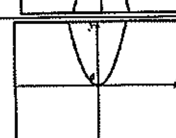
Function: $y = \sin x$
Domain: $(-\infty,\infty)$
Range: $[-1,1]$



Function: $y = \cos x$
Domain: $(-\infty,\infty)$
Range: $[-1,1]$

E. Transformation of Graphs

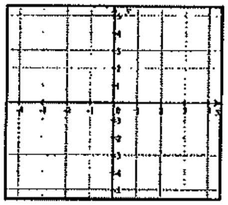
A curve in the form $y = f(x)$, which is one of the basic common functions from section C can be transformed in a variety of ways. The shape of the resulting curve stays the same but zeros and y-intercepts might change and the graph could be reversed. The table below describes transformations to a general function $y = f(x)$ with the parabolic function $f(x) = x^2$ as an example.

Notation	How $f(x)$ changes	Example with $f(x) = x^2$
$f(x)+a$	Moves graph up a units	
$f(x)-a$	Moves graph down a units	
$f(x+a)$	Moves graph a units left	
$f(x-a)$	Moves graph a units right	
$a \cdot f(x)$	$a > 1$: Vertical Stretch	
$a \cdot f(x)$	$0 < a < 1$: Vertical shrink	
$f(ax)$	$a > 1$: Horizontal compress (same effect as vertical stretch)	
$f(ax)$	$0 < a < 1$: Horizontal elongated (same effect as vertical shrink)	
$-f(x)$	Reflection across x-axis	
$f(-x)$	Reflection across y-axis	

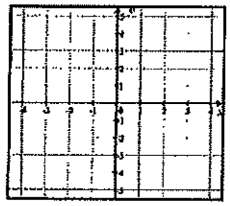
E. Transformation of Graphs Assignment

• Sketch the following equations:

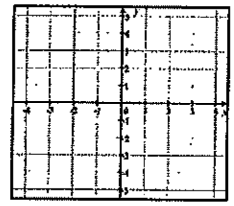
1. $y=-x^2$



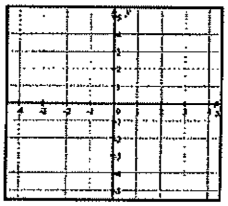
2. $y=2x^2$



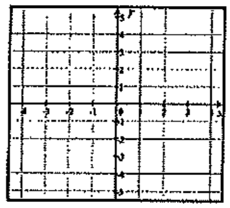
3. $y=(x-2)^2$



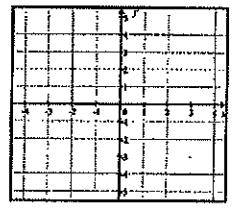
4. $y=2-\sqrt{x}$



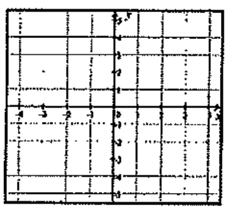
5. $y=\sqrt{x+1}+1$



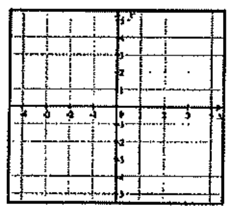
6. $y=\sqrt{4x}$



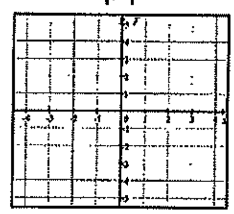
7. $y=|x+1|-3$



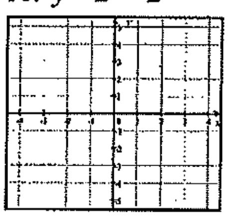
8. $y=-2|x-1|+4$



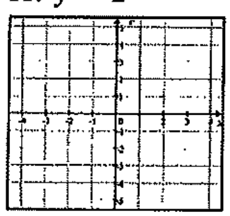
9. $y=-\left|\frac{x}{2}\right|-1$



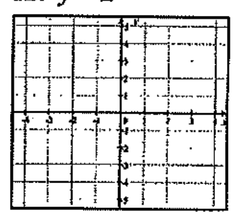
10. $y=2^x-2$



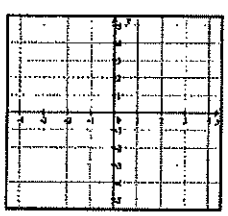
11. $y=-2^{x+2}$



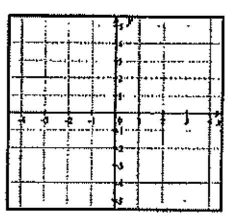
12. $y=2^{-2x}$



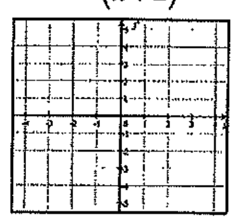
13. $y=\frac{1}{x-2}$



14. $y=\frac{-2}{x+1}$



15. $y=\frac{1}{(x+2)^2}-3$



F. Special Factorization

While factoring skills were more important in the days when A topics were specifically tested, students still must know how to factor. The special forms that occur most regularly are:

Common factor: $x^3+x^2+x=x(x^2+x+1)$
Difference of squares: $x^2-y^2=(x+y)(x-y)$ or $x^{2n}-y^{2n}=(x^n+y^n)(x^n-y^n)$
Perfect squares: $x^2+2xy+y^2=(x+y)^2$
Perfect squares: $x^2-2xy+y^2=(x-y)^2$
Sum of cubes: $x^3+y^3=(x+y)(x^2-xy+y^2)$ - Trinomial unfactorable
Difference of cubes: $x^3-y^3=(x-y)(x^2+xy+y^2)$ - Trinomial unfactorable
Grouping: $xy+xb+ay+ab=x(y+b)+a(y+b)=(x+a)(y+b)$

The term "factoring" usually means that coefficients are rational numbers. For instance, x^2-2 could technically be factored as $(x+\sqrt{2})(x-\sqrt{2})$ but since $\sqrt{2}$ is not rational, we say that x^2-2 is not factorable. It is important to know that x^2+y^2 is unfactorable.

