Day 1

Factoring Trinomials

Vocabulary-

1. **Term**
   A single number or variable or a combination of the two multiplied.

2. **Monomial**
   A mathematical expression made of 1 term.

3. **Trinomial**
   A mathematical expression made of 3 terms.

4. **Quadratic Form**
   A trinomial with the degree of 2, written in standard form as $ax^2+bx+c$; where $a$, $b$, and $c$ are constants.

Why do we Factor?

Factoring a polynomial, an expression with many monomials, allows us to “undo” the multiplication that has taken place. This in return allows us to simplify our problems and when setting it equal to zero allows us to even solve the polynomials for its roots/zeros/x-intercepts.
Factoring a Quadratic Function when \( a=1 \)

This is the easiest case there is to be able to factor. When \( a \) is already equal to 1 then you are looking for 2 numbers that multiply to equal \( c \), but add to equal \( b \). Let’s look at an example.

1. Find the factors of \( c \)

\[ x^2+5x+6 \]

Factors of 6:
- \( 1 \times 6 = 6 \)
- \( -1 \times -6 = 6 \)
- \( 2 \times 3 = 6 \)
- \( -2 \times -3 = 6 \)

Factors of 6 that add to be 5:
- \( 1 + 6 = 7 \)
- \( -1 + -6 = -7 \)
- \( 2 + 3 = 5 \)
- \( -2 + -3 = -5 \)

2. Once we know the factors \( c \), we need to make sure they add to \( b \).

\[ x^2+5x+6 \]

3. After finding the factors that multiply to give you \( c \) and add to give you \( b \), we can rewrite our quadratic as 2 binomials that are being multiplied.

\[ (x+2)(x+3) \]

*Notice that the two factors are added to \( x \)
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Examples:
1. \( x^2-x-6 \)

<table>
<thead>
<tr>
<th>Factors of -6</th>
<th>Factors of -6 that add to be -1</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1 \times 6 = -6</td>
<td>-1 \times 6 = 5</td>
</tr>
<tr>
<td>1 \times -6 = -6</td>
<td>1 \times -6 = -5</td>
</tr>
<tr>
<td>-2 \times 3 = -6</td>
<td>-2 \times 3 = 1</td>
</tr>
<tr>
<td>2 \times -3 = -6</td>
<td>2 \times -3 = -1</td>
</tr>
</tbody>
</table>

\((x+2)(x-3)\)

2. \( x^2+7x-18 \)

<table>
<thead>
<tr>
<th>Factors of -18</th>
<th>Factors of -6 that add to be 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1 \times 18 = -18</td>
<td>-1 + 18 = 17</td>
</tr>
<tr>
<td>1 \times -18 = -18</td>
<td>1 + -18 = -17</td>
</tr>
<tr>
<td>-2 \times 9 = -18</td>
<td>-2 + 9 = 7</td>
</tr>
<tr>
<td>2 \times -9 = -18</td>
<td>2 + -9 = -7</td>
</tr>
<tr>
<td>-3 \times 6 = -18</td>
<td>-3 + 6 = 3</td>
</tr>
<tr>
<td>3 \times -6 = -18</td>
<td>3 + -6 = -3</td>
</tr>
</tbody>
</table>

\((x-2)(x+9)\)

Practice:
1. \( x^2+6x+8 \)
2. \( x^2-4x-12 \)
3. \( x^2-9x-36 \)
4. \( x^2-10x+16 \)
5. \( x^2+4x-21 \)
Solving a Quadratic Function when \( a=1 \)

Now that we know how to factor a quadratic we need to know how to solve it when it is set equal to 0. The first steps are exactly the same find factors that multiple to give you \( c \), but add to equal \( b \). Once you have the factored form written, using the multiplicative property of zero we can determine the values of \( x \) when \( y=0 \).

