AP Calculus BC Summer Assignment 2015

Part 1

Textbook – Appendix A: Pages A1 to A6

Review, Study, Comprehend

Do problems: Page A11 to A14 #1-41 odd

Page A6 to Page A10: Memorize formulas 11, 12, 13, 40, 41, 43, 44, 45, 46

Part 2

Textbook Chapter 1-1

Problems: Page 11 to Page 14 Quick Check Exercises #1, 3

Exercise Set 1.1 : 1, 3, 9, 11***, 17, 18, 25

Textbook Chapter 1-2

Problems: Page 24 to Page 26 Exercise Set 1.2 : 9-16, 19, 21, 27, 28***, 29

Textbook Chapter 1-3

Problems: Page 36 to Page 39: 9, 13, 17, 21, 27, 31, 33, 39, 43, 45*, 55, 63, 65, 67, 71

Textbook Chapter 1-4

Problems: Page 49 to Page 51: 3, 11, 13, 19, 21, 25, 29

Textbook Chapter 1-5

Problems: Page 62 to Page 65: 1,3, 7, 11, 15, 17, 23, 25, 37, 41, 48***, 55

Textbook Chapter 1-6

Problems: Page 74 to Page 76: 1, 2, 3, 5, 7, 9, 11, 13, 15, 17-33 odd, 49,

Textbook Chapter 1-8

Problems: Page 93 to Page 96: 1, 5, 7, 9, 11, 15, 23, 27, 35, 41,

Chapter 1 Review – Look at and attempt 4 challenging problems

Textbook Chapter 2-1

Problems: Page 110 to Page 113: 3, 9, 11, 15, 25

Textbook Chapter 2-2

Problems: Page 121 to Page 122: 7-31 odd, 37,

Textbook Chapter 2-3

Problems: Page 131 to Page 133: 15-27 odd, 33, 47-55 odd, 63-77 odd

Part 3

Master MathMentor packet

Part 4

AP Calculus Problem Book

Should you have any questions please contact me:
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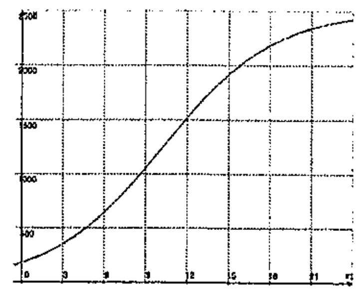
Part 3

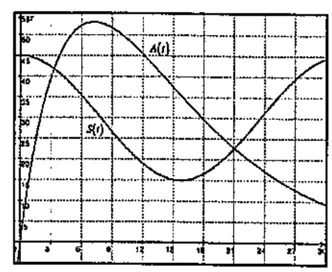
1 – Introduction

Before we get into what calculus is, here are several examples of what you could do BC (before calculus) and what you will be able to do at the end of this course.



Example 1: On April 15, many people mail in their taxes to the Internal Revenue Service. The town of Newton monitors the number of tax forms that are mailed from their post office. The total number of tax forms that are mailed from the Newton post office on April 15 is modeled by the function: $M(t)=\dfrac{2500}{1+13e^{-0.25t}}$ where t is the number of hours from 12 midnight on April 14 through 12 midnight on April 15 and $M(t)$ is the total number of letters mailed that day from the Newton post office.

What you can do with precalculus	What you will be able to do with calculus
Find the number of tax forms mailed by 9 PM on April 15. **Answer**: $M(21)\approx 2{,}340$	Find the average rate of tax forms mailed between 6 PM and 9 PM. **Answer**: 51.9 forms per hour
	Find the rate that the tax forms are coming into the post office at 9 PM. **Answer**: 37.3 forms/hour.
	Find the time of day when the letters are coming into the post office at the fastest rate and what is that rate? **Answer**: Approximately 10:15 AM at the rate of 156.25 forms/hour.
	What is the total number of hours that all the forms sit in the post office? **Answer**: 33,939 hours.



Example 2: A new car called the Sexus has its plant next to its only dealership. Cars are sold directly from the plant. Suppose at the start of the month of May there are 100 cars on the lot waiting to be sold. Cars come off the assembly line at the rate of $A(t)=20t\left(2^{-0.2t}\right)$ and cars are sold at the rate of $S(t)=30+15\cos(0.2t)$ where t represents the day of the month (for May 1, $t=0$ and for May 31, $t=30$).

What you can do with precalculus	What you will be able to do with calculus
Find the rate of cars that are produced on May 15. Answer: $A(14)=40.2$ cars/day	Find the total number of cars available to be sold in the month of May. **Answer**: Approx. 1,057 cars
Find the rate of cars that are sold on May 15. Answer: $S(14)=15.9$ cars/day	Find the total numbers of cars sold in May. **Answer**: 879 cars.
	Find the average number of cars produced per day in May. **Answer**: 31.9 cars/day.
	On what day will there be a maximum number of cars waiting to be sold and approximately what will that number be? **Answer**: May 22 and 355 cars will be on the lot.
	On what day will there be a minimum number of cars waiting to be sold and approximately what will that number be? **Answer**: May 4 and 164 cars will be on the lot.

All the math courses you have ever taken have been, in a sense, precalculus. In these courses, you are analyzing numbers and expressions and determining their current state. (ex: Find the value of x^2 when $x = 4$). In calculus, you analyze how things **change**. When real-life quantities change, they will either change in a positive direction, or in a negative direction. Your first assignment is more an English one than math. You are to find words that are used to denote positive change, negative change, or no change. I have started you off with one. Find as many as you can. When stuck, think of real life areas where change occurs such as academics, sports, economics, biology, music, etc. Get your dictionaries out!

Positive Change	**Negative Change**	**No Change**
1. Increasing.	1. Decreasing	1. Constant
2.	2.	2.
3.	3.	3.
4.	4.	4.
5.	5.	5.
6.	6.	6.
8.	8.	8.
9.	9.	9.
10.	10.	10.
11.	11.	
12.	12.	
13.	13.	
14.	14.	
15.	15.	

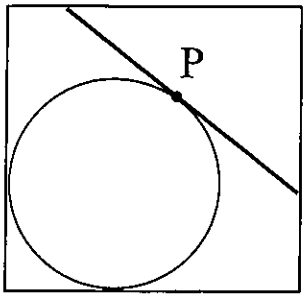
Since calculus is the study of change, let's talk about change around us. Give several things that are changing about **YOU** right now.

1.________________ 2. ________________ 3. ________________ 4. ________________

Choose one. How do we know the change is occurring? Is the change a constant change?

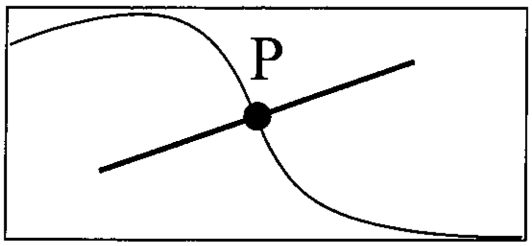
In calculus, we study four topics: 1) limits, 2) derivatives, 3) integrals (one kind) and 4) integrals (another kind). All of these 4 topics are related to the concept of change. Everything we do in this course will be related to these 4 concepts. Although we will be involved in many details, everything comes down to these 4 concepts. Your job in this course will be to answer the question "which of these 4 topics does the problem I am attempting to solve apply to." Although we will deal with many little details, we always need to see the big picture. The introductory lessons, found in the Non-Essentials section will focus on the **derivative** and the **definite integral** and meant to give you an idea what this fascinating course is all about.

2. Tangent Lines – Classwork



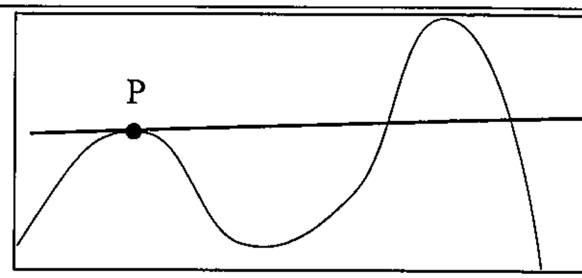
In plane geometry, we say that a line is tangent to a circle if it intersects the circle at one point. In the figure to the left, the line is tangent to the circle at point P. However, for more general curves, we need a better definition. The idea of tangent lines is crucial to your understanding of differential calculus so we must have an accurate idea of its meaning. There are many "rough" ideas of what a tangent line is that are subtly incorrect. Without looking at the rest of the page, write below what your definition of a tangent line is: A line is tangent to a curve if____

____.



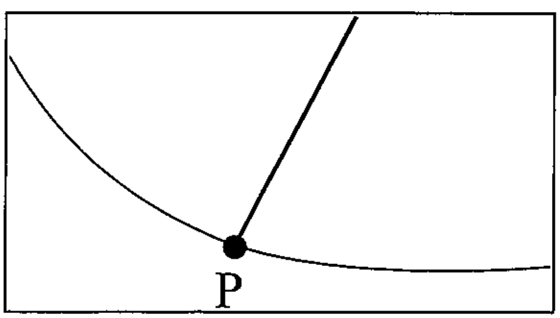
Misconception 1: "A line is tangent to a curve if it crosses the curve at one point."

This is wrong. The line crossed the curve at point P and is not a tangent line.



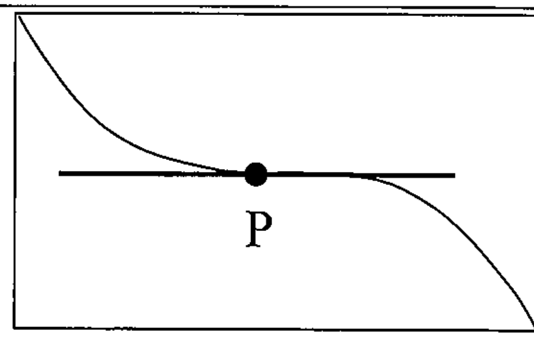
Misconception 2: "A tangent line to a curve must cross the curve only once."

This is wrong. The line is tangent to the curve at point P but it also crosses it at two other points.



Misconception 3: "A line is tangent to a curve if it touches the curve at one point but does not cross the curve."

This is also wrong. The line touches the curve at point P but it is clearly not a tangent line.

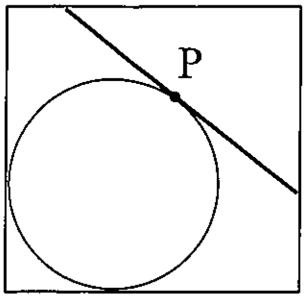


Misconception 4: "A tangent line to a curve is a line that just grazes the curve at a point but does not cross the curve."

This is also wrong. The line is tangent to the curve at point P but it does cross the curve as well.

It will be some time before we get a clear definition of a tangent line. At this point, we simply wish to give you some inkling of the difficulty of creating a definition. We have to rely on our general knowledge to draw a tangent line.

2. Tangent Lines – Classwork



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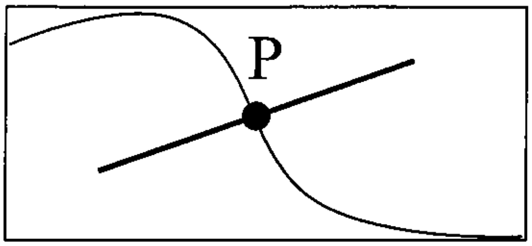

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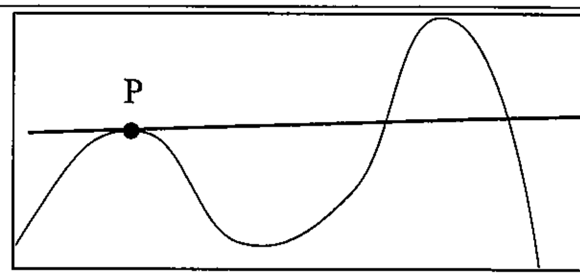

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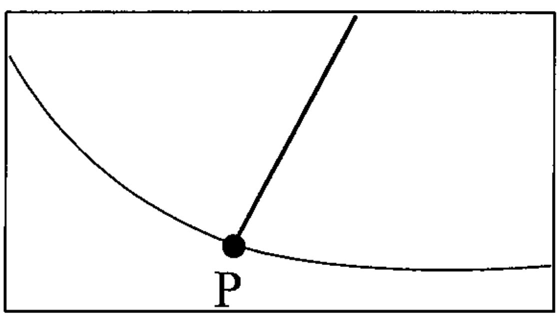

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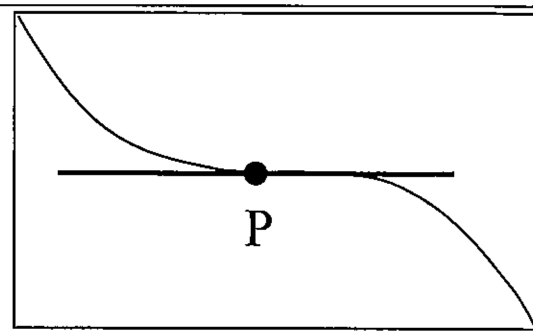

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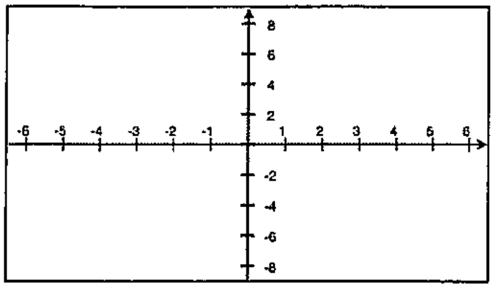
It will be some time before we get a clear definition of a tangent line. At this point, we simply wish to give you some inkling of the difficulty of creating a definition. We have to rely on our general knowledge to draw a tangent line.

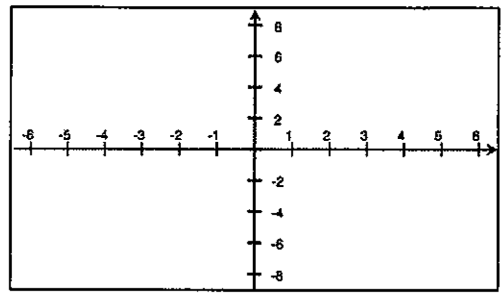
2. Tangent Lines – Homework

For each equation, graph in an appropriate window and draw the tangent line at the indicated x-value.

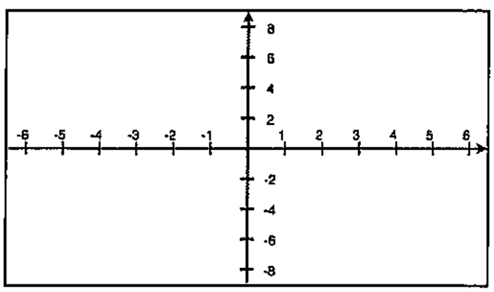
1. $y=x^2 \quad (x=2)$



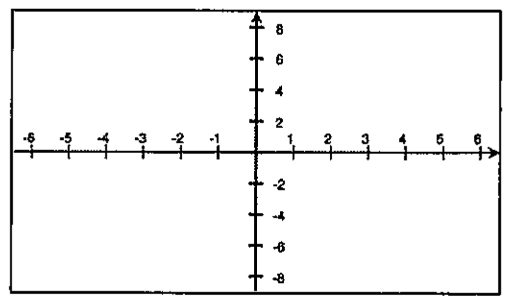
2. $y=\frac{x}{3} \quad (x=-2)$



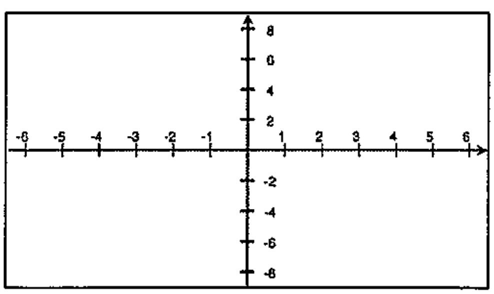
3. $y=x^3 \quad (x=0)$



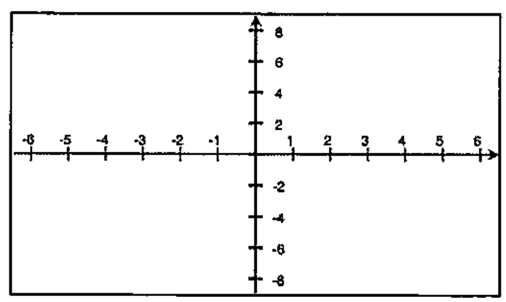
4. $y=x^4-x^2-2 \quad (x=1)$



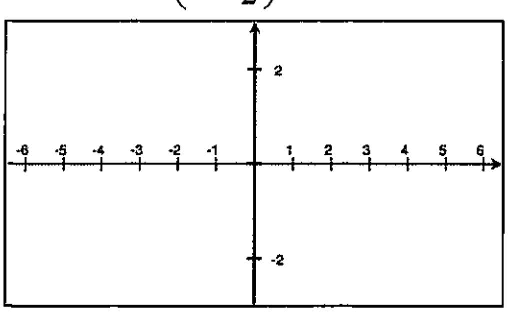
5. $y=|x| \quad (x=0)$



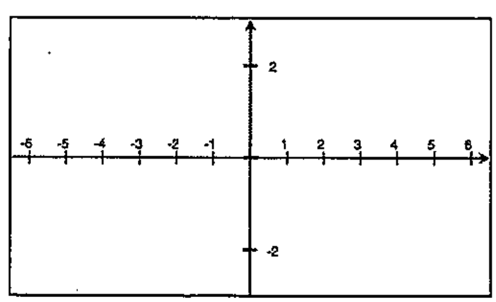
6. $y=2^x \quad (x=0)$



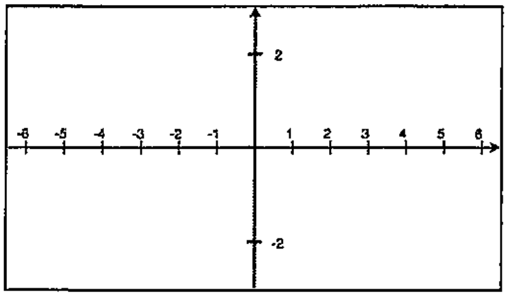
7. $y=\cos x \quad \left(x=\frac{\pi}{2}\right)$