• Completely factor the following expressions.

1. $4a^2+2a$
   $2a(2a+1)$

2. $x^2+16x+64$
   $(x+8)^2$

3. $4x^2-64$
   $4(x+4)(x-4)$

4. $5x^4-5y^4$
   $5(x^2+y^2)(x+y)(x-y)$

5. $16x^2-8x+1$
   $(4x-1)^2$

6. $9a^4-a^2b^2$
   $a^2(3a+b)(3a-b)$

7. $2x^2-40x+200$
   $2(x-10)^2$

8. x^3-8
   $(x-2)(x^2+2x+4)$

9. $8x^3+27y^3$
   $(2x+3y)(4x^2-6xy+9y^2)$

10. x^4-11x^2-80
    $(x+4)(x-4)(x^2+5)$

11. x^4-10x^2+9
    $(x+1)(x-1)(x+3)(x-3)$

12. $36x^2-64$
    $4(3x+4)(3x-4)$

13. x^3-x^2+3x-3
    $x^2(x-1)+3(x-1)$
    $(x-1)(x^2+3)$

14. $x^3+5x^2-4x-20$
    $x^2(x+5)-4(x+5)$
    $(x+5)(x-2)(x+2)$

15. $9-(x^2+2xy+y^2)$
    $9-(x+y)^2$
    $(3+x+y)(3-x-y)$

F. Special Factorization - Assignment

• Completely factor the following expressions

1. x^3-25x
2. $30x-9x^2-25$
3. $3x^3-5x^2+2x$
4. $3x^8-3$
5. $16x^4-24x^2y+9y^2$
6. $9a^4-a^2b^2$
7. $4x^4+7x^2-36$
8. $250x^3-128$
9. $\frac{8x^3}{125}+\frac{64}{y^3}$
10. x^5+17x^3+16x
11. $144+32x^2-x^4$
12. $16x^{4a}-y^{8a}$
13. $x^3-xy^2+x^2y-y^3$
14. $x^6-9x^4-81x^2+729$
15. $x^2-8xy+16y^2-25$
16. $x^5+x^3+x^2+1$
17. x^6-1
18. x^6+1

G. Linear Functions

Probably the most important concept from precalculus that is required for differential calculus is that of linear functions. The formulas you need to know backwards and forwards are:

Slope: Given two points (x_1, y_1) and (x_2, y_2), the slope of the line passing through the points can be written as: $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$.

Slope intercept form: the equation of a line with slope m and y-intercept b is given by $y = mx + b$.

Point-slope form: the equation of a line passing through the points (x_1, y_1) and slope m is given by $y - y_1 = m(x - x_1)$. While you might have preferred the simplicity of the $y = mx + b$ form in your algebra course, the $y - y_1 = m(x - x_1)$ form is far more useful in calculus.

Intercept form: the equation of a line with x-intercept a and y-intercept b is given by $\frac{x}{a} + \frac{y}{b} = 1$.

General form: $Ax + By + C = 0$ where A, B and C are integers. While your algebra teacher might have required your changing the equation $y - 1 = 2(x - 5)$ to general form $2x - y - 9 = 0$, you will find that on the AP calculus test, it is sufficient to leave equations for a lines in point-slope form and it is recommended not to waste time changing it unless you are specifically told to do so.

Parallel lines Two distinct lines are parallel if they have the same slope: $m_1 = m_2$.

Normal lines: Two lines are normal (perpendicular) if their slopes are negative reciprocals: $m_1 \cdot m_2 = -1$.

Horizontal lines have slope zero. **Vertical lines** have no slope (slope is undefined).

1. Find the equation of the line in slope-intercept form, with the given slope, passing through the given point.

a. $m = -4, (1,2)$

$y - 2 = -4(x - 1) \Rightarrow y = -4x + 6$

b. $m = \frac{2}{3}, (5,1)$

$y - 1 = \frac{2}{3}(x - 5) \Rightarrow y = \frac{2x}{3} - \frac{7}{3}$

c. $m = 0, \left(-\frac{1}{2}, \frac{3}{4}\right)$

$y = -\frac{3}{4}$

2. Find the equation of the line in slope-intercept form, passing through the following points.

a. $(4,5)$ and $(-2,-1)$

$m = \frac{5+1}{4+2} = 1$

$y - 5 = x - 4 \Rightarrow y = x + 1$

b. $(0,-3)$ and $(-5,3)$

$m = \frac{3+3}{-5-0} = \frac{-6}{5}$

$y + 3 = \frac{-6}{5}x \Rightarrow y = \frac{-6}{5}x - 3$

c. $\left(\frac{3}{4}, -1\right)$ and $\left(1, \frac{1}{2}\right)$

$m = \left(\frac{\frac{1}{2}+1}{1-\frac{3}{4}}\right)\left(\frac{4}{4}\right) = \frac{2+4}{4-3} = 6$

$y - \frac{1}{2} = 6(x - 1) \Rightarrow y = 6x - \frac{11}{2}$

3. Write equations of the line through the given point a) parallel and b) normal to the given line.

a. $(4,7)$, $4x - 2y = 1$

$y = 2x - \frac{1}{2} \Rightarrow m = 2$

a) $y - 7 = 2(x - 4)$ b) $y - 7 = \frac{-1}{2}(x - 4)$

b. $\left(\frac{2}{3}, 1\right)$, $x + 5y = 2$

$y = \frac{-1}{5}x + 2 \Rightarrow m = \frac{-1}{5}$

a) $y - 1 = \frac{-1}{5}\left(x - \frac{2}{3}\right)$ b) $y - 1 = 5\left(x - \frac{2}{3}\right)$

