Name: ______________________________________ Date: ______________________

Calculus 2: Summer Assignment

The Fundamental Theorem of Calculus

If a function f is continuous on the closed interval [a,b], and F is an antiderivative of f on the interval [a,b], then $\int_a^b f(x)dx = F(b) - F(a)$.

Examples:

1. $\int_{-1}^{2}\left(6x^2+5\right)dx =$

2. $\int_{\frac{5\pi}{6}}^{2\pi}\sin\theta d\theta =$

3. $\int_{1}^{e}\frac{2}{x}dx =$

4. $\int_{2}^{5}\frac{3x^3-2x}{x}dx =$

5. $\int_{\frac{\pi}{4}}^{\frac{\pi}{6}}4\csc^2 x dx =$

U-Substitution

Consider the example: $\int \frac{1}{2} e^{2x} dx$

1. Set one part of *f(x)* equal to *u*
2. Take the derivative of both sides of *u*.
3. Substitute *u* and *du* into the integral so the integral is only in terms of *u* and simplify the integrand.
4. Find the antiderivative
5. Substitute back to *x*.

Examples: (indefinite integrals)

1. $\int \frac{9r^2 dr}{\sqrt{1-r^3}}$

2. $\int \left(x^2+2x-3\right)^2 (x+1)\, dx$

3. $\int 4xe^{x^2} dx$

4. $\int 28(7x-2)^3\, dx$

5. $\int x\cos(2x^2)dx$

6. $\int (1-\cos\frac{t}{2})^2 \sin\frac{t}{2}\ dt$

Examples: (definite integrals)

1. $\int_{-\pi/4}^{0} \tan x \sec^2 x\ \ dx$

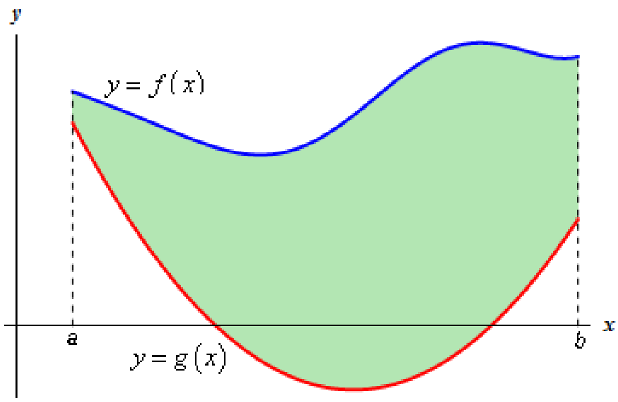
2. $\int_{-\pi}^{\pi} \frac{\cos x}{\sqrt{4+3\sin x}}\ dx$

3. $\int_0^1 \sqrt{t^5+2t}\ (5t^4+2)\ dt$

4. $\int_0^{\pi/6} \frac{\sin 2\theta}{\cos^3 2\theta}\ d\theta$

Area Between Two Curves

Case 1: $A = \int_a^b f(x) - g(x)dx$

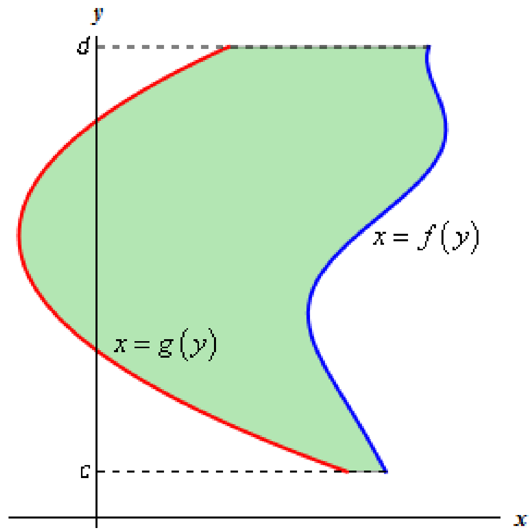


$$A = \int_a^b \begin{pmatrix} \text{upper} \\ \text{function} \end{pmatrix} - \begin{pmatrix} \text{lower} \\ \text{function} \end{pmatrix} dx, \qquad a \le x \le b$$

Example 1: Determine the area of the region enclosed by $y = x^2$ and $y = \sqrt{x}$

Example 2: Determine the area of the region bounded by $y = xe^{-x^2}$, $y = x + 1$, $x = 2$ and the *y*-axis.

Case 2: $A = \int_c^d f(y) - g(y) dy$



$$A = \int_c^d \begin{pmatrix} \text{right} \\ \text{function} \end{pmatrix} - \begin{pmatrix} \text{left} \\ \text{function} \end{pmatrix} dy, \qquad c \le y \le d$$

Example 3: Determine the area of the region enclosed by $x = \frac{1}{2}y^2 - 3$ and $y = x - 1$.

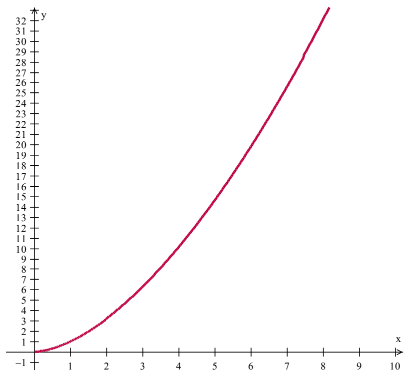
Example 4: Determine the area of the region bounded by $x = -y^2 + 10$ and $x = (y - 2)^2$.

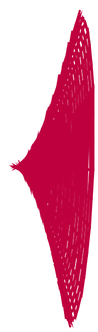
Curves that Intersect at More than two Points

Example 5: Find the area of the region between the graphs of $y = 3x^3 - x^2 - 10x$ and $y = -x^2 + 2x$.

Disk Method $V = \pi \int_a^b R^2 \, dx$ $V = \pi \int_c^d R^2 \, dy$

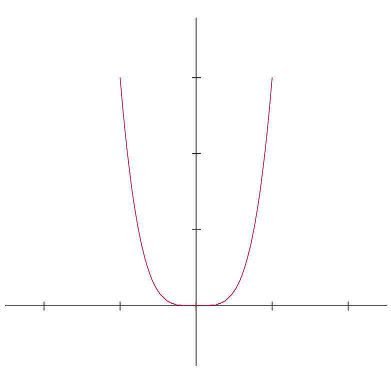
1. The region between the graph of $y = x^{5/3}$, $x = 1$ and $x = 8$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

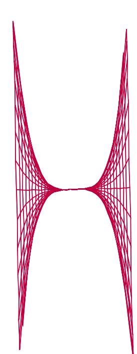




2. Find the volume of the region enclosed by the triangle with vertices (0,1), (0,0) and (1,0) if the region is revolved around the y-axis.

3. Find the volume of the solid generated by revolving the region bounded by the x-axis, the curve $y = 3x^4$, and the lines $x = -1$ and $x = 1$ about the x-axis.





Washer Method

$$A = \pi\left(\begin{pmatrix}\text{outer}\\ \text{radius}\end{pmatrix}^2 - \begin{pmatrix}\text{inner}\\ \text{radius}\end{pmatrix}^2\right)$$

1. Determine the volume of the solid obtained by rotating the portion of the region bounded by $y = \sqrt[3]{x}$ and $y = \frac{x}{4}$ that lies in the first quadrant about the y-axis.

2. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x} + 3$, $y = 1$ and $x = 4$ if the region is revolved about the x-axis.

3. Determine the volume of the solid obtained by rotating the region bounded by $y = 2\sqrt{x-1}$ and $y = x - 1$ about the line $x = -1$.