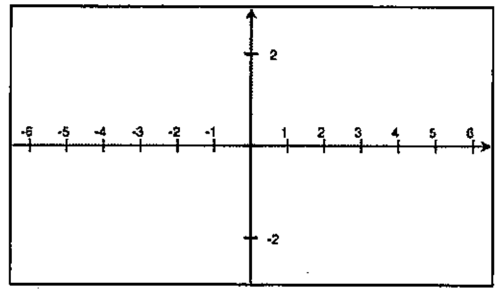
8. $y=\sqrt{x} \quad (x=0)$



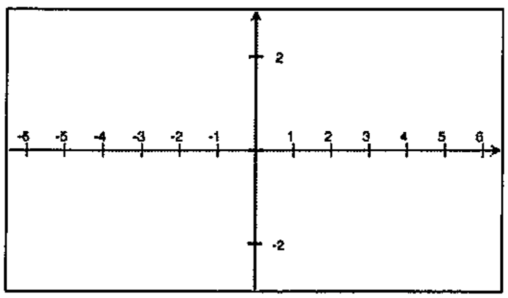
9. $y=\sqrt[3]{x} \quad (x=0)$



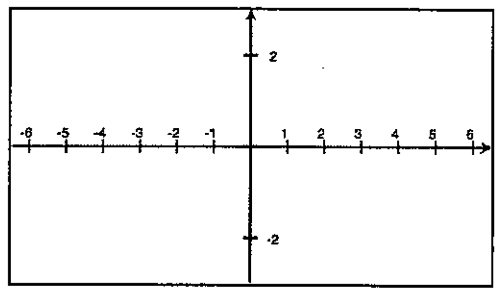
10. $y=x^{2/3} \quad (x=0)$



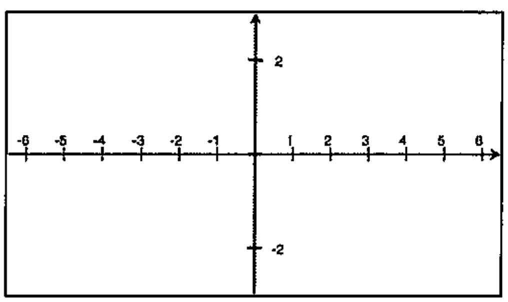
11. $y=\dfrac{x}{x-2} \quad (x=2)$



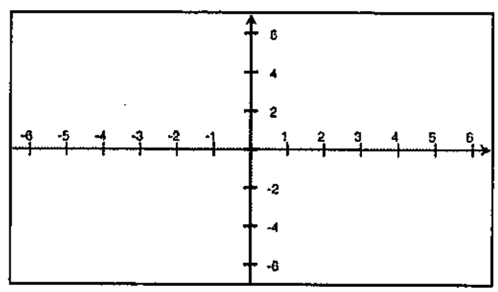
12. $y=\tan x \quad \left(x=\dfrac{\pi}{2}\right)$



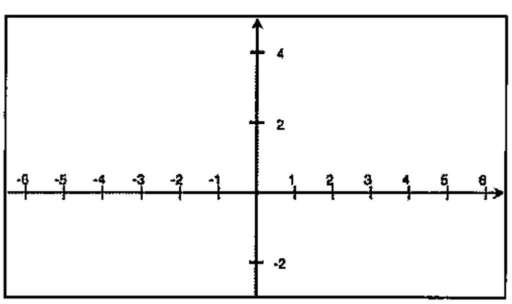
13. $y=\ln x \quad (x=e)$



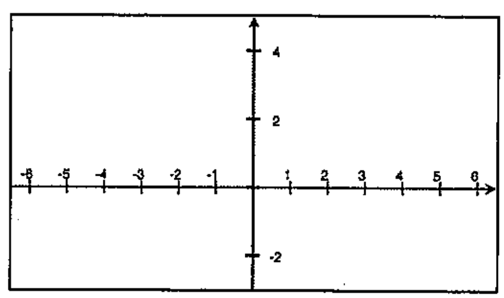
14. $y=x+\dfrac{1}{x} \quad (x=1)$



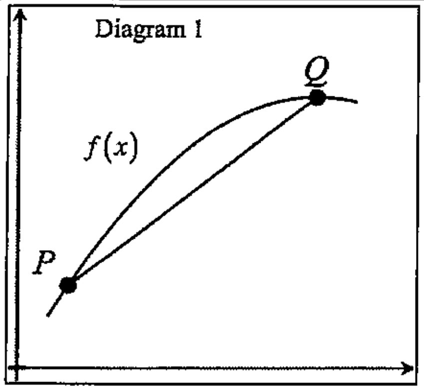
15. $y=\sqrt{16-x^2} \quad (x=0)$



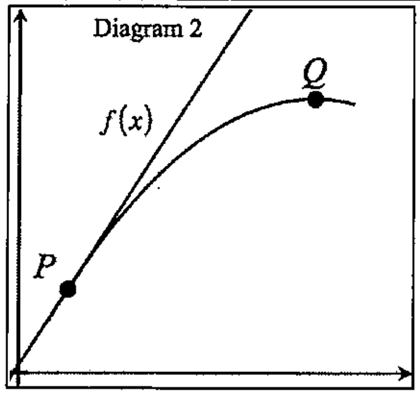
16. $y=\sqrt{16-x^2} \quad (x=4)$



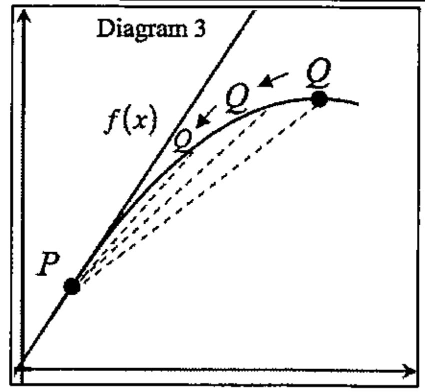
3. Slopes of Secants and Tangent Lines – Classwork



A line is drawn through points P and Q, both on $f(x)$. That line is called the **secant line** through P and Q.



A line is drawn that touches $f(x)$ at only point P. That line is called the **tangent line** through P.



We draw the secant line through PQ. Point Q moves along $f(x)$ towards point P. The closer that Q gets to P, the more the secant line starts to look like the tangent line at P.

We will say that as Q gets closer and closer to P, that the secant line PQ gets closer to the tangent line through P and thus the slope of the secant line (called m_{sec}) approaches the slope of the tangent line (called $m_{\tan}$).

As an example, let $f(x) = x^2$. Let us try and find the slope of the secant line between $x = 1$ and a value of x greater than, but very close to 1. Complete the chart. Set your calculator to maximum decimal place accuracy.

x	**2**	**1.5**	**1.1**	**1.05**	**1.01**	**1.001**	**1**
$f(x)$							
Rise							
Run							
m_{sec}							

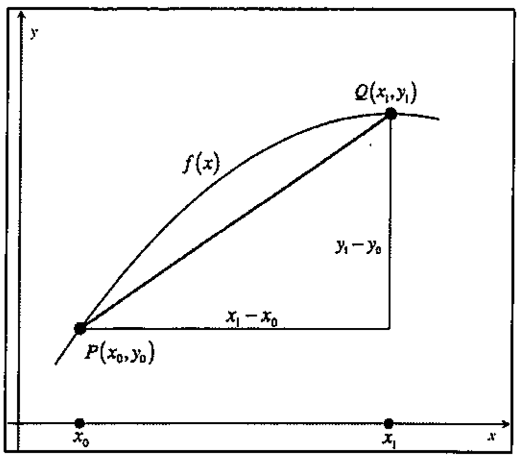
Why can't you use this method to find the slope of the tangent line at $x = 1$? ____________________
Secant lines use how many points? ______Tangent lines use how many points? _____ Using the fact that the closer x gets to 1, the slope of the secant line approaches the slope of the tangent line, what is your best guess for the slope of the tangent line to $f(x)$ at $x = 1$ ________.

Let us use an analogy. You leave home at 10 AM on a trip and arrive at 12 noon. The trip is a total of 100 miles. How fast are you going at 11 AM? ________

We cannot find the actual velocity at 11 AM (called the **instantaneous velocity**). But we can find the **average velocity** between 10 AM and any other time, using the fact that average velocity = $\dfrac{\text{total distance}}{\text{total time}}$. The average velocity between 10 AM and 12 noon is ______. Complete the chart on the next page.

Between 11 AM &	11:30	11:15	11:10	11:05	11:01	11:00:30	11:00:01	11:00
Distance traveled	24 miles	13 miles	10 miles	5.5 miles	0.8 miles	0.5 miles	80 feet	0
Time duration (hrs)								
Average velocity								
Instantaneous velocity at 11 AM								

Why can't you actually use this technique to find the instantaneous velocity at 11 AM. ___________
Average velocity uses how many times? _______ Instantaneous velocity uses how many times? _______.
As the time duration after 11:00 becomes small, the instantaneous velocity at 11 AM is much more likely to be very close to which of these values? _____ Why? ______________________________



Let us now take the two points and give them general coordinates $P(x_0,y_0)$ and $Q(x_1,y_1)$.

We draw the right triangle below the curve (the rise and the run). The length of the rise is y_1-y_0 and the length of the run is x_1-x_0. We now find the slope of the secant line *PQ* and denote it as $m_{\sec}=\frac{\text{rise}}{\text{run}}=\frac{y_1-y_0}{x_1-x_0}$. Since $y=f(x)$, we can also say that $m_{\sec}=\frac{\text{rise}}{\text{run}}=\frac{f(x_1)-f(x_0)}{x_1-x_0}$.

Concentrating on the tangent line at point *P*, we have shown that the above formula does not work? Why not? ____________________. Define the variable *h* as the horizontal distance between the two points *P* and *Q*:

Since $h=x_1-x_0$ it follows that $x_1=x_0+h$.

Since $m_{\sec}=\frac{\text{rise}}{\text{run}}=\frac{f(x_1)-f(x_0)}{x_1-x_0}$, it follows that $m_{\sec}=\frac{f(x_0+h)-f(x_0)}{h}$.

The important step: We showed that the tangent line at *P* is defined as the secant line between *P* and *Q* and *Q* gets closer and closer to *P*. As *Q* gets closer to *P*, x_1 gets closer to x_0, so *h* (the horizontal distance between *P* and *Q*) gets close to zero.

So we have the starting point for differential calculus:

$m_{\tan}=\frac{f(x_0+h)-f(x_0)}{h}$ as *h* gets infinitely close to zero. Note that *h* cannot actually equal zero? Why not?

__________________________.

Three formulas you need to know:

- • the slope of the secant line between $(x_0, f(x_0))$ and $(x_1, f(x_1))$: $m_{sec} = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$
- the slope of the tangent line at $(x_0, f(x_0))$: $m_{tan} = \dfrac{f(x_0 + h) - f(x_0)}{h}$ as h gets infinitely close to zero
- the point-slope equation of a line passing through $(x_0, f(x_0))$: $y - y_0 = m(x - x_0)$

Example 1) For the function $f(x) = x^2 + 1$, find the following: Confirm c) on your calculator.

a) the slope of the secant line between $x = 1$ and $x = 3$

b) the slope of the tangent line at $x = 2$

c) the equation of the tangent line at $x = 2$

Example 2) For the function $f(x) = 5x + 2$, find the following: Confirm c) on your calculator.

a) the slope of the secant line between $x = -1$ and $x = 4$

b) the slope of the tangent line at $x = 2$

c) the equation of the tangent line at $x = 2$

Example 3) For the function $f(x) = x^2 + 4x - 1$, find the following: Confirm c) on your calculator.

a) the slope of the secant line between $x = -3$ and $x = -1$

b) the slope of the tangent line at $x = -2$

c) the equation of the tangent line at $x = -2$

Example 4) For the function $f(x) = 2x^2 - 5x + 3$, find the following: Confirm c) on your calculator.

a) the slope of the secant line between $x = 0$ and $x = 1$

b) the slope of the tangent line at $x = 2$

c) the equation of the tangent line at $x = 2$

Example 5) For the function $f(x) = x^3 - x^2 + 1$, find the following: Confirm c) on your calculator.

a) the slope of the secant line between $x = -1$ and $x = 1$

b) the slope of the tangent line at $x = 1$

c) the equation of the tangent line at $x = 1$

Example 6) For the function $f(x) = \frac{2}{x+1}$, find the following: Confirm c) on your calculator.

a) the slope of the secant line between $x = 1$ and $x = 4$

b) the slope of the tangent line at $x = 2$

c) the equation of the tangent line at $x = 2$

Suppose that an object is traveling along a straight line according to the formula $s(t)=2t+3$ where t is measured in seconds and $s(t)$ is measured in feet. Complete the table.

t	0	1	2	3	4	5
$s(t)=2t+3$						

To calculate the average velocity between $t = 0$ and $t = 4$, we know average velocity = $\dfrac{\text{total distance}}{\text{total time}}$. So the average velocity is _____, measured in ________.

> Two formulas you need to know: Given $s(t)$ as the distance traveled in time t,
>
> - Average velocity between t_1 and t_2 — Avg. vel. = $\dfrac{s(t_2)-s(t_1)}{t_2-t_1}$
> - Instantaneous velocity at t_1 — Inst. vel. = $\dfrac{s(t_1+h)-s(t_1)}{h}$ as h gets infinitely close to zero

So in the example above where $s(t)=2t+3$, the instantaneous velocity at $t = 4$ can be found:

In this case, the average velocity between $t = 0$ and $t = 4$ is the same as the instantaneous velocity at $t = 4$. In a car, if your average velocity is the same as the instantaneous velocity, what is that called? _______________

Example 7) If $s(t)=3t+1$ is a measure of feet traveled with t measured in seconds, find

a) the average velocity between $t = 0$ and $t = 3$ b) the instantaneous velocity at $t = 2$ seconds

Example 8) If $s(t)=t^2-2t$ is a measure of feet traveled with t measured in seconds, find

a) the average velocity between $t = 0$ and $t = 2$ b) the instantaneous velocity at $t = 2$ seconds

Example 9) If $s(t)=t^3+t^2-t-1$ is a measure of feet traveled with t measured in seconds, find

a) the average velocity between $t = 1$ and $t = 2$ b) the instantaneous velocity at $t = 1$ second

3. Slopes of Secants and Tangent Lines – Homework

1. For the function $f(x)=5x+3$, find the following: Confirm c) on your calculator.

   a. the slope of the secant line between $x=1$ and $x=5$

   b. the slope of the tangent line at $x=2$

   c. the equation of the tangent line at $x=2$

2. For the function $f(x)=x^2-3$, find the following: Confirm c) on your calculator.

   a) the slope of the secant line between $x=0$ and $x=3$

   b) the slope of the tangent line at $x=1$

   c) the equation of the tangent line at $x=1$

3. For the function $f(x)=x^2-5x+4$, find the following: Confirm c) on your calculator.

   a. the slope of the secant line between $x=3$ and $x=6$

   b. the slope of the tangent line at $x=3$

   c. the equation of the tangent line at $x=3$

4. For the function $f(x)=2x^2-7x+8$, find the following: Confirm c) on your calculator.

   a. the slope of the secant line between $x=-1$ and $x=4$

   b. the slope of the tangent line at $x=-1$

   c. the equation of the tangent line at $x=-1$

5. For the function $f(x) = x^3 + x - 2$, find the following: Confirm c) on your calculator.

a. the slope of the secant line between $x = -3$ and $x = 3$

b. the slope of the tangent line at $x = 2$

c. the equation of the tangent line at $x = 2$

6. For the function $f(x) = \frac{5}{x-3}$, find the following: Confirm c) on your calculator.

a. the slope of the secant line between $x = 4$ and $x = 6$

b. the slope of the tangent line at $x = 1$

c. the equation of the tangent line at $x = 1$

7. For the function $f(x) = \frac{x-2}{x+1}$, find the following: Confirm c) on your calculator.

a. the slope of the secant line between $x = 1$ and $x = 5$

b. the slope of the tangent line at $x = 1$

c. the equation of the tangent line at $x = 1$

8. If $s(t)=4t+1$ is a measure of feet traveled with t measured in seconds, find

a. the average velocity between $t=1$ and $t=5$ b. the instantaneous velocity at $t=2$ seconds

9. If $s(t)=t^2+4$ is a measure of feet traveled with t measured in seconds, find

a. the average velocity between $t=0$ and $t=4$ b. the instantaneous velocity at $t=1$ second

10. If $s(t)=t^2-3t+2$ is a measure of miles traveled with t measured in hours, find

a. the average velocity between $t=0$ and $t=4$ b. the instantaneous velocity at $t=1$ hour

11. If $s(t)=t^3+t-1$ is a measure of meters traveled with t measured in seconds, find

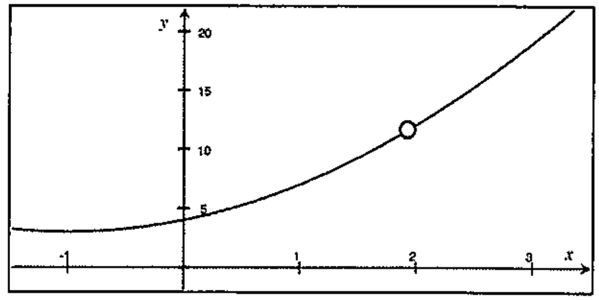
a. the average velocity between $t=2$ and $t=7$ b. the instantaneous velocity at $t=2$ seconds

12. If $s(t)=\dfrac{6}{t+2}$ is a measure of feet traveled with t measured in seconds, find

a. the average velocity between $t=1$ and $t=7$ b. the instantaneous velocity at $t=4$ seconds

4. Graphical Approach to Limits – Classwork

To the right is the graph of $f(x)=\dfrac{x^3-8}{x-2}$. For all values of x not equal to 2, you can use standard curve sketching techniques. But the function is not defined at $x = 2$. There is a hole in the graph. So let's get an idea of the behavior of the graph close to $x = 2$. Set your calculator to 4-decimal place accuracy and complete the table.