G. Linear Functions - Assignment

1. Find the equation of the line in slope-intercept form, with the given slope, passing through the given point.

   a. $m = -7, (-3,-7)$ b. $m = \frac{-1}{2}, (2,-8)$ c. $m = \frac{2}{3}, \left(-6, \frac{1}{3}\right)$

2. Find the equation of the line in slope-intercept form, passing through the following points.

   a. $(-3,6)$ and $(-1,2)$ b. $(-7,1)$ and $(3,-4)$ c. $\left(-2, \frac{2}{3}\right)$ and $\left(\frac{1}{2}, 1\right)$

3. Write equations of the line through the given point a) parallel and b) normal to the given line.

   a. $(5,-3)$, $x + y = 4$ b. $(-6,2)$, $5x + 2y = 7$ c. $(-3,-4)$, $y = -2$

4. Find an equation of the line containing $(4,-2)$ and parallel to the line containing $(-1,4)$ and $(2,3)$. Put your answer in general form.

5. Find k if the lines $3x - 5y = 9$ and $2x + ky = 11$ are a) parallel and b) perpendicular.

H. Solving Quadratic Equations

Solving quadratics in the form of $ax^2+bx+c=0$ usually show up on the AP exam in the form of expressions that can easily be factored. But occasionally, you will be required to use the quadratic formula. When you have a quadratic equation, factor it, set each factor equal to zero and solve. If the quadratic equation doesn't factor or if factoring is too time-consuming, use the quadratic formula:

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$. The **discriminant** b^2-4ac will tell you how many solutions the quadratic has:

$$b^2-4ac\begin{cases}>0,\ 2\text{ real solutions (if a perfect square, the solutions are rational)}\\=0,\ 1\text{ real solution}\\<0,\ 0\text{ real solutions (or 2 imaginary solutions, but AP calculus does not deal with imaginaries)}\end{cases}$$

1. Solve for x.

a. $x^2+3x+2=0$

$(x+2)(x+1)=0$
$x=-2, x=-1$

b. $x^2-10x+25=0$

$(x-5)^2=0$
$x=5$

c. $x^2-64=0$

$(x-8)(x+8)=0$
$x=8, x=-8$

d. $2x^2+9x=18$

$(2x-3)(x+6)=0$
$x=\frac{3}{2}, x=-6$

e. $12x^2+23x=-10$

$(4x+5)(3x+2)=0$
$x=-\frac{5}{4}, x=-\frac{2}{3}$

f. $48x-64x^2=9$

$(8x-3)^2=0$
$x=\frac{3}{8}$

g. $x^2+5x=2$

$x=\dfrac{-5\pm\sqrt{25+8}}{2}$
$x=\dfrac{-5\pm\sqrt{33}}{2}$

h. $8x-3x^2=2$

$x=\dfrac{8\pm\sqrt{64-24}}{6}$
$x=\dfrac{8\pm2\sqrt{10}}{6}=\dfrac{4\pm\sqrt{10}}{3}$

i. $6x^2+5x+3=0$

$x=\dfrac{-5\pm\sqrt{25-72}}{12}=\dfrac{-5\pm\sqrt{-47}}{12}$
No real solutions

j. $x^3-3x^2+3x-9=0$

$x^2(x-3)-3(x-3)=0$
$(x-3)(x^2-3)=0$
$x=3,\ x=\pm\sqrt{3}$

k. $\dfrac{x}{3}-\dfrac{5}{2}=\dfrac{-3}{x}$

$6x\left(\dfrac{x}{3}-\dfrac{5}{2}=\dfrac{-3}{x}\right)$
$2x^2-15x+18=0$
$(2x-3)(x-6)=0$
$x=\frac{3}{2},\ x=6$

l. $x^4-7x^2-8=0$

$(x^2-8)(x^2+1)=0$
$x=\pm\sqrt{8}=\pm2\sqrt{2}$

2. If $y=5x^2-3x+k$, for what values of k will the quadratic have two real solutions?

$(-3)^2-4(5)k>0\Rightarrow 9-20k>0\Rightarrow k<\dfrac{9}{20}$

H. Solving Quadratic Equations Assignment

1. Solve for x.

a. $x^2+7x-18=0$

b. $x^2+x+\frac{1}{4}=0$

c. $2x^2-72=0$

d. $12x^2-5x=2$

e. $20x^2-56x+15=0$

f. $81x^2+72x+16=0$

g. $x^2+10x=7$

h. $3x-4x^2=-5$

i. $7x^2-7x+2=0$

j. $x+\frac{1}{x}=\frac{17}{4}$

k. $x^3-5x^2+5x-25=0$

l. $2x^4-15x^3+18x^2=0$

2. If $y=x^2+kx-k$, for what values of k will the quadratic have two real solutions?

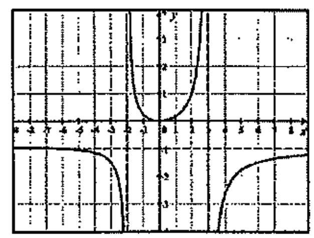
3. Find the domain of $y=\dfrac{2x-1}{6x^2-5x-6}$.

I. Asymptotes

Rational functions in the form of $y = \dfrac{p(x)}{q(x)}$ possibly have vertical asymptotes, lines that the graph of the curve approach but never cross. To find the **vertical asymptotes**, factor out any common factors of numerator and denominator, reduce if possible, and then set the denominator equal to zero and solve.

