Solving Equations and Inequalities

Before we begin reviewing solving equations and inequalities, let’s review what an equation is. An equation is a statement that uses an equal symbol to show that two quantities are equal, while an inequality, use an inequality symbol, to show a relationship between the two quantities.

For example: \(2x + 3 = 5x - 7\), is an equation and \(3x - 7 \geq 4x + 10\) is an inequality statement.

Inequality symbols:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>meaning</th>
<th>Inequality Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;</td>
<td>greater than</td>
<td>Remember the open part of the symbol faces the larger quantity, while the pointed part points to the smaller quantity.</td>
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<tr>
<td>&lt;</td>
<td>less than</td>
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<td>≤</td>
<td>less than or equal to</td>
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<tr>
<td>≥</td>
<td>greater than or equal to</td>
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Now let’s review the order of operations.

**Parenthesis ⇒ Exponents⇒Multiply & Divide (left to right)⇒Add & Subtract (left to right)** You may know it as PEMDAS (Please excuse my dear aunt Sally). When you are asked to simplify a numerical expressions such as \((3 \times 2^2) - 4 + 3 \times 8 \div 6\),

- The first thing you want to look for are parentheses. We have a set \((3 \times 2^2)\), so we want to simplify that first.
- Going inside the parentheses we have \(2^2\), so we know that means \(2 \times 2\), so \(2^2 = 4\).
- Now we need to multiply the 4 by 3 and the expression inside the parentheses becomes 12.

So we now have simplified our original equation to \(12 - 4 + 3 \times 8 \div 6\). Now, if we look at our equation we only have adding, subtracting, multiplying and dividing.

- Looking at the order of operations, we know we need to start with multiplying and dividing, from left to right. So we’re going to look at \(3 \times 8\).
- We know \(3 \times 8 = 24\), so the \(3 \times 8\) will be replaced by 24.
- Looking again, we now have division. So we will divide 24 by 6, getting 4. Replacing \(12 \div 6\) by 4. All we need to do is to subtract 4 from 12 giving us 6.
Solving Linear Equations
One Step

Let’s begin by looking at \( x - 2 = 7 \). You can actually solve this equations by just asking yourself the following question? What number can I take 2 from and get 7? Answer 9. Since we are not always going to have such simple equations to solve we’re going to look at a mathematical process for solving.

Looking at \( x - 2 = 7 \), we want to solve for \( x \). In other words, we want to get \( x \) by itself. In order to get \( x \) by itself we would need to get rid of the \(-2\). In order to get rid of \(-2\), we would add 2, to the \(-2\). If we do that to one side we need to also do it to the other side.

\[
x - 2 = 7
\]

So:

\[
+2 = +2
\]

\[
x = 9
\]

In solving equations, we’re going to always use the opposite operations.

<table>
<thead>
<tr>
<th>What it looks like</th>
<th>Operation</th>
<th>Opposite operation</th>
<th>What it look like*</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x - 2 )</td>
<td>Subtracting 2</td>
<td>Adding 2</td>
<td>( x - 2 + 2 )</td>
</tr>
<tr>
<td>( x + 2 )</td>
<td>Adding 2</td>
<td>Subtracting 2</td>
<td>( x + 2 - 2 )</td>
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<tr>
<td>( 2x )</td>
<td>Multiplying by 2</td>
<td>Dividing by 2</td>
<td>( \frac{2x}{2} )</td>
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<td>( \frac{x}{2} )</td>
<td>Dividing by 2</td>
<td>Multiplying by 2</td>
<td>( \frac{x}{2} )</td>
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* Remember what you do to one side you must do to the other side.