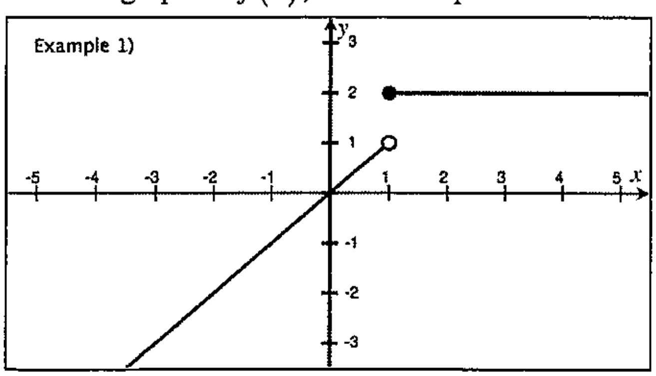
x	1.75	1.9	1.99	1.999	2	2.001	2.01	2.1	2.5
$f(x)$									

It should be obvious that as x gets closer and closer to 2, the value of $f(x)$ becomes closer and closer to 12. Note that $f(x)$ never actually equals 12, just gets closer to it. This is expressed with the concept of a **limit**. We say: the limit of $f(x)$ as x approaches 2 equals 12 and the notation we use is: $\lim\limits_{x\to 2} f(x)=12$ or $\lim\limits_{x\to 2}\dfrac{x^3-8}{x-2}=12$.

The informal definition of a limit answers the question: "what happens to y as x gets close to a certain value." In order for a limit to exist, we must be approaching the same y-values as x approaches some number c from either the left or right side of c. If this does not occur, we say that the limit does not exist (DNE) as we approach c.

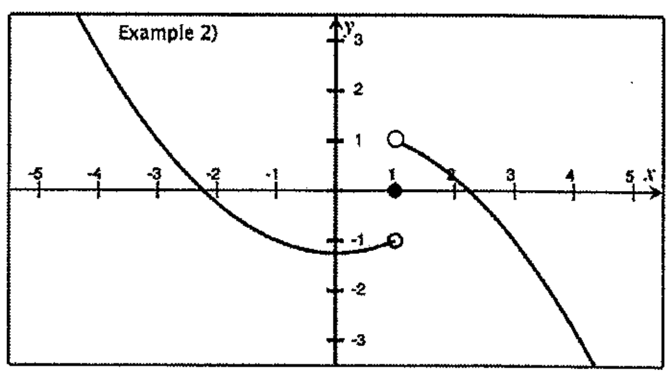
> The limit of $f(x)$ as x approaches some value of c from the left side (left-hand limit) is written: $\lim\limits_{x\to c^-} f(x)$.
> The limit of $f(x)$ as x approaches some value of c from the right side (right-hand limit) is written: $\lim\limits_{x\to c^+} f(x)$.
> In order for a limit to exist at $x = c$, $\lim\limits_{x\to c^-} f(x)$ must equal $\lim\limits_{x\to c^+} f(x)$ and we then say $\lim\limits_{x\to c} f(x)=L$.
> It is important to understand that $\lim\limits_{x\to c} f(x)$ and $f(c)$ do not have to equal each other.

For each graph of $f(x)$, find the required information.



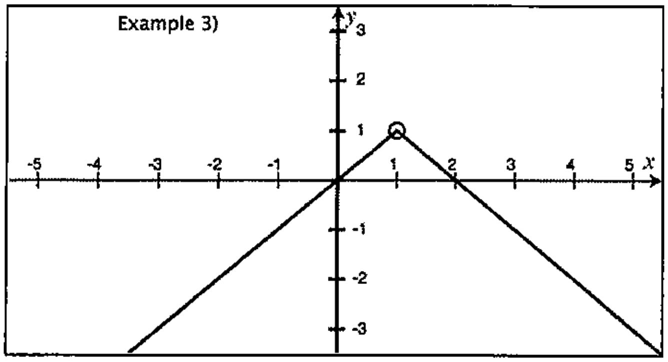
$\lim\limits_{x\to 1^-} f(x)=$ ____ $\lim\limits_{x\to 1^+} f(x)=$ ____

$\lim\limits_{x\to 1} f(x)=$ ____ $f(1)=$ ____



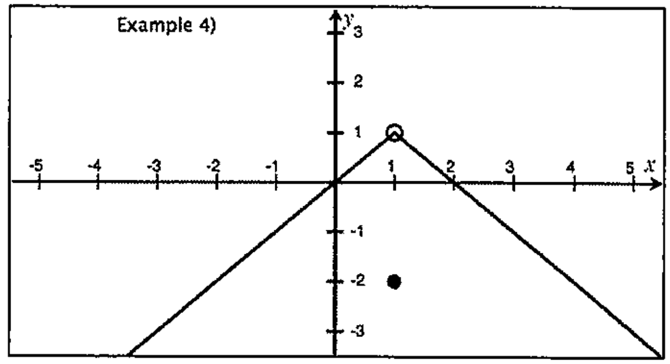
$\lim\limits_{x\to 1^-} f(x)=$ ____ $\lim\limits_{x\to 1^+} f(x)=$ ____

$\lim\limits_{x\to 1} f(x)=$ ____ $f(1)=$ ____



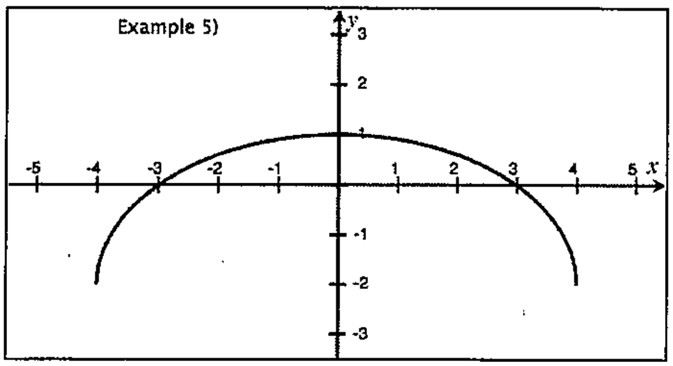
$\lim_{x\to 1^-} f(x) =$ ____ $\lim_{x\to 1^+} f(x) =$ ____

$\lim_{x\to 1} f(x) =$ ____ $f(1) =$ ____



$\lim_{x\to 1^-} f(x) =$ ____ $\lim_{x\to 1^+} f(x) =$ ____

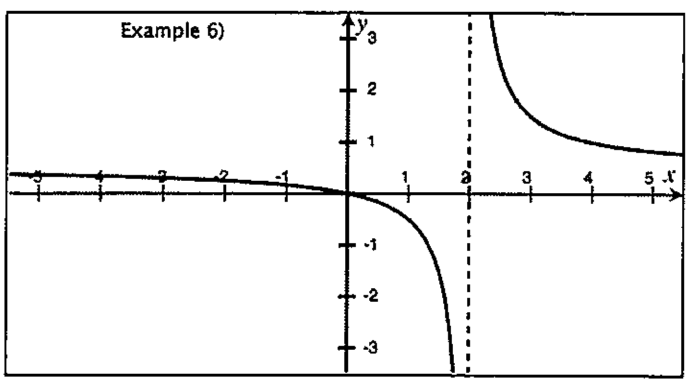
$\lim_{x\to 1} f(x) =$ ____ $f(1) =$ ____



$\lim_{x\to 0^-} f(x) =$ ____ $\lim_{x\to 0^+} f(x) =$ ____

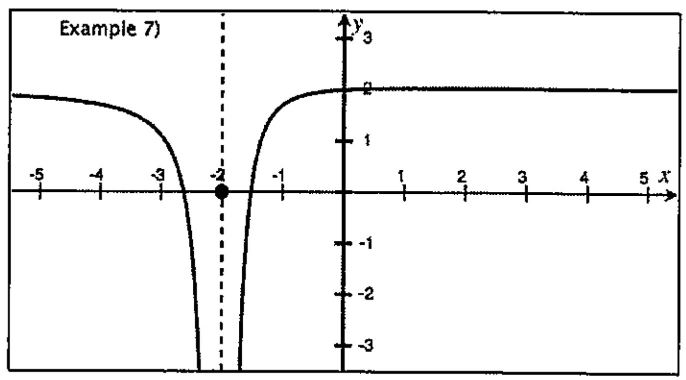
$\lim_{x\to 0} f(x) =$ ____ $f(0) =$ ____

When there is a vertical asymptote at $x = c$, $\lim_{x\to c} f(x)$ does not exist. If the function is going up approaching the vertical asymptote, we say that $\lim_{x\to c} f(x) = \infty$ which is saying that there is no limit. If the function is going down approaching the vertical asymptote, we say that $\lim_{x\to c} f(x) = -\infty$ which again says that there is no limit.



$\lim_{x\to 2^-} f(x) =$ ____ $\lim_{x\to 2^+} f(x) =$ ____

$\lim_{x\to 2} f(x) =$ ____ $f(2) =$ ____



$\lim_{x\to -2^-} f(x) =$ ____ $\lim_{x\to -2^+} f(x) =$ ____

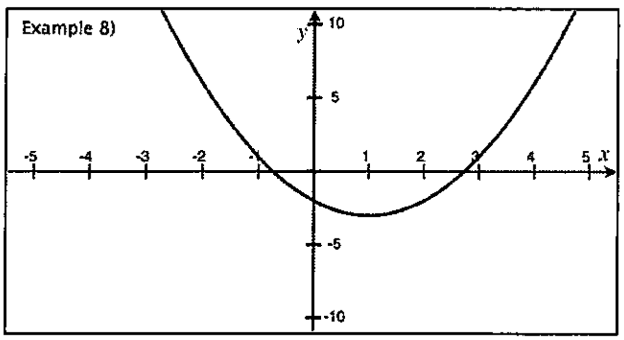
$\lim_{x\to -2} f(x) =$ ____ $f(-2) =$ ____

We also are interested the limit of a function as x approaches positive or negative infinity. This answers the question: "what happens to y as x gets infinitely far to the right or to the left?" The terminology we use for these limits is: $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

Although we use the term "as x approaches infinity", realize that x cannot approach infinity as infinity does not exist. The term "x approaches infinity" is just a convenient way to express the notion that x is getting infinitely far to the right of the y-axis. Note also that it makes no sense to talk about $\lim_{x\to\infty^+} f(x)$ or $\lim_{x\to-\infty^-} f(x)$ as we can only get infinitely far to the right if we approach from the left, and we can only get infinitely far to the left if we approach from the right.

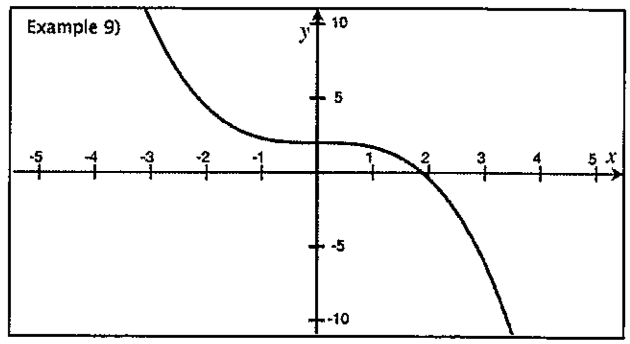
There are only 5 possibilities for $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

- The curve can go up forever. In this case, the limit does not exist. For convenience's sake, we say that $\lim_{x\to\infty} f(x) = \infty$.



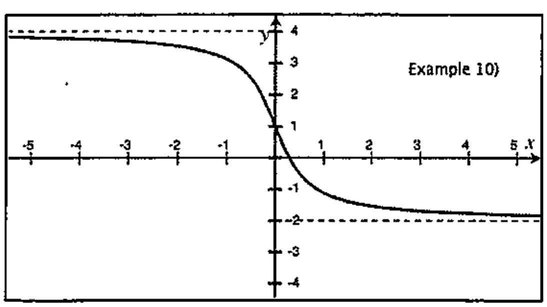
$\lim_{x\to-\infty} f(x) =$ ___ $\lim_{x\to\infty} f(x) =$ ___

- The curve can go down forever. Again, in that case, the limit does not exist. For convenience sake, we say that $\lim_{x\to\infty} f(x) = -\infty$.



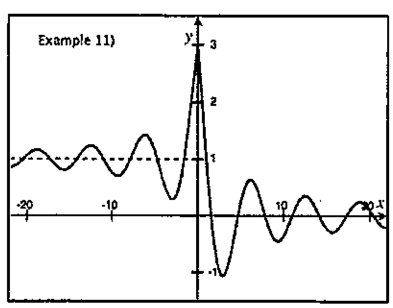
$\lim_{x\to-\infty} f(x) =$ ___ $\lim_{x\to\infty} f(x) =$ ___

- The curve can become asymptotic to a line. In this case, the limit is a value. We say $\lim_{x\to\infty} f(x) = L$.



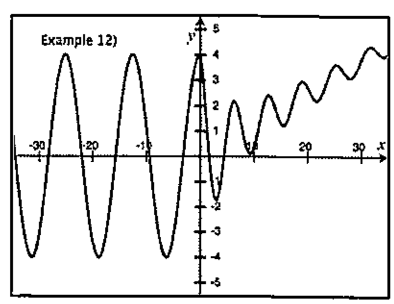
$\lim_{x\to-\infty} f(x) =$ __ $\lim_{x\to\infty} f(x) =$ __

- the curve can oscillate but get closer to a value. Again, we say $\lim_{x\to\infty} f(x) = L$.



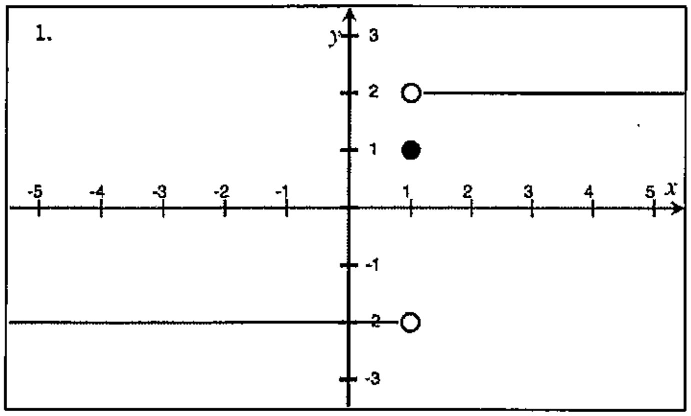
$\lim_{x\to-\infty} f(x) =$ __ $\lim_{x\to\infty} f(x) =$ __

- the curve can oscillate but not get closer to a value We say $\lim_{x\to\infty} f(x)$ does not exist. Also $\lim_{x\to\infty} f(x) = \infty$ as the curve will eventually be higher than any value k.