Horizontal asymptotes are lines that the graph of the function approaches when x gets very large or very small. While you learn how to find these in calculus, a rule of thumb is that if the highest power of x is in the denominator, the horizontal asymptote is the line $y = 0$. If the highest power of x is both in numerator and denominator, the horizontal asymptote will be the line $y = \dfrac{\text{highest degree coefficient in numerator}}{\text{highest degree coefficient in denominator}}$. If the highest power of x is in the numerator, there is no horizontal asymptote, but a slant asymptote which is not used in calculus.

1) Find any vertical and horizontal asymptotes for the graph of $y = \dfrac{-x^2}{x^2 - x - 6}$.

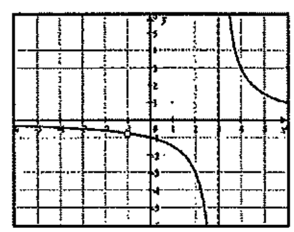


$$y = \frac{-x^2}{x^2 - x - 6} = \frac{-x^2}{(x-3)(x+2)}$$

Vertical asymptotes: $x - 3 = 0 \Rightarrow x = 3$ and $x + 2 = 0 \Rightarrow x = -2$
Horizontal asymptotes: Since the highest power of x is 2 in both numerator and denominator, there is a horizontal asymptote at $y = -1$.

This is confirmed by the graph to the right. Note that the curve actually crosses its horizontal asymptote on the left side of the graph.

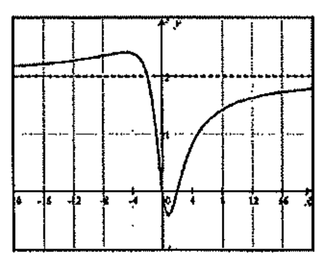
2) Find any vertical and horizontal asymptotes for the graph of $y = \dfrac{3x+3}{x^2 - 2x - 3}$.



$$y = \frac{3x+3}{x^2 - 2x - 3} = \frac{3(x+1)}{(x-3)(x+1)} = \frac{3}{x-3}$$

Vertical asymptotes: $x - 3 = 0 \Rightarrow x = 3$. Note that since the $(x+1)$ cancels, there is no vertical asymptote at $x = 1$, but a hole (sometimes called a removable discontinuity) in the graph.
Horizontal asymptotes: Since there the highest power of x is in the denominator, there is a horizontal asymptote at $y = 0$ (the x-axis). This is confirmed by the graph to the right.

3) Find any vertical and horizontal asymptotes for the graph of $y = \dfrac{2x^2 - 4x}{x^2 + 4}$.



$$y = \frac{2x^2 - 4x}{x^2 + 4} = \frac{2x(x-2)}{x^2 + 4}$$

Vertical asymptotes: None. The denominator doesn't factor and setting it equal to zero has no solutions.
Horizontal asymptotes: Since the highest power of x is 2 in both numerator and denominator, there is a horizontal asymptote at $y = 2$. This is confirmed by the graph to the right.

I. Asymptotes - Assignment

• Find any vertical and horizontal asymptotes and if present, the location of holes, for the graph of

1. $y=\dfrac{x-1}{x+5}$

2. $y=\dfrac{8}{x^2}$

3. $y=\dfrac{2x+16}{x+8}$

4. $y=\dfrac{2x^2+6x}{x^2+5x+6}$

5. $y=\dfrac{x}{x^2-25}$

6. $y=\dfrac{x^2-5}{2x^2-12}$

7. $y=\dfrac{4+3x-x^2}{3x^2}$

8. $y=\dfrac{5x+1}{x^2-x-1}$

9. $y=\dfrac{1-x-5x^2}{x^2+x+1}$

10. $y=\dfrac{x^3}{x^2+4}$

11. $y=\dfrac{x^3+4x}{x^3-2x^2+4x-8}$

12. $y=\dfrac{10x+20}{x^3-2x^2-4x+8}$

13. $y=\dfrac{1}{x}-\dfrac{x}{x+2}$ (hint: express with a common denominator)

J. Negative and Fractional Exponents

In calculus, you will be required to perform algebraic manipulations with **negative exponents** as well as **fractional exponents.** You should know the definition of a negative exponent: $x^{-n} = \dfrac{1}{x^n}, x \neq 0$. Note that negative powers do not make expressions negative; they create fractions. Typically expressions in multiple-choice answers are written with positive exponents and students are required to eliminate negative exponents. Fractional exponents create roots. The definition of $x^{1/2} = \sqrt{x}$ and $x^{a/b} = \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a$.

As a reminder: when we multiply, we add exponents: $\left(x^a\right)\left(x^b\right) = x^{a+b}$.

When we divide, we subtract exponents: $\dfrac{x^a}{x^b} = x^{a-b}, x \neq 0$

When we raise powers, we multiply exponents: $\left(x^a\right)^b = x^{ab}$

In your algebra course, leaving an answer with a radical in the denominator was probably not allowed. You had to rationalize the denominator: $\dfrac{1}{\sqrt{x}}$ changed to $\left(\dfrac{1}{\sqrt{x}}\right)\left(\dfrac{\sqrt{x}}{\sqrt{x}}\right) = \dfrac{\sqrt{x}}{x}$. In calculus, you will find that it is not necessary to rationalize and it is recommended that you not take the time to do so.

- Simplify and write with positive exponents. Note: # 12 involves complex fractions, covered in section K.