After finding the answer we should check to make sure it works. To check to see if it works, substitute your answer back in for the variable you solved for and use the order of operations to see if you two expressions are equal.

\[
x - 2 = 7
\]

So in our problem: \( +2 = +2 \), we will replace the \( x \) with 9 and see if we get 7.

\[
x = 9
\]

\[
x - 2 = 7
\]

\[
9 - 2 = 7
\]

\[
7 = 7
\]

Since the expressions on both sides of the equation are equal, our solution \( x = 9 \) is correct. If the expressions on both sides of the equal sign were not equal, our answer would not have been correct and we should redo our problem to get the correct answer.
### Example 1
\[
\begin{align*}
x - 7 &= 14 \\
+7 &= +7 \\
x &= 21 \\
\end{align*}
\]
\[
\begin{align*}
x &= 21 \\
14 &= 14 \\
\end{align*}
\]

### Example 2
\[
\begin{align*}
x - 7 &= 14 \\
+7 &= +7 \\
21 - 7 &= 14 \checkmark \\
\end{align*}
\]
\[
\begin{align*}
x &= 7 \\
14 &= 14 \\
\end{align*}
\]

### Example 3
\[
\begin{align*}
7x &= 14 \\
\frac{7x}{7} &= \frac{14}{7} \\
x &= 2 \\
\end{align*}
\]
\[
\begin{align*}
7x &= 14 \\
7(2) &= 14 \checkmark \\
x &= 2 \\
\end{align*}
\]

### Example 4
\[
\begin{align*}
x &= 14 \\
\frac{x}{7} &= \frac{14}{7} \\
\frac{1}{1} &= \frac{7}{7} \checkmark \\
98 &= 14 \\
x &= 98 \\
14 &= 14 \\
\end{align*}
\]

### YOUR TURN

Directions: Solve and check each of the following problems on the attached sheet for Day 1 which is at the end of Day 1.

1. \( z + 6 = 10 \)
2. \( 8y = 48 \)
3. \( q - 12 = 1 \)
4. \( 18 = \frac{a}{2} \)
5. \( 11 = m - 4 \)
6. \( \frac{r}{3} = 7 \)
7. \( t - 19 = 2 \)
8. \( 1 + s = 6 \)
9. \( 24 = 4c \)
10. \( \frac{v}{5} = 9 \)
Solving Linear Equations
Two Step

The main difference between one-step and two-step equations is that there is one more step you need to do in order to solve a two-step equation.

Let’s look at solving: \(4x - 7 = 13\).

Remember that we want to solve for \(x\).

So the first thing that we’re going to do is to add 7 to both sides so that we will have \(4x\) by itself.

If you look at the equation now, you have a one-step equation. We can see that we are multiplying \(x\) by 4, so we need to divide both sides of the equation by 4.

\[
\begin{align*}
4x - 7 + 7 &= 13 + 7 \\
4x &= 20 \\
\frac{4x}{4} &= \frac{20}{4} \\
x &= 5
\end{align*}
\]

We’ve got an answer, but we have one more thing to do. We need to make sure our answer is correct, so we will substitute 5, back into the original equation for \(x\). If the expressions on both side of the equal sign are correct, we know our answer is correct.

Check:

\[
\begin{align*}
4(5) - 7 &= 13 \\
20 - 7 &= 13 \\
13 &= 13
\end{align*}
\]

<table>
<thead>
<tr>
<th>Example 1</th>
<th>Example 2</th>
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<tbody>
<tr>
<td>(-8 + b) = 4</td>
<td>(-15n + 4 = 199)</td>
</tr>
<tr>
<td>(-8 + b \left(\frac{5}{1}\right) = 4(5))</td>
<td>(-15n + 4 - 4 = 199 - 4)</td>
</tr>
<tr>
<td>(-8 + b = 20)</td>
<td>(-15n = 195)</td>
</tr>
<tr>
<td>(-8 + 8 + b = 20 + 8)</td>
<td>(-8 + 28 = 4)</td>
</tr>
<tr>
<td>(b = 28)</td>
<td>(\frac{20}{5} = 4)</td>
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<td>(4 = 4)</td>
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YOUR TURN

Directions: Solve and check each of the following problems on the attached sheet for Day 1, which starts on the next page.

1. $3p - 10 = 62$

2. $12 + \frac{x}{40} = 13$

3. $-16n + 7 = -41$

4. $8 - 19x = 350$

5. $\frac{b + 7}{25} = 1$

6. $-8 + \frac{n}{-2} = -17$
DAY 1 – WORK/ANSWER SHEET

One-Step Equations

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## Two Step Equations

<table>
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Day 2

Solving Equations with Variables on Both Side

Before beginning to solve an equation with variables on both sides, simplify both sides if possible. For example, if you have and equations that looks like this: \(3x + 7 - 2 = 4x - 8 - 3x\), the first thing we would like to do is simplify both sides of the equation. We know that \(7 - 2 = 5\) and \(4x - 3x = x\). So, the equation can be written as: \(3x + 5 = x - 8\).