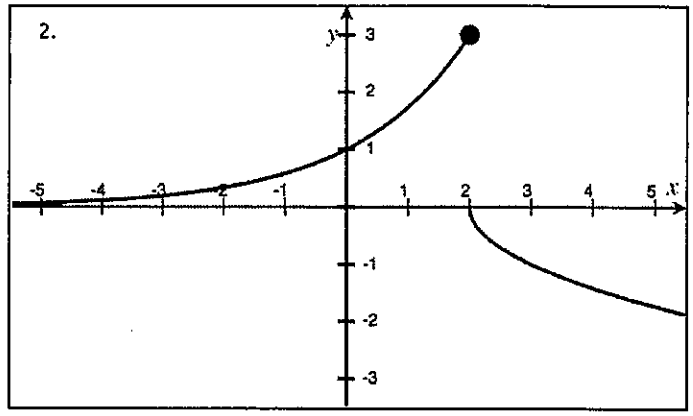


$\lim_{x\to-\infty} f(x) =$ __ $\lim_{x\to\infty} f(x) =$ __

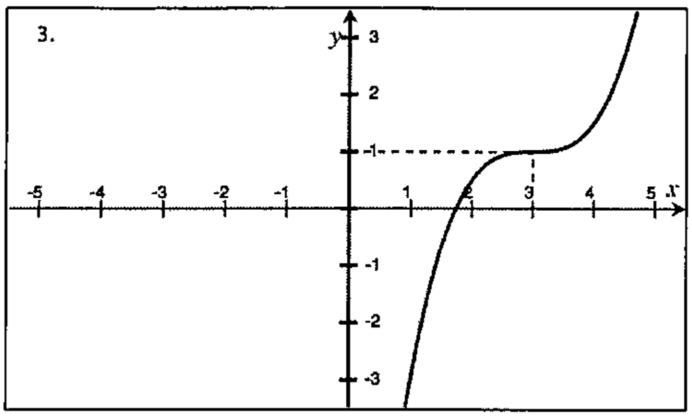
4. Graphical Approach to Limits – Homework



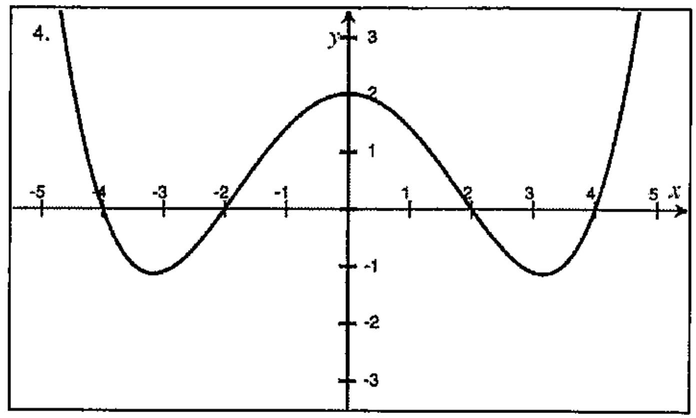
a) $\lim\limits_{x\to 1^-} f(x) =$ __ b) $\lim\limits_{x\to 1^+} f(x) =$ __ c) $\lim\limits_{x\to 1} f(x) =$ __
d) $f(1) =$ __ e) $\lim\limits_{x\to -\infty} f(x) =$ __ f) $\lim\limits_{x\to \infty} f(x) =$ __



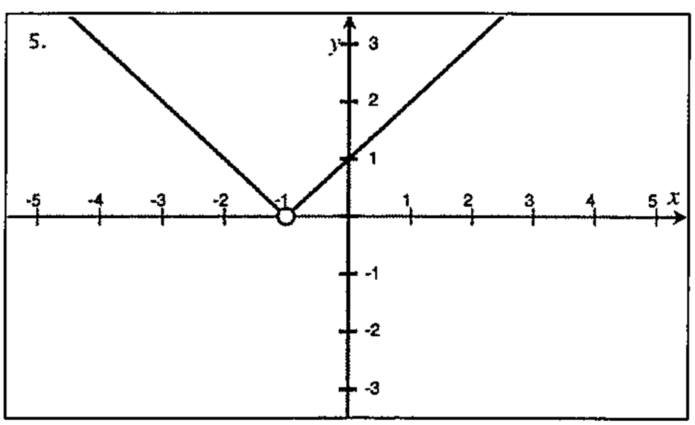
a) $\lim\limits_{x\to 2^-} f(x) =$ __ b) $\lim\limits_{x\to 2^+} f(x) =$ __ c) $\lim\limits_{x\to 2} f(x) =$ __
d) $f(2) =$ __ e) $\lim\limits_{x\to -\infty} f(x) =$ __ f) $\lim\limits_{x\to \infty} f(x) =$ __



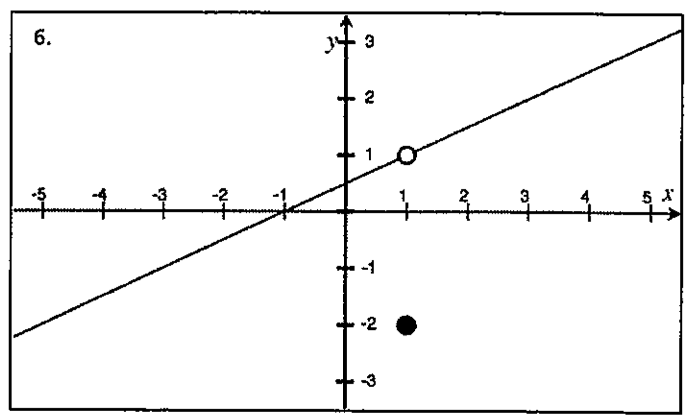
a) $\lim\limits_{x\to 3^-} f(x) =$ __ b) $\lim\limits_{x\to 3^+} f(x) =$ __ c) $\lim\limits_{x\to 3} f(x) =$ __
d) $f(3) =$ __ e) $\lim\limits_{x\to -\infty} f(x) =$ __ f) $\lim\limits_{x\to \infty} f(x) =$ __



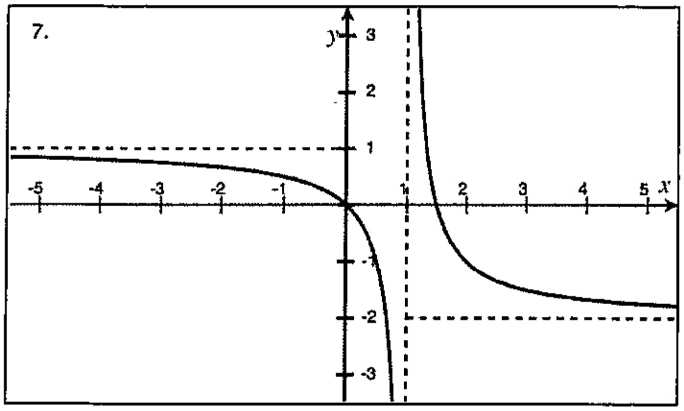
a) $\lim\limits_{x\to 0^-} f(x) =$ __ b) $\lim\limits_{x\to 0^+} f(x) =$ __ c) $\lim\limits_{x\to 0} f(x) =$ __
d) $f(0) =$ __ e) $\lim\limits_{x\to -\infty} f(x) =$ __ f) $\lim\limits_{x\to \infty} f(x) =$ __



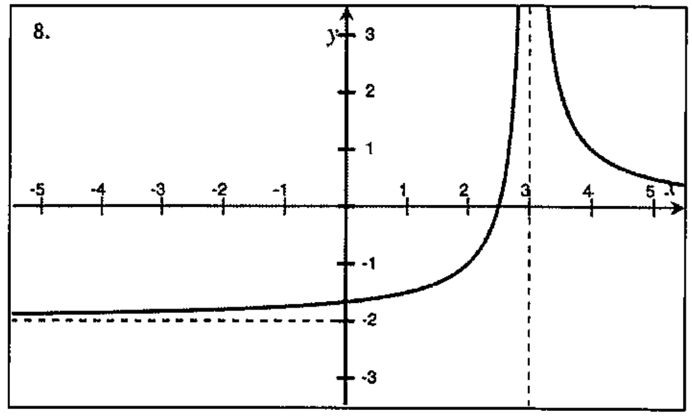
a) $\lim\limits_{x\to -1^-} f(x) =$ __ b) $\lim\limits_{x\to -1^+} f(x) =$ __ c) $\lim\limits_{x\to -1} f(x) =$ __
d) $f(-1) =$ __ e) $\lim\limits_{x\to -\infty} f(x) =$ __ f) $\lim\limits_{x\to \infty} f(x) =$ __



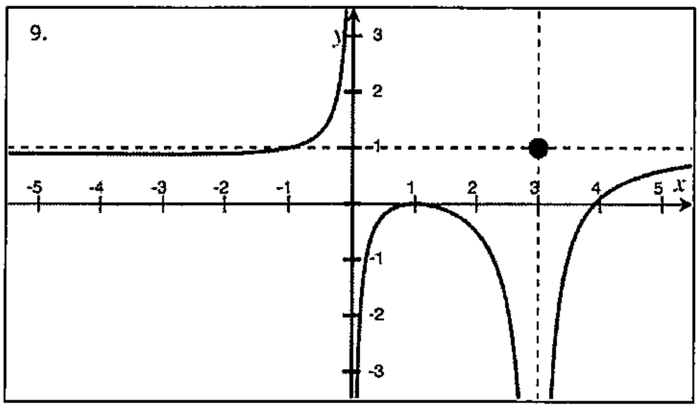
a) $\lim\limits_{x\to 1^-} f(x) =$ __ b) $\lim\limits_{x\to 1^+} f(x) =$ __ c) $\lim\limits_{x\to 1} f(x) =$ __
d) $f(1) =$ __ e) $\lim\limits_{x\to -\infty} f(x) =$ __ f) $\lim\limits_{x\to \infty} f(x) =$ __



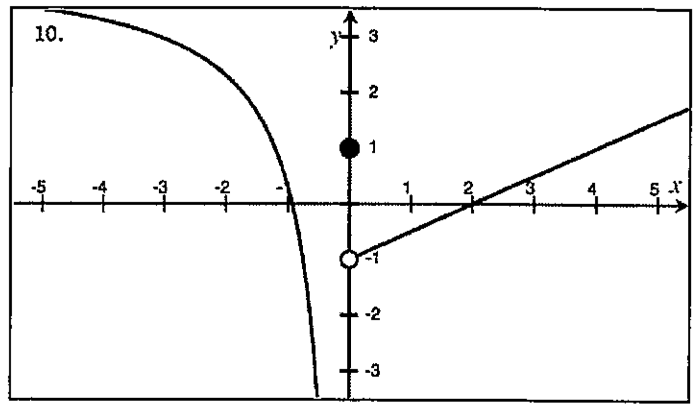
a) $\lim_{x\to 1^-} f(x) =$ __ b) $\lim_{x\to 1^+} f(x) =$ __ c) $\lim_{x\to 1} f(x) =$ __
d) $f(1) =$ __ e) $\lim_{x\to -\infty} f(x) =$ __ f) $\lim_{x\to \infty} f(x) =$ __



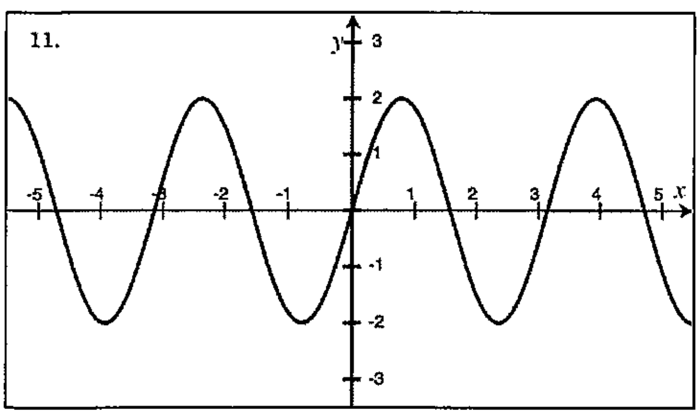
a) $\lim_{x\to 3^-} f(x) =$ __ b) $\lim_{x\to 3^+} f(x) =$ __ c) $\lim_{x\to 3} f(x) =$ __
d) $f(3) =$ __ e) $\lim_{x\to -\infty} f(x) =$ __ f) $\lim_{x\to \infty} f(x) =$ __



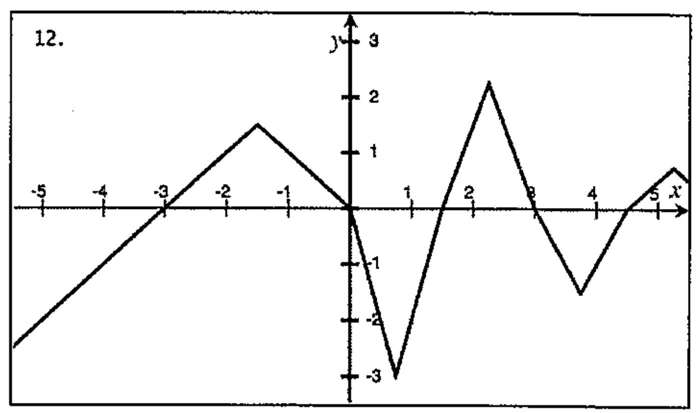
a) $\lim_{x\to 3^-} f(x) =$ __ b) $\lim_{x\to 3^+} f(x) =$ __ c) $\lim_{x\to 3} f(x) =$ __
d) $f(3) =$ __ e) $\lim_{x\to -\infty} f(x) =$ __ f) $\lim_{x\to \infty} f(x) =$ __



a) $\lim_{x\to 0^-} f(x) =$ __ b) $\lim_{x\to 0^+} f(x) =$ __ c) $\lim_{x\to 0} f(x) =$ __
d) $f(0) =$ __ e) $\lim_{x\to -\infty} f(x) =$ __ f) $\lim_{x\to \infty} f(x) =$ __

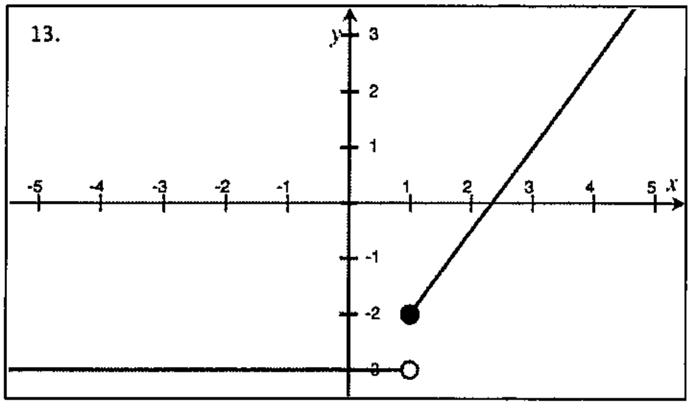


a) $\lim_{x\to 0^-} f(x) =$ __ b) $\lim_{x\to 0^+} f(x) =$ __ c) $\lim_{x\to 0} f(x) =$ __
d) $f(0) =$ __ e) $\lim_{x\to -\infty} f(x) =$ __ f) $\lim_{x\to \infty} f(x) =$ __



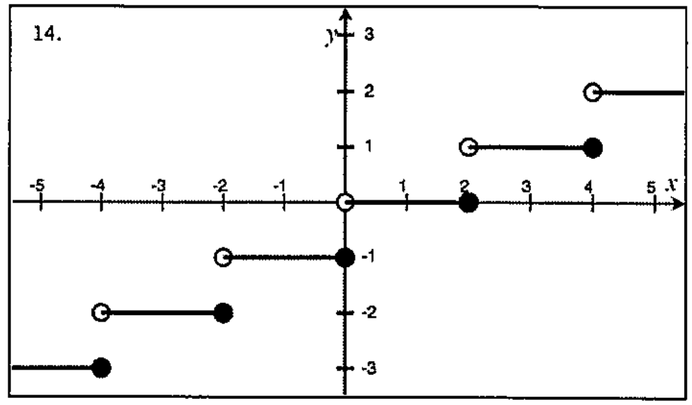
a) $\lim_{x\to 0^-} f(x) =$ __ b) $\lim_{x\to 0^+} f(x) =$ __ c) $\lim_{x\to 0} f(x) =$ __
d) $f(0) =$ __ e) $\lim_{x\to -\infty} f(x) =$ __ f) $\lim_{x\to \infty} f(x) =$ __

13.



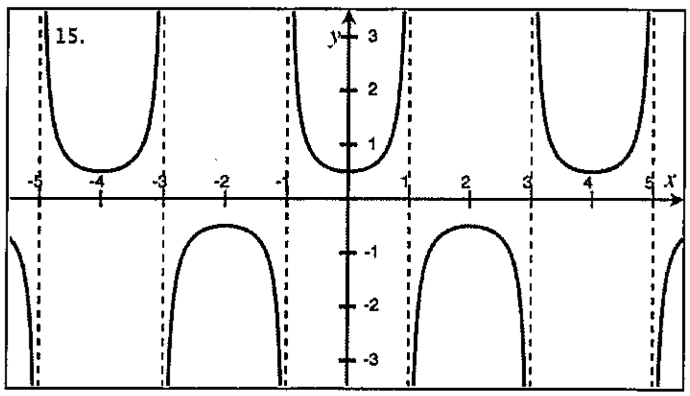
a) $\lim_{x\to1^-} f(x)=$ __ b) $\lim_{x\to1^+} f(x)=$ __ c) $\lim_{x\to1} f(x)=$ __
d) $f(1)=$ __ e) $\lim_{x\to-\infty} f(x)=$ __ f) $\lim_{x\to\infty} f(x)=$ __

14.



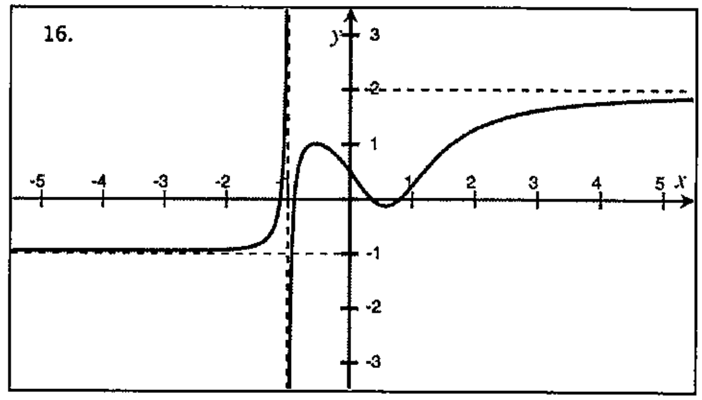
a) $\lim_{x\to0^-} f(x)=$ __ b) $\lim_{x\to0^+} f(x)=$ __ c) $\lim_{x\to0} f(x)=$ __
d) $f(0)=$ __ e) $\lim_{x\to-\infty} f(x)=$ __ f) $\lim_{x\to\infty} f(x)=$ __

15.