1. $-8x^{-2}$

$\boxed{\dfrac{-8}{x^2}}$

2. $\left(-5x^3\right)^{-2}$

$\boxed{(-5)^{-2}x^{-6} = \dfrac{1}{(-5)^2 x^6} = \dfrac{1}{25x^6}}$

3. $\left(\dfrac{-3}{x^4}\right)^{-2}$

$\boxed{\dfrac{(-3)^{-2}}{\left(x^4\right)^{-2}} = \dfrac{1}{(-3)^2 x^{-8}} = \dfrac{x^8}{9}}$

4. $\left(36x^{10}\right)^{1/2}$

$\boxed{6x^5}$

5. $\left(27x^3\right)^{-2/3}$

$\boxed{\dfrac{1}{\left(27x^3\right)^{2/3}} = \dfrac{1}{9x^2}}$

6. $\left(16x^{-2}\right)^{3/4}$

$\boxed{16^{3/4}x^{-4/3} = \dfrac{8}{x^{4/3}}}$

7. $\left(x^{1/2} - x\right)^{-2}$

$\boxed{\dfrac{1}{\left(x^{1/2}-x\right)^2} = \dfrac{1}{x - 2x^{3/2} + x^2}}$

8. $\left(4x^2 - 12x + 9\right)^{-1/2}$

$\boxed{\dfrac{1}{\left[(2x-3)^2\right]^{1/2}} = \dfrac{1}{2x-3}}$

9. $\left(x^{1/3}\right)\left(\dfrac{1}{2}x^{-1/2}\right) + \left(x^{1/2}+1\right)\left(\dfrac{1}{3}x^{-1/3}\right)$

$\boxed{\dfrac{x^{1/3}}{2x^{1/2}} + \dfrac{x^{1/2}+1}{3x^{1/3}} = \dfrac{1}{2x^{1/6}} + \dfrac{x^{1/2}+1}{3x^{1/3}}}$

10. $\dfrac{-2}{3}(8x)^{-5/3}(8)$

$\boxed{\dfrac{-16}{3(8x)^{5/3}} = \dfrac{-16}{3(32)x^{5/3}} = -\dfrac{1}{6x^{5/3}}}$

11. $\dfrac{(x+4)^{1/2}}{(x-4)^{-1/2}}$

$\boxed{(x+4)^{1/2}(x-4)^{1/2} = \left(x^2-16\right)^{1/2}}$

12. $\left(x^{-1} + y^{-1}\right)^{-1}$

$\boxed{\left(\dfrac{1}{\dfrac{1}{x}+\dfrac{1}{y}}\right)\left(\dfrac{xy}{xy}\right) = \dfrac{xy}{y+x}}$

J. Negative and Fractional Exponents - Assignment

Simplify and write with positive exponents.

1. -12^2x^{-5}
2. $\left(-12x^5\right)^{-2}$
3. $\left(4x^{-1}\right)^{-1}$
4. $\left(\frac{-4}{x^4}\right)^{-3}$
5. $\left(\frac{5x^3}{y^2}\right)^{-3}$
6. $\left(x^3-1\right)^{-2}$
7. $\left(121x^8\right)^{1/2}$
8. $\left(8x^2\right)^{-4/3}$
9. $\left(-32x^{-5}\right)^{-3/5}$
10. $\left(x+y\right)^{-2}$
11. $\left(x^3+3x^2+3x+1\right)^{-2/3}$
12. $x\left(x^{1/2}-x\right)^{-2}$
13. $\frac{1}{4}\left(16x^2\right)^{-3/4}(32x)$
14. $\frac{\left(x^2-1\right)^{-1/2}}{\left(x^2+1\right)^{1/2}}$
15. $\left(x^{-2}+2^{-2}\right)^{-1}$

K. Eliminating Complex Fractions

Calculus frequently uses **complex fractions**, which are fractions within fractions. Answers are never left with complex fractions and they must be eliminated. There are two methods to eliminate complex fractions:

When the problem is in the form of $\frac{\frac{a}{b}}{\frac{c}{d}}$, we can "flip the denominator" and write it as $\frac{a}{b}\cdot\frac{d}{c}=\frac{ad}{bc}$.

However, this does not work when the numerator and denominator are not single fractions. The best way is to eliminate the complex fractions in all cases is to find the LCD (lowest common denominator) of all the fractions in the complex fraction. Multiply all terms by this LCD and you are left with a fraction that is magically no longer complex. **Important:** Note that $\frac{x^{-1}}{y^{-1}}$ can be written as $\frac{y}{x}$ but $\frac{1+x^{-1}}{y^{-1}}$ must be written as $\frac{1+\frac{1}{x}}{\frac{1}{y}}$.

• Eliminate the complex fractions.

1. $\frac{\frac{2}{3}}{\frac{5}{6}}$

$$\boxed{\left(\frac{\frac{2}{3}}{\frac{5}{6}}\right)\left(\frac{6}{6}\right)=\frac{4}{5}}$$

2. $\frac{1+\frac{2}{3}}{1+\frac{5}{6}}$

$$\boxed{\left(\frac{1+\frac{2}{3}}{1+\frac{5}{6}}\right)\left(\frac{6}{6}\right)=\frac{6+4}{6+5}=\frac{10}{11}}$$

3. $\frac{\frac{3}{4}+\frac{5}{3}}{2-\frac{1}{6}}$

$$\boxed{\left(\frac{\frac{3}{4}+\frac{5}{3}}{2-\frac{1}{6}}\right)\left(\frac{12}{12}\right)=\frac{9+20}{24-2}=\frac{29}{22}}$$

4. $\frac{1+\frac{1}{2}x^{-1}}{1+\frac{1}{3}x^{-1}}$

$$\boxed{\left(\frac{1+\frac{1}{2x}}{1+\frac{1}{3x}}\right)\left(\frac{6x}{6x}\right)=\frac{6x+3}{6x+2}}$$

