Carteret

High

School

Honors Pre-Calculus

Summer Project, 2018

Name: _____

Date: _____

HONORS PRE-CALCULUS SUMMER PACKET

Complete all steps of each problem in a well-organized notebook and draw graphs on grid paper. Be neat and attempt all problems. You should be able to do every problem **without a calculator** (except graphing problems where you might use calculator to create table of values or might want to use the calculator to check your work). You would be **tested on these concepts** sometimes during the first week of class.

1.) Geometry Topics

Equations of a line:

1. Slope intercept: $y = mx + b$

   where $slope = \frac{y_2 - y_1}{x_2 - x_1}$
2. Point slope: $y - y_1 = m(x - x_1)$
3. Standard: $Ax + By + C = 0$

Directions: Solve each problem in the space provided, circling your final answer. Put all answers in standard form.

1. Write the equation of the line parallel to $2x - 6y = -1$ and containing the x-intercept of $4x - 3y = 12$.

2. Write the equation of the line in slope intercept form through the point with coordinates (-4,6) and perpendicular to $3x - 2y = 8$.

3. Find the value of "a" if a line containing the point (a, -2a) has a y-intercept of 6 and slope of -2/3.

4. Write the equation of the perpendicular bisector of the segment joining the points with coordinates of (-3,4) and (5,-2).

2.) Rules of Exponents

Properties:

$$a^m \cdot a^n = a^{m+n} \qquad (a^m)^n = a^{mn} \qquad a^{\frac{p}{r}} = \sqrt[r]{a^p}$$

$$a^{-n} = \frac{1}{a^n} \qquad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \qquad \frac{a^m}{a^n} = a^{m-n}$$

Directions: Simplify each in the space provided, showing all steps. Answers should have positive exponents. Circle your answer.

1. $(2x^2y)^0(3xy)$

2. $a^{-2}b^3a^3$

3. $\dfrac{4^{-5}4^6}{4^2}$

4. $(2x)^{-2}(2y)^3(4x)$

5. $\dfrac{\left(a^{-3}b^2c\right)^{-2}}{\left(ab^{-2}c^3\right)^{-1}}$

6. $\dfrac{2^4 8^3 16^{-2}}{32^{-1}}$

7. $\left(\dfrac{5u^2v}{2uv^2}\right)^2\left(\dfrac{-3uv}{2u^2v}\right)^3$

8. $(3^{-1}+2^{-1})^2$

9. $\dfrac{a^{-1}-3a^{-2}}{2^{-2}}$

3.) Factoring

Strategies to use:
1. Greatest Common Factor (GCF)
2. Difference of Squares
3. Trinomials
4. Sum and Difference of Cubes
   $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
   $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
5. Grouping

Directions: Factor each of the following completely, circling your final answer.

→ Trinomials

a. $2x^2 + 5x + 3$ *b.* $3x^2 + 7x + 2$ *c.* $5x^2 - 7x + 2$

d. $6x^2 - 11x + 3$ *e.* $6x^2 - 13x - 5$ *f.* $4x^2 - 11x - 3$

g. $7x^2 + 9x + 2$ *h.* $7x^2 - 10x + 3$ *i.* $2x^2 - 9x - 5$

j. $2x^2 + xy - 6y^2$ *k.* $3y^2 - 17xy - 6x^2$ *l.* $8x^2 - 27x - 20$

→ Differences of perfect squares.

a. $9x^2 - 16y2$ *b.* $225x^4 - 64y^8$ *c.* $16a^4 - 81y^8$

→ Sums and difference of perfect cubes.

a. $27x^3 + 1$ *b.* $8a^3 - y^3$

c. $y^6 + 216$ *d.* $m^6 - 2$

→ **Combinations of perfect squares and cubes.**

a. $x^6 - 1$ *b.* $x^6 + y^{12}$ *c.* $x^{12} - y^{24}$

→ **Polynomials in quadratic form**

a. $16x^4 - 40x + 25$ b. $2x^9 + 10x^6 + 12x^3$ c. $x^8 - 5x^4 - 36$

→ **Factor by grouping**

a. $2ax + 6xc + ba + 3bc$ b. $3my + 7x + 7m + 3xy$

c. $a^2 - 2ab + a - 2b$ d. $4ax - 14bx + 35by - 10ay$

e. $x^3 + 2x^2 - x - 2$ f. $x^2 + 6x + 9 - a^2$

g. $n^2 + 2nx - 1 + x^2$ h. $b^2 - y^2 - 2yp - p^2$

i. $a^2 + 2ab + b^2 - 9$ j. $x^3 + y^3 - x^2y - xy^2$

4.) Function Notation

Directions: Find the value of each in the space provided, showing all steps. Circle your answer.