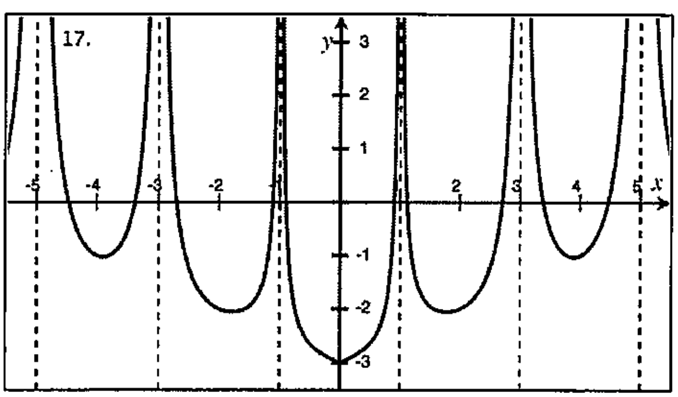
a) $\lim_{x\to1^-} f(x)=$ __ b) $\lim_{x\to1^+} f(x)=$ __ c) $\lim_{x\to1} f(x)=$ __
d) $f(1)=$ __ e) $\lim_{x\to-\infty} f(x)=$ __ f) $\lim_{x\to\infty} f(x)=$ __

16.



a) $\lim_{x\to-1^-} f(x)=$ __ b) $\lim_{x\to-1^+} f(x)=$ __ c) $\lim_{x\to-1} f(x)=$ __
d) $f(-1)=$ __ e) $\lim_{x\to-\infty} f(x)=$ __ f) $\lim_{x\to\infty} f(x)=$ __

17.



a) $\lim_{x\to0^-} f(x)=$ __ b) $\lim_{x\to0^+} f(x)=$ __ c) $\lim_{x\to0} f(x)=$ __
d) $f(0)=$ __ e) $\lim_{x\to-\infty} f(x)=$ __ f) $\lim_{x\to\infty} f(x)=$ __

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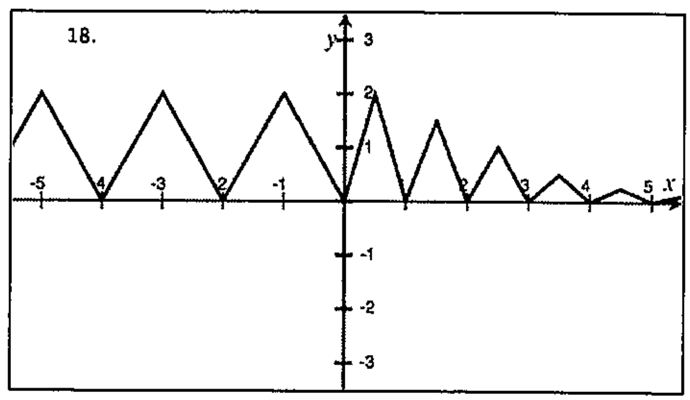


a) $\lim_{x\to0^-} f(x)=$ __ b) $\lim_{x\to0^+} f(x)=$ __ c) $\lim_{x\to0} f(x)=$ __
d) $f(0)=$ __ e) $\lim_{x\to-\infty} f(x)=$ __ f) $\lim_{x\to\infty} f(x)=$ __

5. Algebraic Approach to Limits – Classwork

While determining limits based on graphs is important, we don't want to spend time graphing complex functions to determine limits. Luckily, there are purely algebraic techniques that don't depend on the graph.

There are 4 easy steps to find $\lim_{x \to a} f(x)$. They will be further explained below:			
1. Plug in a	2. Factor/cancel	3. $\infty, -\infty$, or does not exist	4) Indeterminate

1. Plug in a to $f(x)$; that is find $f(a)$. If $f(a)$ exists, that is the limit.

Example 1a) find $\lim_{x \to -2} (x^2 - 4x + 1)$

Example 1b) find $\lim_{x \to -1} \frac{2x-6}{x-1}$

Example 1c) find $\lim_{x \to \pi} \frac{2\cos x - x}{x+2}$

2. If, when finding $f(a)$, you get $\frac{0}{0}$, $\lim_{x \to a} f(x)$ may or may not exist. Factor both numerator and/or denominator of $f(x)$, do any cancellation and go back to step 1.

Example 2a) find $\lim_{x \to -2} \frac{x^2 - 2x - 8}{x+2}$

Example 2b) find $\lim_{x \to 0} \frac{5x^3 - 5x}{x^2 + x}$

Example 2c) find $\lim_{x \to 1} \frac{x^2 - 2x + 1}{x^3 - 1}$

3. When you find $f(a)$ and get $\frac{k}{0}$ or you get $\frac{k}{0}$ after factoring/canceling, $\lim_{x \to a} f(x)$ does not exist. Split your problem into 2 limits, $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$. Each of these will be either $+\infty$ or $-\infty$. To determine which, plug in numbers close to a on the left and the right. We are not interested in the value, just the sign.

Only if these two limits are the same (either $+\infty$ or $-\infty$), can you go the extra step and say that $\lim_{x \to a} f(x) = \infty$ or $\lim_{x \to a} f(x) = -\infty$. If not, just say that $\lim_{x \to a} f(x)$ does not exist. Remember, if $\lim_{x \to a} f(x) = \pm\infty$, it is true that $\lim_{x \to a} f(x)$ does not exist. But if $\lim_{x \to a} f(x)$ does not exist, that does not necessarily mean that $\lim_{x \to a} f(x) = \pm\infty$.

Example 3a) Find $\lim_{x \to 2} \frac{2x+5}{x-2}$

Example 3b) Find $\lim_{x \to 0} \frac{4}{x^2}$

Example 3c) Find $\lim_{x \to -3} \frac{x^2+2x-3}{x^2+6x+9}$

Example 3d) Find $\lim_{x \to 2} \frac{4-2x}{x^3-6x^2+12x-8}$

4. Sometimes, when trying to find $\lim_{x \to a} f(x)$, you plug in, possible factor/cancel and still get $\frac{0}{0}$. If no more canceling can be done, you have an indeterminate form. This means that $\lim_{x \to a} f(x)$ may or may not exist. There are techniques to determine what the limit is if it indeed exists, (and they are covered in chapter 24 of this manual), for right now we simply call them indeterminate (we cannot determine whether or not the limit exists).

Example 4a) find $\lim_{x \to 0} \frac{\sin x}{x}$

Example 4b) find $\lim_{x \to 0} \frac{e^x - 1}{x^2 - x}$

Example 4c) find $\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x}$

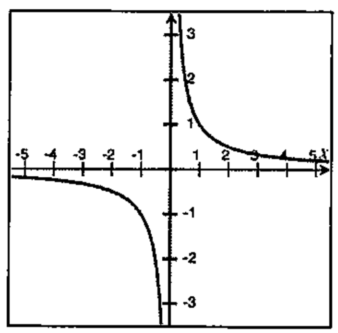
When taking limits involving piecewise functions, you are usually asked to find $\lim_{x \to a} f(x)$ where a is the value of x for which the formula for $f(x)$ changes. These problems are simple. You find $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ both by the techniques above. Only if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$, does $\lim_{x \to a} f(x)$ exist.

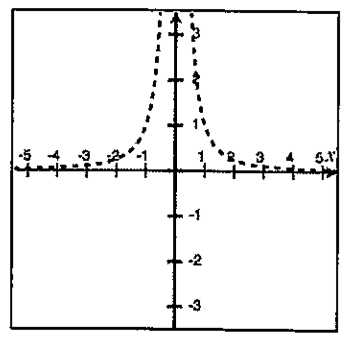
Ex. 5a) If $f(x) = \begin{cases} x^2 - 4, & x \le 1 \\ -3x + 1, & x > 1 \end{cases}$ find $\lim_{x \to 1} f(x)$

Ex. 5b) If $f(x) = \begin{cases} x^2 + 3x - 4, & x < 2 \\ 4, & x = 2 \\ x^3 - x, & x > 2 \end{cases}$ find $\lim_{x \to 2} f(x)$

Ex. 5c) If $f(x) = \begin{cases} \dfrac{x}{x+3}, & x \le -3 \\ \dfrac{x+4}{x+3}, & x > -3 \end{cases}$ find $\lim_{x \to -3} f(x)$

We need to examine techniques for finding limits of functions as x approaches $\pm\infty$. We first examine the graphs of $f(x) = \dfrac{1}{x}$ (solid curve) and $f(x) = \dfrac{1}{x^2}$ (dashed curve). We see that both these functions are asymptotic to the x-axis. So $\lim_{x \to \pm\infty} \dfrac{1}{x} = 0$ and $\lim_{x \to \pm\infty} \dfrac{1}{x^2} = 0$. Consequently, we can say $\lim_{x \to \pm\infty} \dfrac{a}{x^n} = 0$ for any positive value of $n > 1$ where a is a constant.





Ex. 6a) Find $\lim_{x \to \infty} \dfrac{4x - 3}{8x + 1}$

Ex. 6b) Find $\lim_{x \to -\infty} \dfrac{6x - 7}{x^2 - 3}$

Ex. 6c) Find $\lim_{x \to \infty} \dfrac{2x^4}{5x^3 + 1}$

We can make this process easier by using the shortcut below for rational functions $f(x)$.

To find $\lim_{x\to\infty} f(x)$ or $\lim_{x\to-\infty} f(x)$, write $f(x)$ as a fraction.

1) If the highest power of x appears in the denominator (bottom-heavy), $\lim_{x\to\infty} f(x)=0$ and $\lim_{x\to-\infty} f(x)=0$.
2) If the highest power of x appears in both the numerator and denominator (powers-equal), $\lim_{x\to\pm\infty} f(x)=\dfrac{\text{coefficient of numerator's highest power}}{\text{coefficient of denominator's highest power}}$.
3) If the highest power of x appears in the numerator (top-heavy), $\lim_{x\to\pm\infty} f(x)=\pm\infty$. To determine the limit is $+\infty$ or $-\infty$, plug in a large number to $f(x)$ for $\lim_{x\to\infty} f(x)$ and determine the sign. Plug in a small number to $f(x)$ for $\lim_{x\to-\infty} f(x)$ and determine the sign.

Ex. 7a) Find $\lim_{x\to\infty}\dfrac{4x^2+50}{x^3-85}$

Ex. 7b) Find $\lim_{x\to\infty}\dfrac{4x^3-5x^2+3x-1}{5x^3-7x-25}$

Ex. 7c) Find $\lim_{x\to\infty}\dfrac{2x^4}{5x^3+1}$

Ex. 7d) Find $\lim_{x\to-\infty}\dfrac{\sqrt{1-x}}{x}$

Ex. 7e) Find $\lim_{x\to\infty}\dfrac{\sqrt{x^2-3x}}{2x+1}$

Ex. 7f) Find $\lim_{x\to-\infty}\dfrac{\sqrt{x^2-3x}}{2x+1}$

If $f(x)$ is not a rational function and you wish to find $\lim_{x\to\infty} f(x)$ or $\lim_{x\to-\infty} f(x)$, use common sense. Attempt to see whether the numerator is growing faster than the denominator or vice versa. Plug in large or small numbers to confirm your theory.

Ex. 8a) Find $\lim_{x\to\infty}\dfrac{x}{e^x}$

Ex. 8b) Find $\lim_{x\to\infty}\dfrac{e^x+4}{e^x-4}$

Ex. 8c) Find $\lim_{x\to\infty}\dfrac{\sin x}{x}$

5. Algebraic Approach to Limits – Homework

Find the following limits.

1. $\lim_{x \to 5} 12$

2. $\lim_{x \to 0} 2\pi$

3. $\lim_{x \to 2} 4x$

4. $\lim_{x \to 5} 3x^2 - 4x - 1$

5. $\lim_{x \to 0} 5x^3 - 7x^3 + 2^x - 2$

6. $\lim_{y \to -1} 3y^4 - 6y^3 - 2y$

7. $\lim_{x \to 4} \frac{2x-4}{x-1}$

8. $\lim_{x \to -2} \frac{x^2+4x+4}{x^2}$

9. $\lim_{x \to 1} \frac{2x-2}{x-1}$

10. $\lim_{x \to 4} \frac{x^2-16}{x-4}$

11. $\lim_{t \to -2} \frac{t^3+8}{t+2}$

12. $\lim_{x \to 2} \frac{x^2-4x+4}{x^2+x-6}$

13. $\lim_{x \to -1} \frac{x^2+6x+5}{x^2-3x-4}$

14. $\lim_{x \to 2} \frac{x^3+x^2-4x-4}{x^4-16}$

15. $\lim_{x \to 3} \frac{x}{x-3}$

16. $\lim_{x \to 5} \frac{x}{x^2-25}$

17. $\lim_{y \to 6} \frac{y+6}{y^2-36}$

18. $\lim_{x \to 4} \frac{3-x}{x^2-2x-8}$

19. $\lim_{x \to 1} \frac{4}{x^2-2x+1}$

20. $\lim_{x \to 5} \frac{x}{|x-5|}$

21. $\lim_{x \to 3} \frac{-x^2}{(x-3)^2}$

22. $f(x)=\begin{cases} x-1, & x\le 3 \\ 2x-3, & x>3 \end{cases}$ find $\lim\limits_{x\to 3} f(x)$. Show work.

23. $f(x)=\begin{cases} \cos x-\sin x, & x\le \pi \\ x-\pi-1, & x>\pi \end{cases}$ find $\lim\limits_{x\to \pi} f(x)$. Show work.

24. $f(x)=\begin{cases} x^3+x, & x\le -1 \\ -2^{-x}, & x>-1 \end{cases}$ find $\lim\limits_{x\to -1} f(x)$. Show work.

25. $f(x)=\begin{cases} \dfrac{x-2}{x-1}, & x<1 \\ \dfrac{x}{x-1}, & x>1 \end{cases}$ find $\lim\limits_{x\to 1} f(x)$ Show work.

26. Find $\lim\limits_{x\to 0}\dfrac{\sqrt{x+4}-2}{x}$

27. Find $\lim\limits_{x\to 4}\dfrac{x^2-2x-8}{\cos \pi x(x-4)}$

28. If $f(x)=\begin{cases} x^2-2x-3, & x\ne 2 \\ k-3, & x=2 \end{cases}$ find k such that $\lim\limits_{x\to 2} f(x)=f(2)$

29. $f(x)=\begin{cases} \dfrac{x^2-49}{x-7}, & x\ne 7 \\ k^2-2, & x=7 \end{cases}$ find k such that $\lim\limits_{x\to 7} f(x)=f(7)$

30. Find $\lim\limits_{x\to\infty} 6$

31. Find $\lim\limits_{x\to-\infty}(11-2x)$

32. $\lim\limits_{x\to\infty}\left(0.2x^4-x^2-9\right)$

33. $\lim\limits_{x\to\infty} 2^{-x}$

34. Find $\lim\limits_{x\to\infty}\dfrac{2x-3}{4x+5}$

35. Find $\lim\limits_{x\to-\infty}\dfrac{7-3x^3}{2x^3+1}$

36. Find $\lim\limits_{x\to\infty}\dfrac{2}{5x-4}$

37. Find $\lim\limits_{x\to-\infty}\dfrac{4x^5}{1-5x^3}$

38. Find $\lim\limits_{x\to\infty}\dfrac{x}{\sqrt{x^2+4}}$

39. $\lim\limits_{x\to-\infty}\dfrac{x}{\sqrt{x^2+4}}$

40. $\lim\limits_{x\to-\infty}\dfrac{\sqrt{3x^2+x}}{x^2-1}$

41. $\lim\limits_{x\to-\infty}\dfrac{\sqrt{3x^2+x}}{x-1}$

Part 4

CHAPTER 1

LIMITS

1.1 Graphs of Functions

DESCRIBE THE GRAPHS OF EACH OF THE FOLLOWING FUNCTIONS USING ONLY ONE OF THE FOLLOWING TERMS: *line, parabola, cubic, hyperbola, semicircle.*

1. $y = x^3 + 5x^2 - x - 1$

2. $y = \dfrac{1}{x}$

3. $y = 3x + 2$

4. $y = -x^3 + 500x$

5. $y = \sqrt{9 - x^2}$

6. $y = x^2 + 4$

7. $y = \dfrac{-3}{x - 5}$

8. $y = 9 - x^2$

9. $y = -3x^3$

10. $y = 34x - 5^2$

11. $y = 34x^2 - 52$

12. $y = \sqrt{1 - x^2}$

GRAPH THE FOLLOWING FUNCTIONS ON YOUR CALCULATOR ON THE WINDOW $-3 \le x \le 3$, $-2 \le y \le 2$. SKETCH WHAT YOU SEE. CHOOSE ONE OF THE FOLLOWING TO DESCRIBE WHAT HAPPENS TO THE GRAPH AT THE ORIGIN: A) GOES VERTICAL; B) FORMS A CUSP; C) GOES HORIZONTAL; OR D) STOPS AT ZERO.

13. $y = x^{1/3}$

14. $y = x^{2/3}$

15. $y = x^{4/3}$

16. $y = x^{5/3}$

17. $y = x^{1/4}$

18. $y = x^{5/4}$

19. $y = x^{1/5}$

20. $y = x^{2/5}$

21. Based on the answers from the problems above, find a pattern for the behavior of functions with exponents of the following forms: $x^{\text{even/odd}}$, $x^{\text{odd/odd}}$, $x^{\text{odd/even}}$.

GRAPH THE FOLLOWING FUNCTIONS ON YOUR CALCULATOR IN THE STANDARD WINDOW AND SKETCH WHAT YOU SEE. AT WHAT VALUE(S) OF x ARE THE FUNCTIONS EQUAL TO ZERO?