5. $\frac{x-\frac{1}{2x}}{x^2+\frac{1}{4x^2}}$

$$\boxed{\left(\frac{x-\frac{1}{2x}}{x^2+\frac{1}{4x^2}}\right)\left(\frac{4x^2}{4x^2}\right)=\frac{4x^3-2x}{4x^4+1}}$$

6. $\frac{\frac{2}{5}x^{5/3}}{\frac{5}{3}}$

$$\boxed{\left(\frac{\frac{2}{5}x^{5/3}}{\frac{5}{3}}\right)\left(\frac{15}{15}\right)=\frac{6x^{5/3}}{25}}$$

7. $\frac{x^{-3}+x}{x^{-2}+1}$

$$\boxed{\left(\frac{\frac{1}{x^3}+x}{\frac{1}{x^2}+1}\right)\left(\frac{x^3}{x^3}\right)=\frac{1+x^4}{x+x^3}}$$

8. $\frac{\frac{1}{2}(2x+5)^{-2/3}}{\frac{-2}{3}}$

$$\boxed{\left(\frac{\frac{1}{2}}{\frac{-2}{3}(2x+5)^{2/3}}\right)\frac{6}{6}=\frac{-3}{4(2x+5)^{2/3}}}$$

9. $\frac{(x-1)^{1/2}-\frac{x(x-1)^{-1/2}}{2}}{x-1}$

$$\boxed{\left(\frac{(x-1)^{1/2}-\frac{x}{2(x-1)^{1/2}}}{x-1}\right)\left[\frac{2(x-1)^{1/2}}{2(x-1)^{1/2}}\right] \quad \frac{2(x-1)-x}{2(x-1)^{3/2}}=\frac{x-2}{2(x-1)^{3/2}}}$$

K. Eliminating Complex Fractions - Assignment

• Eliminate the complex fractions.

1. $\dfrac{\frac{5}{8}}{\frac{-2}{3}}$

2. $\dfrac{4-\frac{2}{9}}{3+\frac{4}{3}}$

3. $\dfrac{2+\frac{7}{2}+\frac{3}{5}}{5-\frac{3}{4}}$

4. $\dfrac{x-\frac{1}{x}}{x+\frac{1}{x}}$

5. $\dfrac{1+x^{-1}}{1-x^{-2}}$

6. $\dfrac{x^{-1}+y^{-1}}{x+y}$

7. $\dfrac{x^{-2}+x^{-1}+1}{x^{-2}-x}$

8. $\dfrac{\frac{1}{3}(3x-4)^{-3/4}}{\frac{-3}{4}}$

9. $\dfrac{2x(2x-1)^{1/2}-2x^2(2x-1)^{-1/2}}{(2x-1)}$

M. Adding Fractions and Solving Fractional Equations

There are two major problem types with fractions: Adding/subtracting fractions and solving fractional equations. Algebra has taught you that in order to add fractions, you need to find an LCD and *multiply each fraction by one* … in such a way that you obtain the LCD in each fraction. However, when you solve fractional equations (equations that involve fractions), you still find the LCD but you *multiply every term by the LCD.* When you do that, all the fractions disappear, leaving you with an equation that is hopefully solvable. Answers should be checked in the original equation.

1. a. Combine: $\frac{x}{3}-\frac{x}{4}$

LCD: 12 $\quad \frac{x}{3}\left(\frac{4}{4}\right)-\frac{x}{4}\left(\frac{3}{3}\right)$

$\frac{4x-3x}{12}=\frac{x}{12}$

b. Solve: $\frac{x}{3}-\frac{x}{4}=12$

$12\left(\frac{x}{3}\right)-12\left(\frac{x}{4}\right)=12(12)$

$4x-3x=144 \Rightarrow x=144$

$x=144: \frac{144}{3}-\frac{144}{4}=48-36=12$

2. a. Combine $x+\frac{6}{x}$

LCD: x $\quad x\left(\frac{x}{x}\right)+\frac{6}{x}$

$\frac{x^2+6}{x}$

b. Solve: $x+\frac{6}{x}=5$

$x(x)+x\left(\frac{6}{x}\right)=5x$

$x^2+6=5x \Rightarrow x^2-5x+6=0$

$(x-2)(x-3)=0 \Rightarrow x=2, x=3$

$x=2: 2+\frac{6}{2}=2+3=5 \quad x=3: 3+\frac{6}{3}=3+2=5$

3 . a. Combine: $\frac{12}{x+2}-\frac{4}{x}$

LCD: $x(x+2)$ $\quad \left(\frac{12}{x+2}\right)\left(\frac{x}{x}\right)-\frac{4}{x}\left(\frac{x+2}{x+2}\right)$

$\frac{12x-4x-8}{x(x+2)}$

$\frac{8x-8}{x(x+2)}$

b. Solve $\frac{12}{x+2}-\frac{4}{x}=1$

$\frac{12}{x+2}(x)(x+2)-\frac{4}{x}(x)(x+2)=1(x)(x+2)$

$12x-4x-8=x^2+2x \Rightarrow x^2-6x+8=0$

$(x-2)(x-4)=0 \Rightarrow x=2,4$

$x=2: \frac{12}{4}-\frac{4}{2}=3-2=1 \quad x=4: \frac{12}{6}-\frac{4}{4}=2-1=1$

4. a. $\frac{x}{2x-6}-\frac{3}{x^2-6x+9}$

LCD: $2(x-3)^2$

$\frac{x}{2(x-3)}\left(\frac{x-3}{x-3}\right)-\frac{3}{(x-3)^2}\left(\frac{2}{2}\right)$

