Contest # 1

Problem 1-1

The sum of two odd primes is even. Here, the sum of the two primes is 2019, an odd number, so one of the primes is even. The only even prime is 2, so the other prime is
\[ 2019 - 2 = 2017. \]

Problem 1-2

The length of each diagonal of the \(3 \times 4\) rectangle is 5. The side-lengths of each small right triangle are half the side-lengths of the 3-4-5 triangle. Since the 3-4-5 triangle has perimeter 12, the perimeter of each small right triangle is half that, \(\text{[6]}\).

Problem 1-3

If all 3 integers were 0, their sum would be 0. If all 3 were 9, their sum would be 27. Therefore, the number of different possible sums of all 3 integers is \(\text{[28]}\).

Problem 1-4

The length of the base must be 22, and the length of each leg must be 61. In a right triangle with a hypotenuse of length 61 and a leg of length 11, the length of the other leg (the isosceles triangle's altitude) is 60. The area of the triangle is \(11 \times 60 = 660\).

Problem 1-5

The repetition of "97" implies that \(9797 = 101 \times 97\). Now, let's write each factor as a sum of 2 squares. Since \(101 = 10^2 + 1^2\), we can use \((a,b) = (10,1)\) or \((1,10)\). Similarly, since \(97 = 9^2 + 4^2\), we could use \((c,d) = (9,4)\) or \((4,9)\). Hence, \((ac+bd)^2 + (ad-bc)^2 = (90+4)^2 + (40-9)^2 = 94^2 + 31^2\). If we use \((c,d) = (4,9)\), then we get \(49^2 + 86^2\) instead. The four possible answers are \((31,94), (94,31), (49,86), \text{or} (86,49)\).

Any ONE answer gets FULL CREDIT.
Parentheses are NOT required.
[Note: \((ac-bd)^2 + (ad+bc)^2\) can also be used.]

Problem 1-6

With 2 choices (yes or no) for each topping, it follows that, with 6 toppings, there are a total of \(2^6 = 64\) different combinations of toppings (from no topping to all 6 toppings) on each pizza. To answer the question, there are only 3 cases to consider: 1) the 3 pizzas can have the same selection of toppings in 64 different ways; or 2) the 3 pizzas can have completely different selections of toppings in \(\binom{64}{3} = \frac{64 \times 63 \times 62}{3!}\) ways; or 3) 2 pizzas can have the same toppings, different from the third pizza. To count the number of pizzas in this case, start with 2 different topping selections. The third pizza must match either of the other 2, so the number of ways this third case can happen is \(2 \times \binom{64}{2}\). Adding, we find that the total number of ways from all 3 cases is \(43,760\).