22. $y = |x - 1|$

23. $y = |x^2 - 4|$

24. $y = |x^3 - 8|$

25. $y = |4 + x^2|$

26. $y = |x^3| - 8$

27. $y = |x^2 - 4x - 5|$

In the company of friends, writers can discuss their books, economists the state of the economy, lawyers their latest cases, and businessmen their latest acquisitions, but mathematicians cannot discuss their mathematics at all. And the more profound their work, the less understandable it is. —*Alfred Adler*

1.2 The Slippery Slope of Lines

The point-slope form of a line is

$$m(x - x_1) = y - y_1.$$

IN THE FIRST SIX PROBLEMS, FIND THE EQUATION OF THE LINE WITH THE GIVEN PROPERTIES.

28. slope: $\frac{2}{3}$; passes through $(2, 1)$

29. slope: $-\frac{1}{4}$; passes through $(0, 6)$

30. passes through $(3, 6)$ and $(2, 7)$

31. passes through $(-6, 1)$ and $(1, 1)$

32. passes through $(5, -4)$ and $(5, 9)$

33. passes through $(10, 3)$ and $(-10, 3)$

34. A line passes through $(1, 2)$ and $(2, 5)$. Another line passes through $(0, 0)$ and $(-4, 3)$. Find the point where the two lines intersect.

35. A line with slope $-\frac{2}{5}$ and passing through $(2, 4)$ is parallel to another line passing through $(-3, 6)$. Find the equations of both lines.

36. A line with slope -3 and passing through $(1, 5)$ is perpendicular to another line passing through $(1, 1)$. Find the equations of both lines.

37. A line passes through $(8, 8)$ and $(-2, 3)$. Another line passes through $(3, -1)$ and $(-3, 0)$. Find the point where the two lines intersect.

38. The function $f(x)$ is a line. If $f(3) = 5$ and $f(4) = 9$, then find the equation of the line $f(x)$.

39. The function $f(x)$ is a line. If $f(0) = 4$ and $f(12) = 5$, then find the equation of the line $f(x)$.

40. The function $f(x)$ is a line. If the slope of $f(x)$ is 3 and $f(2) = 5$, then find $f(7)$.

41. The function $f(x)$ is a line. If the slope of $f(x)$ is $\frac{2}{3}$ and $f(1) = 1$, then find $f(\frac{3}{2})$.

42. If $f(2) = 1$ and $f(b) = 4$, then find the value of b so that the line $f(x)$ has slope 2.

43. Find the equation of the line that has x-intercept at 4 and y-intercept at 1.

44. Find the equation of the line with slope 3 which intersects the semicircle $y = \sqrt{25 - x^2}$ at $x = 4$.

I hope getting the nobel will improve my credit rating, because I really want a credit card. —*John Nash*

1.3 The Power of Algebra

FACTOR EACH OF THE FOLLOWING COMPLETELY.

45. $y^2 - 18y + 56$

46. $33u^2 - 37u + 10$

47. $c^2 + 9c - 8$

48. $(x-6)^2 - 9$

49. $3(x+9)^2 - 36(x+9) + 81$

50. $63q^3 - 28q$

51. $2\pi r^2 + 2\pi r + hr + h$

52. $x^3 + 8$

53. $8x^2 + 27$

54. $64x^6 - 1$

55. $(x+2)^3 + 125$

56. $x^3 - 2x^2 + 9x - 18$

57. $p^5 - 5p^3 + 8p^2 - 40$

SIMPLIFY EACH OF THE FOLLOWING EXPRESSIONS.

58. $\dfrac{3(x-4)+2(x+5)}{6(x-4)}$

59. $\dfrac{1}{x-y} - \dfrac{1}{y-x}$

60. $3x - \dfrac{5x-7}{4}$

61. $\dfrac{\frac{9x^2}{5x^3}}{\frac{3}{x}}$

62. $\dfrac{y}{1-\frac{1}{y}}$

63. $\dfrac{x}{1-\frac{1}{y}} + \dfrac{y}{1-\frac{1}{x}}$

RATIONALIZE EACH OF THE FOLLOWING EXPRESSIONS.

64. $\dfrac{-3+9\sqrt{7}}{\sqrt{7}}$

65. $\dfrac{3\sqrt{2}+\sqrt{5}}{2\sqrt{10}}$

66. $\dfrac{2x+8}{\sqrt{x+4}}$

67. $\dfrac{2-\sqrt{3}}{4+\sqrt{3}}$

68. $\dfrac{x-6}{\sqrt{x-3}+\sqrt{3}}$

69. $\dfrac{9}{\sqrt{2x+3}-\sqrt{2x}}$

70. $\dfrac{5x}{\sqrt{x+5}-\sqrt{5}}$

71. $\dfrac{2\sqrt{5}-6\sqrt{3}}{4\sqrt{5}+\sqrt{3}}$

72. $\dfrac{x}{\sqrt{x+3}-\sqrt{3}}$

Incubation is the work of the subconscious during the waiting time, which may be several years. Illumination, which can happen in a fraction of a second, is the emergence of the creative idea into the conscious. This almost always occurs when the mind is in a state of relaxation, and engaged lightly with ordinary matters. Illumination implies some mysterious rapport between the subconscious and the conscious, otherwise emergence would not happen. What rings the bell at the right moment? —*John E. Littlewood*

1.4 Functions Behaving Badly

SKETCH A GRAPH OF EACH FUNCTION, THEN FIND ITS DOMAIN.

73. $G(x) = \begin{cases} x^2 & x \geq -1 \\ 2x + 3 & x < -1 \end{cases}$

74. $A(t) = \begin{cases} |t| & t < 1 \\ -3t + 4 & t \geq 1 \end{cases}$

75. $h(x) = x + |x|$

76. $V(r) = \begin{cases} \sqrt{1 - r^2} & -1 \leq r \leq 1 \\ \dfrac{1}{r} & r > 1 \end{cases}$

77. $U(x) = \begin{cases} 1/x & x < -1 \\ x & -1 \leq x \leq 1 \\ 1/x & x > 1 \end{cases}$

78. $f(x) = \dfrac{x}{|x|}$

FOR THE FOLLOWING, FIND A) THE DOMAIN; B) THE y-INTERCEPT; AND C) ALL VERTICAL AND HORIZONTAL ASYMPTOTES.

79. $y = \dfrac{x - 2}{x}$

80. $y = \dfrac{-1}{(x - 1)^2}$

81. $y = \dfrac{x - 2}{x - 3}$

82. $y = \dfrac{x}{x^2 + 2x - 8}$

83. $y = \dfrac{x^2 - 2x}{x^2 - 16}$

84. $y = \dfrac{x^2 - 4x + 3}{x - 4}$

CHOOSE THE BEST ANSWER.

85. Which of the following represents the graph of $f(x)$ moved to the left 3 units?

A) $f(x - 3)$ B) $f(x) - 3$ C) $f(x + 3)$ D) $f(x) + 3$

86. Which of the following represents the graph of $g(x)$ moved to the right 2 units and down 7 units?

A) $g(x - 2) - 7$ B) $g(x + 2) + 7$ C) $g(x + 7) - 2$ D) $g(x - 7) + 2$

FACTOR EACH OF THE FOLLOWING.

87. $49p^2 - 144q^2$

88. $15z^2 + 52z + 32$

89. $x^3 - 8$

90. $8x^3 - 27$

91. $27x^3 + y^3$

92. $2w^3 - 10w^2 + w - 5$

He gets up in the morning and immediately starts to do calculus. And in the evening he plays his bongo drums. —*Mrs. Feyman's reasons cited for divorcing her husband, Richard Feyman, Nobel prize-winning physicist*

1.5 Take It to the Limit

EVALUATE EACH LIMIT.

93. $\lim_{x\to -2}(3x^2 - 2x + 1)$

94. $\lim_{x\to 5} 4$

95. $\lim_{x\to -3}(x^3 - 2)$

96. $\lim_{z\to 8} \dfrac{z^2 - 64}{z - 8}$

97. $\lim_{t\to 1/4} \dfrac{4t - 1}{1 - 16t^2}$

98. $\lim_{x\to -2} \dfrac{x^2 + 5x + 6}{x^2 - 4}$

99. $\lim_{x\to 1/3} \dfrac{3x^2 - 7x + 2}{-6x^2 + 5x - 1}$

100. $\lim_{p\to 4} \dfrac{p^3 - 64}{4 - p}$

101. $\lim_{k\to -1} \sqrt[3]{\dfrac{3k - 5}{25k - 2}}$

102. $\lim_{x\to 2} \sqrt{\dfrac{x^2 - 4}{2x^2 + x - 6}}$

103. $\lim_{x\to 0} \dfrac{x}{\sqrt{x+3} - \sqrt{3}}$

104. $\lim_{y\to 0} \dfrac{\sqrt{3y+2} - \sqrt{2}}{y}$

105. Let $F(x) = \dfrac{3x - 1}{9x^2 - 1}$. Find $\lim_{x\to 1/3} F(x)$. Is this the same as the value of $F\left(\frac{1}{3}\right)$?

106. Let $G(x) = \dfrac{4x^2 - 3x}{4x - 3}$. Find $\lim_{x\to 3/4} G(x)$. Is this the same as the value of $G\left(\frac{3}{4}\right)$?

107. Let $P(x) = \begin{cases} 3x - 2 & x \neq \frac{1}{3} \\ 4 & x = \frac{1}{3}. \end{cases}$ Find $\lim_{x\to 1/3} P(x)$. Is this the same as the value of $P\left(\frac{1}{3}\right)$?

108. Let $Q(x) = \begin{cases} \dfrac{x^2 - 16}{x - 4} & x \neq 4 \\ 3 & x = 4. \end{cases}$ Find $\lim_{x\to 4} Q(x)$. Is this the same as the value of $Q(4)$?

SOLVE EACH SYSTEM OF EQUATIONS.

109. $\begin{cases} 2x - 3y = -4 \\ 5x + y = 7 \end{cases}$

110. $\begin{cases} 6x + 15y = 8 \\ 3x - 20y = -7 \end{cases}$

111. If $F(x) = \begin{cases} 2x - 5 & x > \frac{1}{2} \\ 3kx - 1 & x < \frac{1}{2} \end{cases}$ then find the value of k such that $\lim_{x\to 1/2} F(x)$ exists.

1.6 One-Sided Limits

FIND THE LIMITS, IF THEY EXIST, AND FIND THE INDICATED VALUE. IF A LIMIT DOES NOT EXIST, EXPLAIN WHY.

112. Let $f(x) = \begin{cases} 4x-2 & x>1 \\ 2-4x & x \le 1. \end{cases}$

a) $\lim_{x\to 1^+} f(x)$ b) $\lim_{x\to 1^-} f(x)$ c) $\lim_{x\to 1} f(x)$ d) $f(1)$

113. Let $a(x) = \begin{cases} 3-6x & x>1 \\ -1 & x=1 \\ x^2 & x<1. \end{cases}$

a) $\lim_{x\to 1^+} a(x)$ b) $\lim_{x\to 1^-} a(x)$ c) $\lim_{x\to 1} a(x)$ d) $a(1)$

114. Let $h(t) = \begin{cases} 3t-1 & t>2 \\ -5 & t=2 \\ 1+2t & t<2. \end{cases}$

a) $\lim_{t\to 2^+} h(t)$ b) $\lim_{t\to 2^-} h(t)$ c) $\lim_{t\to 2} h(t)$ d) $h(2)$

115. Let $c(x) = \begin{cases} x^2-9 & x<3 \\ 5 & x=3 \\ 9-x^2 & x>3. \end{cases}$

a) $\lim_{x\to 3^+} c(x)$ b) $\lim_{x\to 3^-} c(x)$ c) $\lim_{x\to 3} c(x)$ d) $c(3)$

116. Let $v(t) = |3t-6|$.

a) $\lim_{t\to 2^+} v(t)$ b) $\lim_{t\to 2^-} v(t)$ c) $\lim_{t\to 2} v(t)$ d) $v(2)$

117. Let $y(x) = \dfrac{|3x|}{x}$.

a) $\lim_{x\to 0^+} y(x)$ b) $\lim_{x\to 0^-} y(x)$ c) $\lim_{x\to 0} y(x)$ d) $y(0)$

118. Let $k(z) = |-2z+4| - 3$.

a) $\lim_{z\to 2^+} k(z)$ b) $\lim_{z\to 2^-} k(z)$ c) $\lim_{z\to 2} k(z)$ d) $k(2)$

EXPLAIN WHY THE FOLLOWING LIMITS DO NOT EXIST.

119. $\lim_{x\to 0} \dfrac{x}{|x|}$

120. $\lim_{x\to 1} \dfrac{1}{x-1}$

1.7 One-Sided Limits (Again)

IN THE FIRST NINE PROBLEMS, EVALUATE EACH LIMIT.

121. $\lim_{x\to 5^+} \frac{x-5}{x^2-25}$

122. $\lim_{x\to 2^+} \frac{2-x}{x^2-4}$

123. $\lim_{x\to 2} \frac{|x-2|}{x-2}$

124. $\lim_{x\to 4^-} \frac{3x}{16-x^2}$

125. $\lim_{x\to 0} \frac{x^2-7}{3x^3-2x}$

126. $\lim_{x\to 0^-} \left(\frac{3}{x^2}-\frac{2}{x}\right)$

127. $\lim_{x\to 2^-} \frac{x+2}{2-x}$

128. $\lim_{x\to 4^+} \frac{3x}{x^2-4}$

129. $\lim_{x\to 0} \frac{x^2}{\sqrt{3x^2+1}-1}$

SOLVE EACH SYSTEM OF EQUATIONS.

130. $\begin{cases} x-y=-7 \\ \frac{1}{2}x+3y=14 \end{cases}$

131. $\begin{cases} 8x-5y=1 \\ 5x-8y=-1 \end{cases}$

132. If $G(x)=\begin{cases} 3x^2-kx+m & x\geq 1 \\ mx-2k & -1<x<1 \\ -3m+4x^3k & x\leq -1 \end{cases}$ then find the values of m and k such that both $\lim_{x\to 1} G(x)$ and $\lim_{x\to -1} G(x)$ exist.

FOR THE FOLLOWING, FIND A) THE DOMAIN; B) THE y-INTERCEPT; AND C) ALL VERTICAL AND HORIZONTAL ASYMPTOTES.

133. $y=\frac{x^3+3x^2}{x^4-4x^2}$

134. $y=\frac{x^5-25x^3}{x^4+2x^3}$

135. $y=\frac{x^2+6x+9}{2x}$

SUPPOSE THAT $\lim_{x\to 4} f(x)=5$ AND $\lim_{x\to 4} g(x)=-2$. FIND THE FOLLOWING LIMITS.

136. $\lim_{x\to 4} f(x)g(x)$

137. $\lim_{x\to 4} (f(x)+3g(x))$

138. $\lim_{x\to 4} \frac{f(x)}{f(x)-g(x)}$

139. $\lim_{x\to 4} xf(x)$

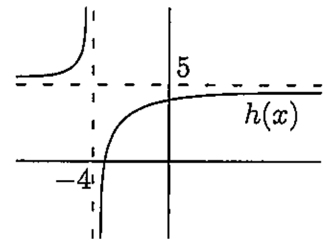
140. $\lim_{x\to 4} (g(x))^2$

141. $\lim_{x\to 4} \frac{g(x)}{f(x)-1}$

How can you shorten the subject? That stern struggle with the multiplication table, for many people not yet ended in victory, how can you make it less? Square root, as obdurate as a hardwood stump in a pasture, nothing but years of effort can extract it. You can't hurry the process. Or pass from arithmetic to algebra; you can't shoulder your way past quadratic equations or ripple through the binomial theorem. Instead, the other way; your feet are impeded in the tangled growth, your pace slackens, you sink and fall somewhere near the binomial theorem with the calculus in sight on the horizon. So died, for each of us, still bravely fighting, our mathematical training; except for a set of people called "mathematicians" – born so, like crooks. —*Stephen Leacock*

1.8 Limits Determined by Graphs

REFER TO THE GRAPH OF $h(x)$ TO EVALUATE THE FOLLOWING LIMITS.



142. $\lim_{x\to -4^+} h(x)$

143. $\lim_{x\to -4^-} h(x)$

144. $\lim_{x\to \infty} h(x)$

145. $\lim_{x\to -\infty} h(x)$

REFER TO THE GRAPH OF $g(x)$ TO EVALUATE THE FOLLOWING LIMITS.

146. $\lim_{x\to a^+} g(x)$

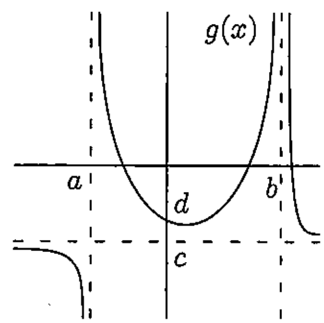
147. $\lim_{x\to a^-} g(x)$

148. $\lim_{x\to 0} g(x)$

149. $\lim_{x\to \infty} g(x)$

150. $\lim_{x\to b^+} g(x)$

151. $\lim_{x\to b^-} g(x)$



REFER TO THE GRAPH OF $f(x)$ TO DETERMINE WHICH STATEMENTS ARE TRUE AND WHICH ARE FALSE. IF A STATEMENT IS FALSE, EXPLAIN WHY.