Remember the goal is to get the variable on one side and the numbers on both sides.

\[
\begin{align*}
3x + 7 - 2 &= 4x - 8 - 3x \\
3x + 5 &= x - 8 \\
\end{align*}
\]

While it’s usually easier to begin the smaller term that contains the variable, it really doesn’t matter. We will subtract \(x\) from both sides to move the \(x\) from the right hand side of the equation to the left hand side of the equation.

Now you have a two-step equation. We want to move the 5 to the other side so we will subtract 5 from both sides.

Next we will divide both sides by 2.

\[
\begin{align*}
2x &= -13 \\
x &= -6\frac{1}{2} \text{ or } -6.5 \\
\end{align*}
\]

Remember now we need to check to see if the solution is correct. We will need to substitute \(-6\frac{1}{2}\) or \(-6.5\) back into the original equation for \(x\) on both sides of the equation.

\[
\begin{align*}
3x + 7 - 2 &= 4x - 8 - 3x \\
3(-6.5) + 7 - 2 &= 4(-6.5) - 8 - 3(-6.5) \\
\text{CHECK:} \\
-19.5 + 7 - 2 &= -26 - 8 + 19.5 \\
-12.5 - 2 &= -34 + 19.5 \\
-14.5 &= -14.5 \\
\end{align*}
\]
YOUR TURN

Directions: Solve and check each of the following problems on the attached sheet for Day 2 which is at the end of Day 2.

1. $8x - 190 = -3x + 85$
2. $-15 - 14x = 185 + 11x$
3. $2x - 90 = -10x + 102$
4. $6x - 252 = -11x + 71$
5. $6x - 168 = 147 + 15x$
6. $11x - 179 = 91 + 2x$

Solving Multi-Step Equations

To solve a multi-step equation:

1. Perform any distributive property shown in the equation.
2. Combine any like terms in the equation (do not cross the equal sign at this point.)
3. Now you have a two-step equation remaining, follow the steps for solving two step equations.
Remember the goal is to get the variable on one side and the numbers on both sides.

Begin by distributing the 2
2(x + 5) = −11
Subtract 10 from both sides
2x + 10 − 10 = −11 − 10
Divide by 2
2x = −21
2x = −21
x = −10 \frac{1}{2} \text{ or } −10.5

Now, well need to check our answer.

CHECK:
2(−5.5) = −11
−11 = −11

<table>
<thead>
<tr>
<th>Example 1</th>
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</tr>
</thead>
</table>
| −13 = 5 + 4x − 2x  
−13 = 5 + 2x  
−13 − 5 = 5 − 5 + 2x  
−18 = 2x  
−18 \frac{2}{2} = x  
x = −9 | CHECK:  
−13 = 5 + 4x − 2x  
−13 = 5 + 4(−9) − 2(−9)  
−13 = 5 − 36 + 18  
−13 = −31 + 18  
−13 = −13 |
| 1 \frac{1}{2}(4x − 10) = −7  
2x − 5 = −7  
2x − 5 + 5 = −7 + 5  
2x = −2  
2x = \frac{−2}{2}  
x = −1 | CHECK:  
1 \frac{1}{2}(4x − 10) = −7  
\frac{1}{2}(4(−1) − 10) = −7  
\frac{1}{2}(−4 − 10) = −7  
\frac{1}{2}(−14) = −7  
−7 = −7 |

YOUR TURN

Directions: Solve and check each of the following problems on the attached sheet for Day 2 which starts on the next page.

1. \(−2(4x − 3) = 10\)  
2. \(−14 = −\frac{1}{2}(8x + 12)\)

3. \(4(2x − 11) − 6x = −4\)  
4. \(−32 = 4 + 7x + 3(x − 2)\)

5. \(−13 + 7x = −3(x + 11)\)  
6. \(2.3 + 0.02(x + 20) − 4.8 = −9\)
Please print your first and last name on the line at right

Please print your teacher’s name and period on the line at right.