*multi\textit{plicative property of zero} states that any number times 0 equals zero*

Example:

\[
x^2-15x+56=0
\]

<table>
<thead>
<tr>
<th>Factors of 56</th>
<th>Factors of 56 that add to be -15</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1 \times 56 = 56 )</td>
<td>( -1 + 56 = 57 )</td>
</tr>
<tr>
<td>( -1 \times -56 = -56 )</td>
<td>( -1 + -56 = -57 )</td>
</tr>
<tr>
<td>( 2 \times 28 = 56 )</td>
<td>( 2 + 28 = 30 )</td>
</tr>
<tr>
<td>( -2 \times -28 = 56 )</td>
<td>( -2 + -28 = -30 )</td>
</tr>
<tr>
<td>( 4 \times 14 = 56 )</td>
<td>( 4 + 14 = 18 )</td>
</tr>
<tr>
<td>( -4 \times -14 = 56 )</td>
<td>( -4 + -14 = -18 )</td>
</tr>
<tr>
<td>( 7 \times 8 = 56 )</td>
<td>( 7 + 8 = 15 )</td>
</tr>
<tr>
<td>( -7 \times -8 = 56 )</td>
<td>( -7 + -8 = -15 )</td>
</tr>
</tbody>
</table>

\((x-7)(x-8)=0\)

*new*

Either \((x-7)=0\) or \((x-8)=0\) because of the multiplicative property of 0.

\[
\begin{align*}
x - 7 & = 0 & \text{or} & \quad x - 8 & = 0 \\
+7 & \quad +7 & \quad +8 & \quad +8 \\
x & = 7 & \text{or} & \quad x & = 8
\end{align*}
\]
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Practice:

1. $x^2+12x+20=0$

2. $x^2-11x-60=0$

3. $x^2-5x-6=0$

4. $x^2-6x+8=0$

5. $x^2+4x-32=0$
Factoring a Quadratic Function when \( a \neq 1 \)

When \( a \) is not equal to 1 then you need to factor out the value before you start looking for numbers that multiply to equal \( c \), but add to equal \( b \). Let’s look at an example.

1. Factor out the \( a \) term if it is not equal to 0 by dividing each term by \( a \).

\[
2x^2-10x+12
\]

\[
\text{a does not equal 0 so factor out a 2 from each term.}
\]

\[
2\left(\frac{2}{2}x^2-\frac{10}{2}x+\frac{12}{2}\right) = 2(x^2-5x+6)
\]

2. Once the \( a \) term is factored out you can now finishing simplifying the quadratic.

\[
2(x^2-5x+6)
\]

\[
2(x-2)(x-3)
\]

Example:

1. \(-x^2+x+6\)
   - \( \text{Factor out } a -1 \quad -1(x^2-x-6) \quad \text{make sure to change signs when factoring out a negative} \)
   - \( \text{Factors of } c \text{ that add to be } b \quad -1(x-2)(x-3) \)

2. \(3x^2+21x-54\)
   - \( \text{Factor out 3 from each term } \quad 3(x^2+7x-18) \quad \text{notice the signs do not change because no negative has been factored out} \)
   - \( \text{Factors of } c \text{ that add to be } b \quad 3(x-2)(x+9) \)
Day 2

Practice:
1. $2x^2+24x+40$
2. $3x^2-33x-180$
3. $4x^2-20x-24$
4. $-2x^2+12x-16$
5. $-x^2-4x+32=0$
6. $3x^2-12x-15=0$
7. $-2x^2-14x+16=0$
Day 3

Mixed Practice Problems: Pick 5 from each section and complete.

**Factor completely.**

1. $3x + 36$
2. $4x^2 + 16x$
3. $x^2 - 14x - 40$
4. $x^2 + 4x - 12$
5. $x^2 - 144$
6. $x^4 - 16$
7. $81x^2 - 49$
8. $50x^2 - 72$
9. $2x^3 - 16x^2 - 18x$
10. $4x^2 + 17x - 15$
11. $-8x^2 - 15x + 2$
12. $x^3 - 3x^2 + 5x - 15$
13. $5rs + 25r - 3s - 15$
14. $125x^3 - 64$
15. $2x^3 + 128y^3$

**Solve the following equations.**

16. $(x - 4)^2 - 9 = 0$
17. $(x - 10)^2 - 48 = 0$
18. $x^2 + 14x + 45 = 0$
19. $x^2 + 6x - 10 = 30$
20. $4x^2 - 100 = 0$
21. $6x^2 - 48x - 54 = 0$
22. $4x^2 + 2x = 12$
23. $9x^2 + 7x - 4 = 0$
24. $3x^2 + 9x - 6 = 0$
25. $x^3 - 5x^2 - 4x + 20 = 0$