Given: $f(x) = 3x - 7$ $g(x) = x^2 + 3$

1. Find f(-1)

2. $f(x + 3)$

3. f(f(x))

4. $g(x + 2) - g(x)$

5. f(g(2))

6. g(f(2))

5.) Rational Expressions

Simplify the following expressions :

1. $\frac{x}{xy^2} - \frac{2x}{x^2}$

2. $\frac{x}{x-3} + \frac{2}{3x+4}$

3. $\frac{x^2 - 5x + 6}{x - 2}$

4. $\frac{1-x}{x-1}$

5. $\frac{2x^2 + x - 6}{x^2 + 4x - 5} \bullet \frac{x^3 - 3x^2 + 2x}{4x^2 - 6x}$

6.) Simplify Complex Fractions

When simplifying complex fractions, multiply both the numerator and denominator by the reciprocal of the denominator. Remember to also look for common factors to simplify.

1. $\dfrac{\dfrac{x^2}{x-1}}{\dfrac{2x}{x-1}}$

2. $\dfrac{\dfrac{3}{x+1}-4}{\dfrac{2x}{x+1}}$

3. $\dfrac{\dfrac{x^2+2x+1}{x^2-4}}{\dfrac{x+1}{x^2-x-6}}$

7.) Radicals

Simplify radicals whenever possible:

ex1) $\sqrt{x^5y^7z^6}$

$\sqrt{x^2\,x^2\,x\cdot y^2\,y^2\,y^2\,y\cdot z^2\,z^2\,z^2}$

$x\cdot x\cdot y\cdot y\cdot y\cdot z\cdot z\cdot z\sqrt{x\bullet y}$

$x^2y^3z^3\sqrt{xy}$

ex2) $\sqrt[3]{-32}$

$\sqrt[3]{-2\cdot-2\cdot-2\cdot2\cdot2}$

$-2\sqrt[3]{4}$

ex3) $3x\sqrt{2x}\bullet 4x^2\sqrt{5x}$

$3x\cdot 4x^2\sqrt{2x\cdot 5x}$

$12x^3\sqrt{10x^2}$

$12x^4\sqrt{10}$

ex4) $5\sqrt{27}+4\sqrt{2}+7\sqrt{3}$

$15\sqrt{3}+4\sqrt{2}+7\sqrt{3}$

$22\sqrt{3}+4\sqrt{2}$

Rationalize fractions with radicals in the denominator:

ex5) $\sqrt{\frac{24}{15}}$

$\sqrt{\frac{8}{5}}$

$\frac{2\sqrt{2}}{\sqrt{5}}$

$\frac{2\sqrt{2}}{\sqrt{5}}\bullet\frac{\sqrt{5}}{\sqrt{5}}$

$\frac{2\sqrt{10}}{5}$

ex6) $\frac{2}{\sqrt[4]{32}}$

$\frac{2}{\sqrt[4]{2\cdot2\cdot2\cdot2\cdot2}}$

$\frac{\cancel{2}}{\cancel{2}\cdot\sqrt[4]{2}}$

$\frac{1}{\sqrt[4]{2}}$, now rationalize

$\frac{1}{\sqrt[4]{2}}\bullet\frac{\sqrt[4]{2\cdot2\cdot2}}{\sqrt[4]{2\cdot2\cdot2}}$

$\frac{\sqrt[4]{8}}{\sqrt[4]{2\cdot2\cdot2\cdot2}}$

$\frac{\sqrt[4]{8}}{2}$

ex7) $\frac{1}{2-\sqrt{3}}$

$\frac{1}{2-\sqrt{3}}\bullet\frac{2+\sqrt{3}}{2+\sqrt{3}}$

$\frac{2+\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2}$

$\frac{2+\sqrt{3}}{4-3}$

$2+\sqrt{3}$

Simplify the following:

1. $\sqrt[3]{24}$
2. $\sqrt[3]{-40x^6y^7}$
3. $\sqrt{75x^3}\bullet\sqrt{5x^3}$
4. $2\sqrt{48}-3\sqrt{27}$

Rationalize the following:

5. $\frac{5}{2\sqrt{3}}$
6. $\frac{2}{\sqrt[3]{5}}$
7. $\frac{2}{3+\sqrt{7}}$

True or False – Explain why.