$\frac{x^2-3x-6}{2(x-3)^2}$

b. Solve $\frac{x}{2x-6}-\frac{3}{x^2-6x+9}=\frac{x-2}{3x-9}$

$\left[\frac{x}{2(x-3)}-\frac{3}{(x-3)^2}=\frac{x-2}{3(x-3)}\right]6(x-3)^2$

$3x(x-3)-18=2(x-3)(x-2)$

$3x^2-9x-18=2x^2-10x+12$

$x^2+x-30=0 \Rightarrow (x+6)(x-5)=0 \Rightarrow x=-6,5$

$x=-6: \frac{-6}{-18}-\frac{3}{81}=\frac{-8}{-27} \quad x=5: \frac{5}{4}-\frac{3}{4}=\frac{3}{6}$

M. Adding Fractions and Solving Fractional Equations - Assignment

1. a. Combine: $\frac{2}{3}-\frac{1}{x}$

   b. Solve: $\frac{2}{3}-\frac{1}{x}=\frac{5}{6}$

2. a. Combine: $\frac{1}{x-3}+\frac{1}{x+3}$

   b. Solve: $\frac{1}{x-3}+\frac{1}{x+3}=\frac{10}{x^2-9}$

3. a. Combine: $\frac{5}{2x}-\frac{5}{3x+15}$

   b. Solve: $\frac{5}{2x}-\frac{5}{3(x+5)}=\frac{5}{x}$

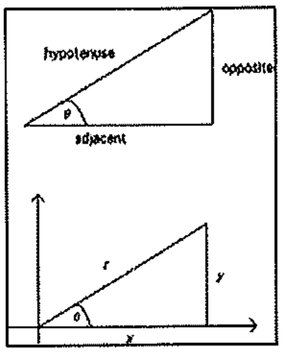
4. a. Combine: $\frac{2x-1}{x-1}-\frac{3x}{2x+1}$

   b. Solve: $\frac{2x-1}{x-1}-\frac{3x}{2x+1}=\frac{x^2+11}{2x^2-x-1}$

Q. Right Angle Trigonometry

Trigonometry is an integral part of AP calculus. Students must know the basic trig function definitions in terms of opposite, adjacent and hypotenuse as well as the definitions if the angle is in standard position.

Given a right triangle with one of the angles named θ, and the sides of the triangle relative to θ named opposite (y), adjacent (x), and hypotenuse (r) we define the 6 trig functions to be:



$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{y}{r}$ $\qquad \csc\theta = \dfrac{\text{hyptotenuse}}{\text{opposite}} = \dfrac{r}{y}$

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{x}{r}$ $\qquad \sec\theta = \dfrac{\text{hyptotenuse}}{\text{adjacent}} = \dfrac{r}{x}$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{y}{x}$ $\qquad \cot\theta = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{x}{y}$

The Pythagorean theorem ties these variables together: $x^2 + y^2 = r^2$. Students should recognize right triangles with integer sides: 3-4-5, 5-12-13, 8-15-17, 7-24-25. Also any multiples of these sides are also sides of a right triangle. Since r is the largest side of a right triangle, it can be shown that the range of $\sin\theta$ and $\cos\theta$ is $[-1,1]$, the range of $\csc\theta$ and $\sec\theta$ is $(-\infty,-1]\cup[1,\infty)$ and the range of $\tan\theta$ and $\cot\theta$ is $(-\infty,\infty)$.

Also vital to master is the signs of the trig functions in the four quadrants. A good way to remember this is A – S – T – C where All trig functions are positive in the 1st quadrant, Sin is positive in the 2nd quadrant, Tan is positive in the 3rd quadrant and Cos is positive in the 4th quadrant.

1. Let be a point on the terminal side of θ. Find the 6 trig functions of θ. (Answers need not be rationalized).

a) $P(-8,6)$

$x=-8, y=6, r=10$

$\sin\theta = \dfrac{3}{5}$ $\qquad \csc\theta = \dfrac{5}{3}$

$\cos\theta = -\dfrac{4}{5}$ $\qquad \sec\theta = -\dfrac{5}{4}$

$\tan\theta = -\dfrac{3}{4}$ $\qquad \cot\theta = -\dfrac{4}{3}$

b. $P(1,3)$

$x=1,\ y=3,\ r=\sqrt{10}$

$\sin\theta = \dfrac{3}{\sqrt{10}}$ $\qquad \csc\theta = \dfrac{\sqrt{10}}{3}$

$\cos\theta = \dfrac{1}{\sqrt{10}}$ $\qquad \sec\theta = \sqrt{10}$

$\tan\theta = 3$ $\qquad \cot\theta = \dfrac{1}{3}$

c. $P(-\sqrt{10},-\sqrt{6})$

$x=-\sqrt{10},\ y=-\sqrt{6},\ r=4$

$\sin\theta = -\dfrac{\sqrt{6}}{4}$ $\qquad \csc\theta = -\dfrac{4}{\sqrt{6}}$

$\cos\theta = -\dfrac{\sqrt{10}}{4}$ $\qquad \sec\theta = -\dfrac{4}{\sqrt{10}}$

$\tan\theta = \sqrt{\dfrac{3}{5}}$ $\qquad \cot\theta = \sqrt{\dfrac{5}{3}}$

2. If $\cos\theta = \dfrac{2}{3}$, θ in quadrant IV, find $\sin\theta$ and $\tan\theta$

$x=2,\ r=3,\ y=-\sqrt{5}$

$\sin\theta = -\dfrac{\sqrt{5}}{3}, \tan\theta = -\dfrac{\sqrt{5}}{2}$

3. If $\sec\theta = \sqrt{3}$ find $\sin\theta$ and $\tan\theta$

θ is in quadrant I or IV

$x=1,\ y=\pm\sqrt{2},\ r=\sqrt{3}$

$\sin\theta = \pm\sqrt{\dfrac{2}{3}},\ \tan\theta = \pm\sqrt{2}$

4. Is $3\cos\theta + 4 = 2$ possible?

$3\cos\theta = -2$

$\cos\theta = -\dfrac{2}{3}$ which is possible.

