**Inverse Functions**

**Inverse function:**
Given an equation in variables $x$ and $y$, the inverse can be found by interchanging $x$ and $y$.

**Given a function $f(x)$, the inverse is denoted $f^{-1}(x)$.**

Find the inverse of $f(x) = 2x + 4$.

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**Ways of thinking about inverse functions.**

The inverse function of $f(x)$ ‘undoes’ what $f(x)$ does:

\[
f(x) = 2x + 4.
\]

\[
f^{-1}(x) = \frac{1}{2}x - 2.
\]

Find the function values of $f(x)$ for the values of $x$ listed in the table.

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Find the function values of $f^{-1}(x)$ for the values of $x$ listed in the table.

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The graph of $f^{-1}(x)$ is a reflection of the graph of $f(x)$ across the line $y = x$.

In the graph above, graph the line $y = x$, $f(x)$ and $f^{-1}(x)$. 
Determining whether a function is one-to-one.

Recall: A graph is a function if it passes the vertical line test.

A function is invertible if it passes the horizontal line test.
(If a graph passes both the vertical and horizontal line tests, we say that it is one-to-one.)

Does the graph pass the horizontal line test?

Find the inverse of the function \( f(x) \) and the indicated value.

1. \( f(x) = 5x - 1 \) Find \( f^{-1}(9) \).

2. \( f(x) = \frac{3}{x + 8} \) Find \( f^{-1}(3) \).

3. \( f(x) = x^3 - 1 \) Find \( f^{-1}(7) \).

4. \( f(x) = \sqrt{x + 1} \) Find \( f^{-1}(3) \).
Find $f^{-1}(x)$ for the one-to-one function $f$ and graph $f$ and $f^{-1}$ on the same coordinate axes.

5. $f(x) = \sqrt{x - 3}$

Find $f^{-1}(x)$.

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Sketch:

6. $f(x) = \frac{1}{2x}$

Find $f^{-1}(x)$.

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Sketch:
7. \( f(x) = x^3 - 4 \)

Find \( f^{-1}(x) \).

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x & y \\
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\end{array}
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x & y \\
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\begin{array}{|c|c|}
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x & y \\
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\begin{array}{|c|c|}
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x & y \\
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\[
\begin{array}{|c|c|}
\hline
x & y \\
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\end{array}
\]

8. Verify that \( f(x) = \sqrt[3]{x-4} \) and \( f(x) = x^3 + 4 \) are inverses.

9. Verify that \( f(x) = \sqrt{x+2} \) and \( f(x) = x^2 - 2; x \geq 0 \) are inverses.
Inverse Relations
Find the inverse for each relation.

10. \{ (1, -3), (-2, 3), (5, 1), (6, 4) \}  
11. \{ (-5, 7), (-6, -8), (1, -2), (10, 3) \}

Finding Inverses
Find an equation for the inverse for each of the following relations.

12. \( y = 3x + 2 \)  
13. \( y = -5x - 7 \)  
14. \( y = 12x - 3 \)

15. \( y = -8x + 16 \)  
16. \( y = \frac{2}{3}x - 5 \)  
17. \( y = -\frac{3}{4}x + 5 \)

18. \( y = -\frac{5}{8}x + 10 \)  
19. \( y = \frac{1}{2}x + 8 \)  
20. \( y = x^2 + 5 \)

21. \( y = x^2 - 4 \)  
22. \( y = (x + 3)^2 \)  
23. \( y = (x - 6)^2 \)

24. \( y = \sqrt{x} - 7, \ y \geq -7 \)  
25. \( y = \sqrt{x + 5}, \ y \geq 0 \)  
26. \( y = \sqrt{x + 8}, \ y \geq 8 \)

Verifying Inverses
Verify that \( f \) and \( g \) are inverse functions.

27. \( f(x) = x + 6, \ g(x) = x - 6 \)  
28. \( f(x) = 5x + 2, \ g(x) = \frac{x - 2}{5} \)
Graph the inverse for each relation below (put your answer on the same graph).

29. \( f(x) = -3x - 9, \ g(x) = \frac{1}{3}x - 3 \)

30. \( f(x) = 2x - 7, \ g(x) = \frac{x + 7}{2} \)

31. \( f(x) = -4x + 8, \ g(x) = -\frac{1}{4}x + 2 \)

32. \( f(x) = \frac{1}{2}x - 7, \ g(x) = 2x + 14 \)

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**Graphing Inverses**

Graph the inverse for each relation below (put your answer on the same graph).

33. 

34. 

35. 

36.