152. $\lim_{x\to -1^+} f(x) = 1$

153. $\lim_{x\to 0^-} f(x) = 0$

154. $\lim_{x\to 0^-} f(x) = 1$

155. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

156. $\lim_{x\to 0} f(x)$ exists

157. $\lim_{x\to 0} f(x) = 0$

158. $\lim_{x\to 0} f(x) = 1$

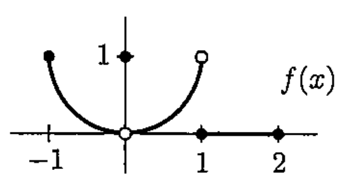
159. $\lim_{x\to 1} f(x) = 1$

160. $\lim_{x\to 1} f(x) = 0$

161. $\lim_{x\to 2^-} f(x) = 2$

162. $\lim_{x\to -1^-} f(x)$ does not exist

163. $\lim_{x\to 2^+} f(x) = 0$



If your experiment needs statistics, you ought to have done a better experiment. —*Ernest Rutherford*

1.9 Limits Determined by Tables

USING YOUR CALCULATOR, FILL IN EACH OF THE FOLLOWING TABLES TO FIVE DECIMAL PLACES. USING THE INFORMATION FROM THE TABLE, DETERMINE EACH LIMIT. (FOR THE TRIGONOMETRIC FUNCTIONS, YOUR CALCULATOR MUST BE IN *radian* MODE.)

164. $\lim\limits_{x\to 0} \dfrac{\sqrt{x+3}-\sqrt{3}}{x}$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$\frac{\sqrt{x+3}-\sqrt{3}}{x}$						

165. $\lim\limits_{x\to -3} \dfrac{\sqrt{1-x}-2}{x+3}$

x	-3.1	-3.01	-3.001	-2.999	-2.99	-2.9
$\frac{\sqrt{1-x}-2}{x+3}$						

166. $\lim\limits_{x\to 0} \dfrac{\sin x}{x}$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$\frac{\sin}{x}$						

167. $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x}$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$\frac{1-\cos x}{x}$						

168. $\lim\limits_{x\to 0} (1+x)^{1/x}$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$(1+x)^{1/x}$						

169. $\lim\limits_{x\to 1} x^{1/(1-x)}$

x	0.9	0.99	0.999	1.001	1.01	1.1
$x^{1/(1-x)}$						

Science is built up with facts, as a house is with stones. But a collection of facts is no more a science than a heap of stones is a house. —*Henri Poincaré*

1.10 The Possibilities Are Limitless...

REFER TO THE GRAPH OF $R(x)$ TO EVALUATE THE FOLLOWING.

170. $\lim_{x\to\infty} R(x)$

171. $\lim_{x\to-\infty} R(x)$

172. $\lim_{x\to a^+} R(x)$

173. $\lim_{x\to a^-} R(x)$

174. $\lim_{x\to a} R(x)$

175. $\lim_{x\to 0} R(x)$

176. $\lim_{x\to b^+} R(x)$

177. $\lim_{x\to b^-} R(x)$

178. $\lim_{x\to b} R(x)$

179. $\lim_{x\to c} R(x)$

180. $\lim_{x\to d} R(x)$

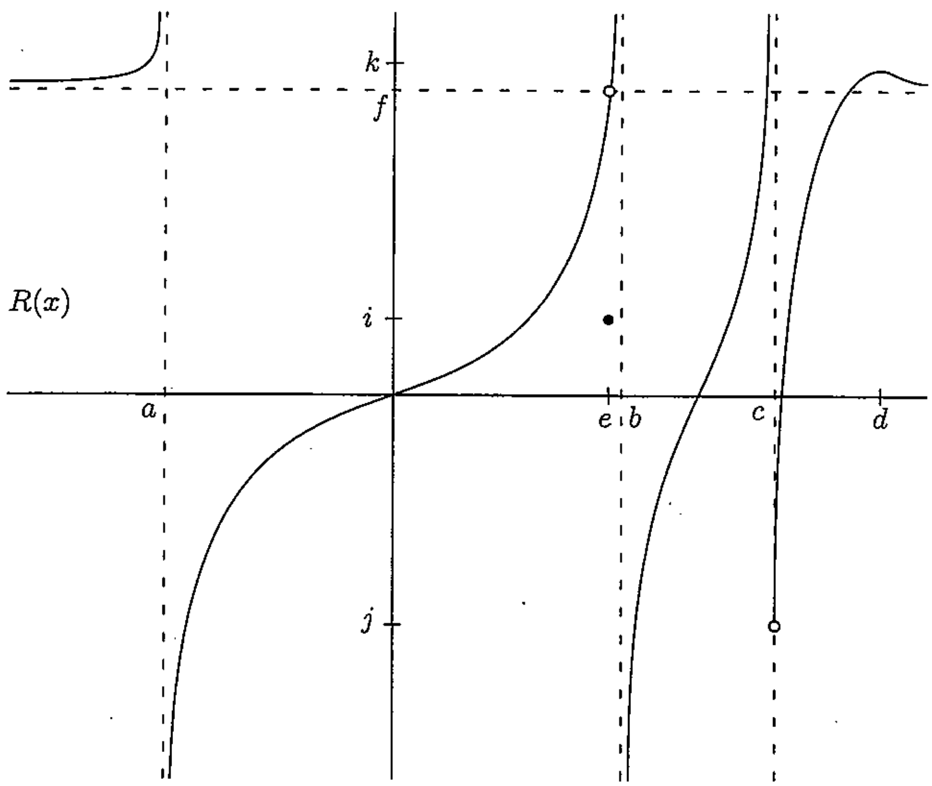
181. $\lim_{x\to e} R(x)$

182. $R(e)$

183. $R(0)$

184. $R(b)$

185. $R(d)$



One of the big misapprehensions about mathematics that we perpetrate in our classrooms is that the teacher always seems to know the answer to any problem that is discussed. This gives students the idea that there is a book somewhere with all the right answers to all of the interesting questions, and that teachers know those answers. And if one could get hold of the book, one would have everything settled. That's so unlike the true nature of mathematics. —*Leon Hankin*

1.11 Average Rates of Change: Episode I

186. Find a formula for the average rate of change of the area of a circle as its radius r changes from 3 to some number x. Then determine the average rate of change of the area of a circle as the radius r changes from

a) 3 to 3.5 b) 3 to 3.2 c) 3 to 3.1 d) 3 to 3.01

187. Find a formula for the average rate of change of the volume of a cube as its side length s changes from 2 to some number x. Then determine the average rate of change of the volume of a cube as the side length s changes from

a) 2 to 3 b) 2 to 2.5 c) 2 to 2.2 d) 2 to 2.1

188. A car is stopped at a traffic light and begins to move forward along a straight road when the light turns green. The distance s, in feet, traveled by a car in t seconds is given by $s(t) = 2t^2$ $(0 \le t \le 30)$. What is the average rate of change of the car from

a) $t = 0$ to $t = 5$ b) $t = 5$ to $t = 10$ c) $t = 0$ to $t = 10$ d) $t = 10$ to $t = 10.1$

IN THE FOLLOWING SIX PROBLEMS, FIND A FORMULA FOR THE AVERAGE RATE OF CHANGE OF EACH FUNCTION FROM $x = 1$ TO SOME NUMBER $x = c$.

189. $f(x) = x^2 + 2x$

190. $f(x) = \sqrt{x}$

191. $f(x) = 2x^2 - 4x$

192. $g(t) = 2t - 6$

193. $p(x) = \dfrac{3}{x}$

194. $F(x) = -2x^3$

1.12 Exponential and Logarithmic Functions

SIMPLIFY THE FOLLOWING EXPRESSIONS.

195. $e^{\ln x + \ln y}$

196. $\ln(e^{3x})$

197. $\log_4(4^{y+3})$

198. $5^{\log_5(x+2y)}$

199. $\ln(e^{5x+\ln 6})$

200. $e^{3\ln x - 2\ln 5}$

FOR THE FOLLOWING FUNCTIONS, FIND THE DOMAIN AND THE y-INTERCEPT.

201. $y = e^{3x-1}\sqrt{x}$

202. $y = x\log_3(5x-2)$

203. $y = e^{3x/(2x-1)}\sqrt[3]{x-7}$

204. $y = \ln(8x^2 - 4)$

205. $y = e^{5x/(3x-2)}\ln e^x$

206. $y = \ln(x^2 - 8x + 15)$

Ludwig Boltzmann, who spent much of his life studying statistical mechanics, died in 1906, by his own hand. Paul Ehrenfest, carrying on the work, died similarly in 1933. Now it is our turn to study statistical mechanics.
—*David L. Goodstein, in the preface to his book* States of Matter

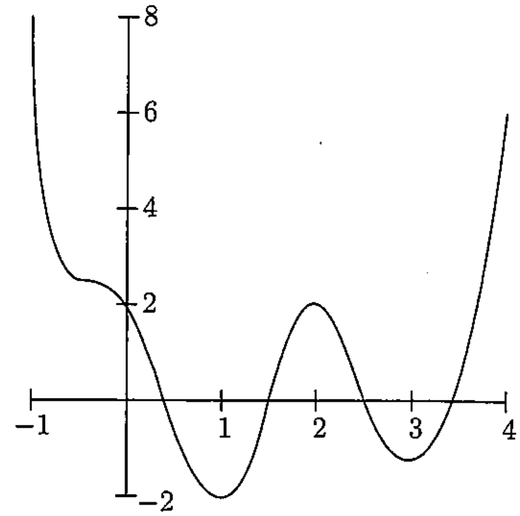
1.13 Average Rates of Change: Episode II

207. The position $p(t)$ is given by the graph at the right.

a) Find the average velocity of the object between times $t = 1$ and $t = 4$.

b) Find the equation of the secant line of $p(t)$ between times $t = 1$ and $t = 4$.

c) For what times t is the object's velocity positive? For what times is it negative?



208. Suppose $f(1) = 2$ and the average rate of change of f between 1 and 5 is 3. Find $f(5)$.

209. The position $p(t)$, in meters, of an object at time t, in seconds, along a line is given by $p(t) = 3t^2 + 1$.

a) Find the change in position between times $t = 1$ and $t = 3$.

b) Find the average velocity of the object between times $t = 1$ and $t = 4$.

c) Find the average velocity of the object between any time t and another time $t + \Delta t$.

210. Let $f(x) = x^2 + x - 2$.

a) Find the average rate of change of $f(x)$ between times $x = -1$ and $x = 2$.

b) Draw the graph of f and the graph of the secant line through $(-1, -2)$ and $(2, 4)$.

c) Find the slope of the secant line graphed in part b) and then find an equation of this secant line.

d) Find the average rate of change of $f(x)$ between any point x and another point $x + \Delta x$.

FIND THE AVERAGE RATE OF CHANGE OF EACH FUNCTION OVER THE GIVEN INTERVALS.

211. $f(x) = x^3 + 1$ over a) $[2, 3]$; b) $[-1, 1]$

212. $R(x) = \sqrt{4x + 1}$ over a) $[0, \frac{3}{4}]$; b) $[0, 2]$

213. $h(t) = \dfrac{1}{\tan t}$ over a) $[\frac{\pi}{4}, \frac{3\pi}{4}]$; b) $[\frac{\pi}{6}, \frac{\pi}{3}]$

214. $g(t) = 2 + \cos t$ over a) $[0, \pi]$; b) $[-\pi, \pi]$

Have lots of ideas and throw away the bad ones. You aren't going to have good ideas unless you have lots of ideas and some sort of principle of selection. —*Linus Pauling*

1.14 Take It To the Limit—One More Time

EVALUATE EACH LIMIT.

215. $\lim\limits_{x\to\infty} \dfrac{5x-3}{3-2x}$

216. $\lim\limits_{y\to\infty} \dfrac{4y-3}{3-2y}$

217. $\lim\limits_{x\to\infty} \dfrac{3x^2+2x+1}{5-2x^2+3x}$

218. $\lim\limits_{x\to\infty} \dfrac{3x+2}{4x^2-3}$

219. $\lim\limits_{x\to\infty} \dfrac{4x^2-3}{3x+2}$

220. $\lim\limits_{x\to\infty} \dfrac{3x^3-1}{4x+3}$

221. $\lim\limits_{x\to\infty} \left(4x+\dfrac{3}{x^2}\right)$

222. $\lim\limits_{z\to\infty} \dfrac{\sqrt{z^2+9}}{z+9}$

223. $\lim\limits_{x\to\infty} \dfrac{3}{x^5}$

224. $\lim\limits_{x\to-2} \dfrac{5x-1}{x+2}$

225. $\lim\limits_{x\to5} \dfrac{-4x+3}{x-5}$

226. $\lim\limits_{x\to0} \left(3-\dfrac{2}{x}\right)$

227. $\lim\limits_{x\to0} \left(3-\dfrac{2}{x^2}\right)$

228. $\lim\limits_{x\to5} \dfrac{3x^2}{x^2-25}$

229. $\lim\limits_{x\to0} \dfrac{\sqrt{x+3}-\sqrt{3}}{x}$

230. $\lim\limits_{x\to-3} \dfrac{x^2-5x+6}{x^2-9}$

231. $\lim\limits_{x\to-3} (3x+2)$

232. $\lim\limits_{x\to2} (-x^2+x-2)$

233. $\lim\limits_{x\to4} \sqrt[3]{x+4}$

234. $\lim\limits_{x\to2} \dfrac{1}{x}$

235. $\lim\limits_{x\to3} \dfrac{\sqrt{x+1}}{x-4}$

236. $\lim\limits_{x\to1} \dfrac{x^2+x-2}{x^2-1}$

237. $\lim\limits_{x\to0} \dfrac{\sqrt{2+x}-\sqrt{2}}{x}$

238. $\lim\limits_{x\to\infty} \dfrac{\sqrt{2+x}-\sqrt{2}}{x}$

FOR THE FOLLOWING, A) SKETCH THE GRAPH OF f AND B) DETERMINE AT WHAT POINTS c IN THE DOMAIN OF f, IF ANY, DOES $\lim\limits_{x\to c} f(x)$ EXIST. JUSTIFY YOUR ANSWER.

239. $f(x)=\begin{cases} 3-x & x<2 \\ \dfrac{x}{2}+1 & x>2 \end{cases}$

240. $f(x)=\begin{cases} 3-x & x<2 \\ 2 & x=2 \\ \dfrac{x}{2} & x>2 \end{cases}$

241. $f(x)=\begin{cases} \dfrac{1}{x-1} & x<1 \\ x^3-2x+5 & x\geq 2 \end{cases}$

242. $f(x)=\begin{cases} 1-x^2 & x\neq -1 \\ 2 & x=-1 \end{cases}$

243. $f(x)=\begin{cases} \sqrt{1-x^2} & 0\leq x<1 \\ 1 & 1\leq x<2 \\ 2 & x=2 \end{cases}$

244. $f(x)=\begin{cases} x & -1\leq x<0 \text{ or } 0<x\leq 1 \\ 1 & x=0 \\ 0 & x<-1 \text{ or } x>1 \end{cases}$

The discovery in 1846 of the planet Neptune was a dramatic and spectacular achievement of mathematical astronomy. The very existence of this new member of the solar system, and its exact location, were demonstrated with pencil and paper; there was left to observers only the routine task of pointing their telescopes at the spot the mathematicians had marked. —*James R. Newman*

1.15 Solving Equations

SOLVE EACH OF THE FOLLOWING EQUATIONS.

245. $1 - \frac{8}{k^3} = 0$

246. $4p^3 - 4p = 0$

247. $x^3 - 2x^2 - 3x = 0$

248. $3x^2 - 10x - 8 = 0$

249. $|4x^3 - 3| = 0$

250. $|w^2 - 6w| = 9$

251. $\frac{3(x-4) - (3x-2)}{(x-4)^2} = 0$

252. $\frac{2x - 3}{2(x^2 - 3x)} = 0$

253. $2 \ln x = 9$

254. $e^{5x} = 7$

255. $\ln(2x - 1) = 0$

256. $e^{3x+7} = 12$

257. $\ln \sqrt[4]{x+1} = \frac{1}{2}$

258. $2^{3x-1} = \frac{1}{2}$

259. $\log_8(x - 5) = \frac{2}{3}$

260. $\log \sqrt{z} = \log(z - 6)$

261. $2 \ln(p + 3) - \ln(p + 1) = 3 \ln 2$

262. $3^{x^2} = 7$

263. $\log_3(3x) = \log_3 x + \log_3(4 - x)$

FIND ALL REAL ZEROS OF THE FOLLOWING FUNCTIONS.

264. $y = x^2 - 4$

265. $y = -2x^4 + 5$

266. $y = x^3 - 3$

267. $y = x^3 - 9x$

268. $y = x^4 + 2x^2$

269. $y = x^3 - 4x^2 - 5x$

270. $y = x^3 - 5x^2 - x + 5$

271. $y = x^3 + 3x^2 - 4x - 12$

272. $y = \frac{x - 2}{x}$

273. $y = \frac{-1}{(x-1)^2}$

274. $y = \frac{1 + x}{1 - x}$

275. $y = \frac{x^3}{1 + x^2}$

276. $y = \frac{x^2 - 2x}{x^2 - 16}$

277. $y = \frac{x^2 - 4x + 3}{x - 4}$

278. $y = \frac{x^3 + 3x^2}{x^4 - 4x^2}$

279. $y = \frac{x^5 - 25x^3}{x^4 + 2x^3}$

280. $y = x^2 + \frac{1}{x}$

281. $y = e^{3x-1}\sqrt{x}$

282. $y = x \log_3(5x - 2)$

283. $y = e^{3x/(2x-1)} \sqrt[3]{x - 7}$

284. $y = \ln(8x^2 - 4)$

285. $y = e^{5x/(3x-2)} \ln e^x$

DETERMINE WHETHER THE FUNCTIONS IN THE PROBLEMS LISTED ARE EVEN, ODD, OR NEITHER.