DAY 2 – WORK/ANSWER SHEET

Solving Equations with Variables on Both Sides

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<th>Problem Number</th>
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</table>
Solving Multi-Step Equations

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Day 3

Solving Inequalities

Solving inequalities is very much like solving equations … we do most of the same things … but we must also pay attention to the direction of the inequality.

Things you can do that will not affect the directions of the inequality:

- Add (or subtract) a number from both sides
- Multiply (or divide) both sides by a positive number
- Simplify a side

\[
3x < 7 + 5 \\
3x < 12
\]

**Example:**

\[
\frac{3x}{3} < \frac{12}{3} \\
x < 4
\]

We can simplify \(7 + 5\) without affecting the inequality sign.

We can divide both sides by 3 without affecting the inequality sign.

**BUT** these things do change the direction of the inequality

- Multiply (or divide) both sides by a negative number
- Swapping left and right hand sides

**Example 1:**

\[
12 > 2y + 7 \\
2y + 7 < 12
\]

When we swap the left and right hand sides, we must also change the direction of the inequality.

**Example 2:**

\[
-3x + 2x \leq 6 \\
-x \leq 6
\]

We can simplify \(-3x + 2x\) without affecting the inequality sign.

But when we divide by \(-1\) we must change the direction of the inequality sign.

\[
\frac{-1x}{-1} \leq \frac{6}{-1} \\
x \geq -6
\]

We should also check our answers to make sure they are true.
Day 3

Example 2, check. Since, \( x \geq -6 \), you can pick any number that is greater than equal to \(-6\). Let’s use 0. (Any time you can use zero, it will make the process easier.) We will substitute 0, back in the original equation everywhere there is an \( x \).

\[
\begin{align*}
-3x + 2x &\leq 6 \\
-3(0) + 2(0) &\leq 6 \\
0 + 0 &\leq 6 \\
0 &\leq 6
\end{align*}
\]

0 is less than 6, so our answer is correct.

Example 3

\[
\begin{align*}
\frac{2x + 5}{3} &> 4x \\
\frac{2 + 5(3)}{3} &> 4x(3) \\
2x + 5 &> 12x \\
2x - 2x + 5 &> 12x - 2x \\
5 &> 10x \\
10x &< 5 \\
10x &< \frac{5}{10} \\
x &< \frac{1}{2} \text{ or } x < 0.5
\end{align*}
\]

Check: Let’s use 0 for the check, since 0 is less than \( \frac{1}{2} \).

\[
\begin{align*}
\frac{2x + 5}{3} &> 4x \\
\frac{2(0) + 5}{3} &> 4(0) \\
\frac{5}{3} &> 0
\end{align*}
\]

This statement is true, so our answer is correct.

Example 4

\[
\begin{align*}
6x + 2 &\leq 2x + 6 \\
6x + 2 - 2 &\leq 2x + 6 - 2 \\
6x &\leq 2x + 4 \\
6x - 2x &\leq 2x - 2x + 4 \\
4x &\leq 4 \\
4x &\leq \frac{1}{4} \\
x &\leq 1
\end{align*}
\]

Check: Let’s use 0 for the check, since 0 is less than 1.

\[
\begin{align*}
6x + 2 &\leq 2x + 6 \\
6(0) + 2 &\leq 2(0) + 6 \\
0 + 2 &\leq 0 + 6 \\
2 &\leq 6
\end{align*}
\]

This statement is true, so our answer is correct.

Directions: Solve and check each of the following problems on the attached sheet for Day 2 which starts on the next page.

1. \( 3(x + 4) > 21 \)
2. \( 4(x - 1) < 8 \)
3. \( 5(x + 2) \leq 100 \)
4. \( 3(x - 2) + 5x \leq 42 \)
5. \( -2(y - 4) + 8y + 2 < 16 \)
6. \( \frac{2}{3}(-3x - 6) + 1 \leq 7 \)
7. \( 4 - (x + 2) > -7 \)
8. \( 4(x + 2) - 10x > 38 \)
9. \( 5x + 3(x - 2) \leq 42 \)
10. \( -5(x + 2) + 6(x - 2) \geq 10 \)
DAY 3 – WORK/ANSWER SHEET

Solving Inequalities

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