8. $\sqrt{x}=\left|\sqrt{x}\right|$ for all x
9. $\sqrt{a^2+b^2}=a+b$
10. $\sqrt[4]{16}=\sqrt[4]{-16}$

8.) Basic Polynomial Graphs -- Translations/Transformations of Parent Functions

Be able to quickly sketch these five basic "parent" graphs from memory:

$y=\sqrt{x}$ $y=x^2$ $y=|x|$ $y=x^3$ $y=\frac{1}{x}$ $y=\log_b(x)$

To translate or move a parent function, use these basic examples:

the equation $y=(x-5)^2-3$ moves the graph 5 units to the right and 3 units down

the equation $y=(x+2)^2+4$ moves the graph 2 units to the left and 4 units up

To transform or stretch/shrink a parent function, use these basic examples:

the equation $y=\frac{1}{3}x^2$ shrinks the graph vertically so the height is $\frac{1}{3}$ as tall as the parent graph.

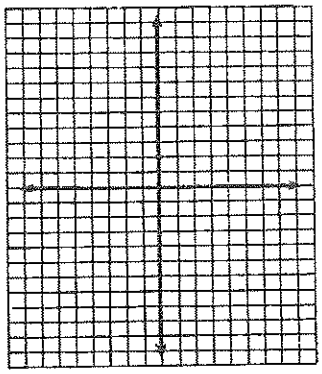
the equation $y=4x^2$ stretches the graph vertically so the height is 4 times as tall as the parent graph.

the equation $y=\left(\frac{1}{2}x\right)^3$ stretches the graph horizontally so it is 2 times as wide as the parent graph.

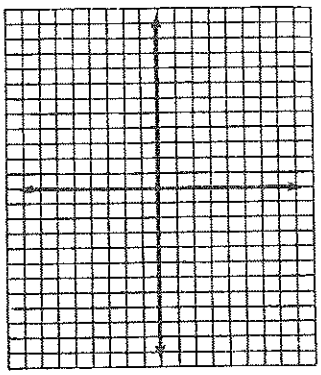
the equation $y=(3x)^3$ shrinks the graph horizontally it is $\frac{1}{3}$ as wide as the parent graph.

Use the equations below to ***ACCURATELY*** sketch the graphs the functions without the aid of your calculator. **Please label at least 3 points, including vertices, and also label the asymptotes if necessary.**

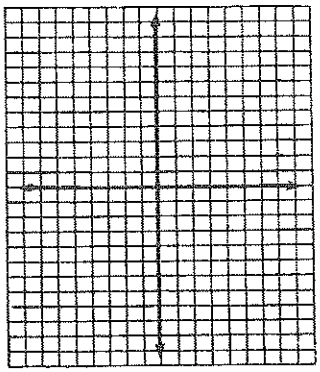
1. $y=|x-3|+2$



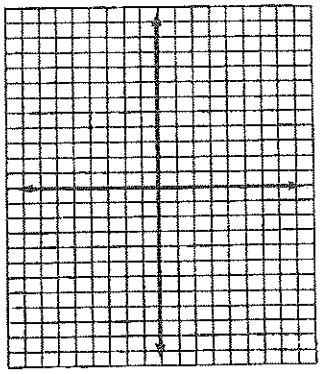
2. $y=\sqrt{x+2}$



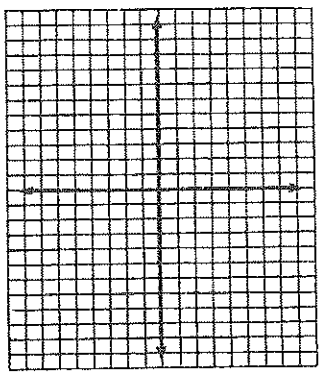
3. $y=\frac{1}{x+2}-2$



4. $y=\frac{1}{2}x^2$



5. $y=3(x-1)^3$



6. $y=\log_4(x+2)$