Q. Right Angle Trigonometry - Assignment

1. Let be a point on the terminal side of θ. Find the 6 trig functions of θ. (Answers need not be rationalized).

   a) $P(15,8)$ b. $P(-2,3)$ c. $P\left(-2\sqrt{5},-\sqrt{5}\right)$

2. If $\tan\theta = \frac{12}{5}$, θ in quadrant III, find $\sin\theta$ and $\cos\theta$

3. If $\csc\theta = \frac{6}{5}$, θ in quadrant II, find $\cos\theta$ and $\tan\theta$

4. $\cot\theta = \frac{-2\sqrt{10}}{3}$ find $\sin\theta$ and $\cos\theta$

5. Find the quadrants where the following is true: Explain your reasoning.

   a. $\sin\theta > 0$ and $\cos\theta < 0$ b. $\csc\theta < 0$ and $\cot\theta > 0$ c. all functions are negative

6. Which of the following is possible? Explain your reasoning.

   a. $5\sin\theta = -2$ b. $3\sin\alpha + 4\cos\beta = 8$ c. $8\tan\theta + 22 = 85$

S. Trigonometric Identities

Trig identities are equalities involving trig functions that are true for all values of the occurring angles. While you are not asked these identities specifically in calculus, knowing them can make some problems easier. The following chart gives the major trig identities that you should know. To prove trig identities, you usually start with the more involved expression and use algebraic rules and the fundamental trig identities. A good technique is to change all trig functions to sines and cosines.

Fundamental Trig Identities
$\csc x = \dfrac{1}{\sin x}$, $\sec x = \dfrac{1}{\cos x}$, $\cot x = \dfrac{1}{\tan x}$, $\tan x = \dfrac{\sin x}{\cos x}$, $\cot x = \dfrac{\cos x}{\sin x}$
$\sin^2 x + \cos^2 x = 1$, $1 + \tan^2 x = \sec^2 x$, $1 + \cot^2 x = \csc^2 x$
Sum Identities
$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\quad$ $\cos(A+B) = \cos A \cos B - \sin A \sin B$
Double Angle Identities
$\sin(2x) = 2\sin x \cos x$ $\quad$ $\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$

• Verify the following identities.

1. $\left(\tan^2 x + 1\right)\left(\cos^2 - 1\right) = -\tan^2 x$

$$\left(\sec^2 x\right)\left(-\sin^2 x\right)$$
$$\left(\frac{1}{\cos^2 x}\right)\left(-\sin^2 x\right)$$
$$-\tan^2 x$$

2. $\sec x - \cos x = \sin x \tan x$

$$\frac{1}{\cos x} - \cos x\left(\frac{\cos x}{\cos x}\right)$$
$$\frac{1-\cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}$$
$$\sin x\left(\frac{\sin x}{\cos x}\right) = \sin x \tan x$$

3. $\dfrac{\cot^2 x}{1+\csc x} = \dfrac{1-\sin x}{\sin x}$

$$\left(\frac{\frac{\cos^2}{\sin^2 x}}{1+\frac{1}{\sin x}}\right)\frac{\sin^2 x}{\sin^2 x} = \frac{\cos^2 x}{\sin^2 x + \sin x}$$
$$\frac{1-\sin^2 x}{\sin x(1+\sin x)} = \frac{(1+\sin x)(1-\sin x)}{\sin x(1+\sin x)}$$
$$\frac{1-\sin x}{\sin x}$$

4. $\dfrac{1+\sin x}{\cos x} + \dfrac{\cos x}{1+\sin x} = 2\sec x$

$$\left(\frac{1+\sin x}{\cos x}\right)\left(\frac{1+\sin x}{1+\sin x}\right) + \left(\frac{\cos x}{1+\sin x}\right)\left(\frac{\cos x}{\cos x}\right)$$
$$\frac{1+2\sin x+\sin^2+\cos^2 x}{\cos x(1+\sin x)}$$
$$\frac{1+2\sin x+1}{\cos x(1+\sin x)} = \frac{2+2\sin x}{\cos x(1+\sin x)}$$
$$\frac{2(1+\sin x)}{\cos x(1+\sin x)} = 2\sec x$$

5. $\cos^4 2x - \sin^4 2x = \cos 4x$

$$\left(\cos^2 2x + \sin^2 2x\right)\left(\cos^2 2x - \sin^2 2x\right)$$
$$1\left[\cos 2(2x)\right]$$
$$\cos 4x$$

6. $\sin(3\pi - x) = \sin x$

$$\sin 3\pi \cos x - \cos 3\pi \sin x$$
$$0(\cos x) - (-1)\sin x = \sin x$$

S. Trig Identities – Assignment

• Verify the following identities.

1. $(1+\sin x)(1-\sin x)=\cos^2 x$

2. $\sec^2 x+3=\tan^2 x+4$

3. $\dfrac{1-\sec x}{1-\cos x}=-\sec x$

4. $\dfrac{1}{1+\tan x}+\dfrac{1}{1+\cot x}=1$

5. $\dfrac{\cos x-\cos y}{\sin x+\sin y}+\dfrac{\sin x-\sin y}{\cos x+\cos y}=0$

6. $\dfrac{\sin^3 x+\cos^3 x}{\sin x+\cos x}=1-\sin x\cos x$

7. $\csc 2x=\dfrac{\csc x}{2\cos x}$

8. $\dfrac{\cos 3x}{\cos x}=1-4\sin^2 x$