286. problem 264

287. problem 268

288. problem 272

289. problem 274

290. problem 275

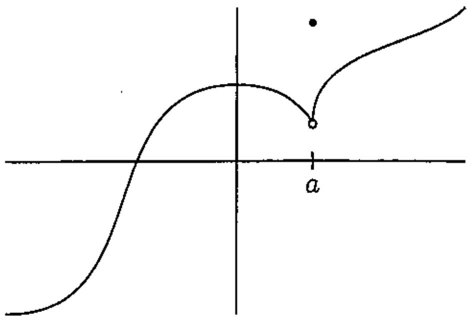
291. problem 280

The chief aim of all investigations of the external world should be to discover the rational order and harmony which has been imposed on it by God and which He revealed to us in the language of mathematics. —*Johannes Kepler*

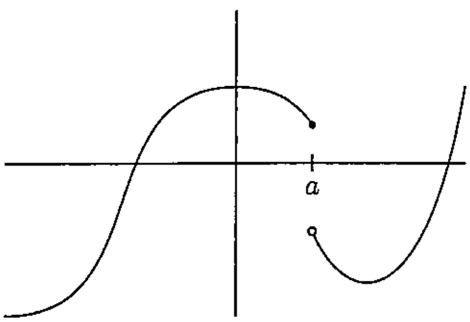
1.16 Continuously Considering Continuity

Examine the graphs of the functions below. Explain why each is discontinuous at $x = a$, and determine the type of discontinuity.

292.



293.



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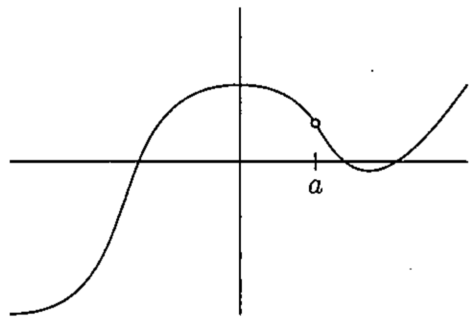

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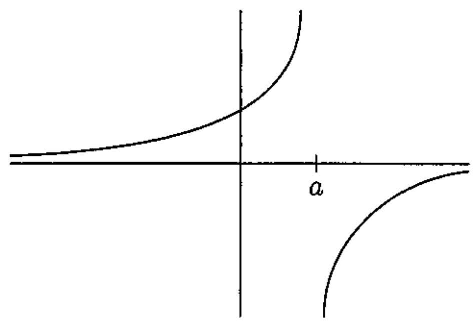

Determine the values of the independent variable for which the function is discontinuous. Justify your answers.

296. $f(x) = \dfrac{x^2 + x - 2}{x - 1}$

297. $d(r) = \dfrac{r^4 - 1}{r^2 - 1}$

298. $A(k) = \dfrac{k^2 - 2}{k^4 - 1}$

299. $q(t) = \dfrac{3}{t + 7}$

300. $m(z) = \begin{cases} \dfrac{z^2 + z - 2}{z - 1} & z \neq 1 \\ 3 & z = 1 \end{cases}$

301. $s(w) = \begin{cases} \dfrac{3}{w + 7} & w \neq -7 \\ 2 & w = -7 \end{cases}$

302. $p(j) = \begin{cases} 4 & j < 0 \\ 0 & j = 0 \\ \sqrt{j} & j > 0 \end{cases}$

303. $b(y) = \begin{cases} y^2 - 9 & y < 3 \\ 5 & y = 3 \\ 9 - y^2 & y > 3 \end{cases}$

Considering how many fools can calculate, it is surprising that it should be thought either a difficult or tedious task for any other fool to learn to master the same tricks. —*Silvanus P. Thompson*

1.17 Have You Reached the Limit?

304. Estimate the value of $\lim_{x\to\infty}(\sqrt{x^2+x+1}-x)$ by graphing or by making a table of values.

305. Estimate the value of $\lim_{x\to\infty}(\sqrt{x^2+x}-\sqrt{x^2-x})$ by graphing or by making a table of values.

306. Consider the function $f(x)=\begin{cases} x^2-1 & -1\le x<0 \\ 2x & 0<x<1 \\ 1 & x=1 \\ -2x+4 & 1<x<2 \\ 0 & 2<x<3. \end{cases}$

a) Graph this function.

b) Does $f(-1)$ exist?

c) Does $\lim_{x\to-1^+} f(x)$ exist?

d) Does $\lim_{x\to-1^+} f(x)=f(-1)$?

e) Is f continuous at $x=-1$?

f) Does $f(1)$ exist?

g) Does $\lim_{x\to1^+} f(x)$ exist?

h) Does $\lim_{x\to1^+} f(x)=f(1)$?

i) Is f continuous at $x=1$?

j) Is f defined at $x=2$?

k) Is f continuous at $x=2$?

l) At what values of x is f continuous?

m) What value should be assigned to $f(2)$ to make the function continuous at $x=2$?

n) To what new value of $f(1)$ be changed to remove the discontinuity?

307. Is $F(x)=\dfrac{|x^2-4|x}{x+2}$ continuous everywhere? Why or why not?

308. Is $F(x)=\dfrac{|x^2+4x|(x+2)}{x+4}$ continuous everywhere? Why or why not?

FIND THE CONSTANTS a AND b SUCH THAT THE FUNCTION IS CONTINUOUS EVERYWHERE.

309. $f(x)=\begin{cases} x^3 & x\le 2 \\ ax^2 & x>2 \end{cases}$

310. $g(x)=\begin{cases} \dfrac{4\sin x}{x} & x<0 \\ a-2x & x\ge 0 \end{cases}$

311. $f(x)=\begin{cases} 2 & x\le -1 \\ ax+b & -1<x<3 \\ -2 & x\ge 3 \end{cases}$

312. $g(x)=\begin{cases} \dfrac{x^2-a^2}{x-a} & x\ne a \\ 8 & x=a \end{cases}$

1.18 Multiple Choice Questions on Limits

313. $\lim_{x\to\infty} \dfrac{3x^4 - 2x + 1}{7x - 8x^5 - 1} =$

A) ∞ B) $-\infty$ C) 0 D) $\frac{3}{7}$ E) $-\frac{3}{8}$

314. $\lim_{x\to 0^-} \dfrac{1}{x} =$

A) ∞ B) $-\infty$ C) 0 D) 1 E) does not exist

315. $\lim_{x\to 1/3} \dfrac{9x^2 - 1}{3x - 1} =$

A) ∞ B) $-\infty$ C) 0 D) 2 E) 3

316. $\lim_{x\to 0} \dfrac{x^3 - 8}{x^2 - 4} =$

A) 4 B) 0 C) 1 D) 3 E) 2

317. In order for the line $y = a$ to be a horizontal asymptote of $h(x)$, which of the following must be true?

A) $\lim_{x\to a^+} h(x) = \infty$

B) $\lim_{x\to a^-} h(x) = -\infty$

C) $\lim_{x\to\infty} h(x) = \infty$

D) $\lim_{x\to -\infty} h(x) = a$

E) $\lim_{x\to -\infty} h(x) = \infty$

318. The function $G(x) = \begin{cases} x - 3 & x > 2 \\ -5 & x = 2 \\ 3x - 7 & x < 2 \end{cases}$ is not continuous at $x = 2$ because

A) $G(2)$ is not defined

B) $\lim_{x\to 2} G(x)$ does not exist

C) $\lim_{x\to 2} G(x) \neq G(2)$

D) $G(2) \neq -5$

E) All of the above

319. $\lim_{x\to 0} \dfrac{3x^2 + 2x}{2x + 1} =$

A) ∞ B) $-\infty$ C) 0 D) 1 E) $\frac{3}{2}$

320. $\lim_{x\to -1/2^-} \dfrac{2x^2-3x-2}{2x+1} =$

A) ∞ B) $-\infty$ C) 1 D) $\frac{3}{2}$ E) $-\frac{5}{2}$

321. $\lim_{x\to -2} \dfrac{\sqrt{2x+5}-1}{x+2} =$

A) 1 B) 0 C) ∞ D) $-\infty$ E) does not exist

322. $\lim_{x\to -\infty} \dfrac{3x^2+2x^3+5}{x^4+7x^2-3} =$

A) 0 B) 2 C) $\frac{3}{7}$ D) ∞ E) $-\infty$

323. $\lim_{x\to 0} \dfrac{-x^2+4}{x^2-1} =$

A) 1 B) 0 C) -4 D) -1 E) ∞

324. The function $G(x) = \begin{cases} x^2 & x>2 \\ 4-2x & x<2 \end{cases}$ is not continuous at $x=2$ because

A) $G(2)$ does not exist

B) $\lim_{x\to 2} G(x)$ does not exist

C) $\lim_{x\to 2} G(x) = G(2)$

D) All three statements A, B, and C

E) None of the above

325. The domain of the function $f(x) = \sqrt{4-x^2}$ is

A) $x < -2$ or $x > 2$ B) $x \le -2$ or $x \ge 2$ C) $-2 < x < 2$ D) $-2 \le x \le 2$ E) $x \le 2$

326. $\lim_{x\to 5} \dfrac{x^2-25}{x-5} =$

A) 0 B) 10 C) -10 D) 5 E) does not exist

327. Find k so that $f(x) = \begin{cases} \dfrac{x^2-16}{x-4} & x\neq 4 \\ k & x=4 \end{cases}$ is continuous for all x.

A) any value B) 0 C) 8 D) 16 E) no value

Insanity means we keep trying the same thing and hope it comes out differently. —*Albert Einstein*

1.19 Sample A.P. Problems on Limits

328. For the function $f(x) = \dfrac{2x-1}{|x|}$, find the following:

a) $\lim_{x\to\infty} f(x)$;

b) $\lim_{x\to-\infty} f(x)$;

c) $\lim_{x\to 0^+} f(x)$;

d) $\lim_{x\to 0^-} f(x)$;

e) All horizontal asymptotes;

f) All vertical asymptotes.

329. Consider the function $h(x) = \dfrac{1}{1-2^{1/x}}$.

a) What is the domain of h?

b) Find all zeros of h.

c) Find all vertical and horizontal asymptotes of h.

d) Find $\lim_{x\to 0^+} h(x)$.

e) Find $\lim_{x\to 0^-} h(x)$.

f) Find $\lim_{x\to 0} h(x)$.

330. Consider the function $g(x) = \dfrac{\sin|x|}{x}$ defined for all real numbers.

a) Is $g(x)$ an even function, an odd function, or neither? Justify your answer.

b) Find the zeros and the domain of g.

c) Find $\lim_{x\to 0} g(x)$.

331. Let $f(x) = \begin{cases} \sqrt{1-x^2} & 0 \le x < 1 \\ 1 & 1 \le x < 2. \\ 2 & x = 2 \end{cases}$

a) Draw the graph of f.

b) At what points c in the domain of f does $\lim_{x\to c} f(x)$ exist?

c) At what points does only the left-hand limit exist?

d) At what points does only the right-hand limit exist?

A.P. Calculus Test One
Section One
Multiple-Choice
No Calculators

Time—30 minutes

Number of Questions—15

The scoring for this section is determined by the formula

$$[C - (0.25 \times I)] \times 1.8$$

where C is the number of correct responses and I is the number of incorrect responses. An unanswered question earns zero points. The maximum possible points earned on this section is 27, which represents 50% of the total test score.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding choice on your answer sheet. Do not spend too much time on any one problem.

Good Luck!

NAME:

1. Which of the following is continuous at $x = 0$?

I. $f(x) = |x|$
II. $f(x) = e^x$
III. $f(x) = \ln(e^x - 1)$

A) I only

B) II only

C) I and II only

D) II and III only

E) none of these

---

2. The graph of a function f is reflected across the x-axis and then shifted up 2 units. Which of the following describes this transformation on f ?

A) $-f(x)$

B) $f(x) + 2$

C) $-f(x+2)$

D) $-f(x-2)$

E) $-f(x) + 2$

---

3. Which of the following functions is *not* continuous for all real numbers x ?

A) $f(x) = x^{1/3}$

B) $f(x) = \dfrac{2}{(x+1)^4}$

C) $f(x) = |x+1|$

D) $f(x) = \sqrt{1 + e^x}$

E) $f(x) = \dfrac{x-3}{x^2+9}$

4. $\lim_{x \to 1} \dfrac{\ln x}{x}$ is

A) 1

B) 0

C) e

D) $-e$

E) nonexistent

---

5. $\lim_{x \to 0} \left(\dfrac{1}{x} + \dfrac{1}{x^2} \right) =$

A) 0

B) $\frac{1}{2}$

C) 1

D) 2

E) ∞

---

6. $\lim_{x \to \infty} \dfrac{x^3 - 4x + 1}{2x^3 - 5} =$

A) $-\frac{1}{5}$

B) $\frac{1}{2}$

C) $\frac{2}{3}$

D) 1

E) Does not exist

7. For what value of k does $\lim_{x\to 4} \dfrac{x^2 - x + k}{x - 4}$ exist?

A) -12

B) -4

C) 3

D) 7

E) No such value exists.

---

8. $\lim_{x\to 0} \dfrac{\tan x}{x} =$

A) -1

B) $-\frac{1}{2}$

C) 0

D) $\frac{1}{2}$

E) 1

---

9. Suppose f is defined as

$$f(x) = \begin{cases} \dfrac{|x| - 2}{x - 2} & x \neq 2 \\ k & x = 2. \end{cases}$$

Then the value of k for which $f(x)$ is continuous for all real values of x is $k =$

A) -2

B) -1

C) 0

D) 1

E) 2

10. The average rate of change of $f(x) = x^3$ over the interval $[a, b]$ is

A) $3b + 3a$

B) $b^2 + ab + a^2$

C) $\dfrac{b^2 + a^2}{2}$

D) $\dfrac{b^3 - a^3}{2}$

E) $\dfrac{b^4 - a^4}{4(b - a)}$

---

11. The function

$$G(x) = \begin{cases} x - 5 & x > 2 \\ -5 & x = 2 \\ 5x - 13 & x < 2 \end{cases}$$

is not continuous at $x = 2$ because

A) $G(2)$ is not defined.

B) $\lim\limits_{x \to 2} G(x)$ does not exist.

C) $\lim\limits_{x \to 2} G(x) \neq G(2)$.

D) $G(2) \neq -5$.

E) None of the above

---

12. $\lim\limits_{x \to -2} \dfrac{\sqrt{2x+5} - 1}{x + 2} =$

A) 1

B) 0

C) ∞

D) $-\infty$

E) does not exist

13. The Intermediate Value Theorem states that given a continuous function f defined on the closed interval $[a, b]$ for which 0 is between $f(a)$ and $f(b)$, there exists a point c between a and b such that

A) $c = a - b$

B) $f(a) = f(b)$

C) $f(c) = 0$

D) $f(0) = c$

E) $c = 0$

---

14. The function $t(x) = 2^x - \dfrac{|x-3|}{x-3}$ has

A) a removable discontinuity at $x = 3$.

B) an infinite discontinuity at $x = 3$.

C) a jump discontinuity at $x = 3$.

D) no discontinuities.

E) a removable discontinuity at $x = 0$ and an infinite discontinuity at $x = 3$.

---

15. Find the values of c so that the function

$$h(x) = \begin{cases} c^2 - x^2 & x < 2 \\ x + c & x \geq 2 \end{cases}$$

is continuous everywhere.

A) $-3, -2$

B) $2, 3$

C) $-2, 3$

D) $-3, 2$

E) There are no such values.

A.P. Calculus Test One
Section Two
Free-Response
Calculators Allowed

Time—45 minutes

Number of Questions—3

Each of the three questions is worth 9 points. The maximum possible points earned on this section is 27, which represents 50% of the total test score. There is no penalty for guessing.

- SHOW ALL YOUR WORK. You will be graded on the methods you use as well as the accuracy of your answers. Correct answers without supporting work may not receive full credit.
- Write all work for each problem in the space provided. Be sure to write clearly and legibly. Erased or crossed out work will not be graded.
- Justifications require that you give mathematical (non-calculator) reasons and that you clearly identify functions, graphs, tables, or other objects that you use.
- You are permitted to use your calculator to solve an equation or graph a function without showing work. However, you must clearly indicate the setup of your problem.
- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $y = x^2$ may not be written as `Y1=X^2`.
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Good Luck!

NAME:

1. Consider the function $f(x) = \dfrac{|x|(x-3)}{9-x^2}$.

a) What is the domain of f ? What are the zeros of f ?

b) Evaluate $\lim_{x\to 3} f(x)$.

c) Determine all vertical and horizontal asymptotes of f.

d) Find all the nonremovable discontinuities of f.

---

2. Consider the function $g(x) = x^x$ with domain $(0, \infty)$.

a) Fill in the following table.

x	0.01	0.1	0.2	0.3	0.4	0.5	1
x^x							

b) What is $\lim_{x\to 1^-} g(x)$? What is $\lim_{x\to 0^+} g(x)$?

c) What do you think the smallest value of $g(x)$ is for values in the interval $(0,1)$? Justify your answer.

d) Find the average rate of change of $g(x)$ from $x = 0.1$ to $x = 0.4$.

---

3. Consider the function $F(x) = (a^{-1} - x^{-1})^{-1}$ where a is a positive real number.

a) What is the domain of F ? What are the zeros of F ?

b) Find all asymptotes of F and discuss any discontinuities of F.

c) Evaluate $\lim_{x\to 0} F(x)$, $\lim_{x\to\infty} F(x)$, and $\lim_{x\to a} F(x)$.

d) For what value of a will $F(6) = 12$?