CONGRUENT AND SIMILAR TRIANGLES

CONGRUENT TRIANGLES
Triangles are congruent when they have exactly the same three sides and exactly the same three angles.

What is “Congruent”…?
It means that one shape can become another using rotations (turns), Flips (reflections) and/or Slides (translations).

Congruent Triangles
When two triangles are congruent they will have exactly the **same three sides** and exactly **the same three angles**. The equal sides and angles may not be in the same position (if there is a turn or a flip), but they are there.

Same Sides – When the sides are the same, the triangles are congruent.

For example:

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\[ \triangle ABC \cong \triangle DEF \]
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Notice all of the sides have the same measure, the only difference is the placement of the triangles.

The two triangles at right are not congruent because two sides do not have the exact same measure.

What if two triangles have the exact same angle measures, will they be congruent? Not always. In this case the angles are the same but one triangle is larger than the other. However, if two triangles are congruent, then their angles will all be the same size.

When two triangles are congruent, we mark corresponding sides and angles like this:

Looking at the triangles, we can tell which sides and angles are congruent by looking at the markings. For Example \( \overline{ED} \) has one mark on it and \( \overline{AB} \) has one mark on it. That means that \( \overline{ED} \cong \overline{AB} \). The symbol \( \cong \) means congruent.
Let’s see if the triangles are congruent. Since we can see that all of the sides and all of the angles are congruent the two triangles are congruent. We can state that by saying, $\triangle ABC \cong \triangle DCE$.

When stating the triangles are congruent, notice that we have matched up the congruent sides and angles. $\triangle ABE \cong \triangle DCE$

**YOU TRY:** For each pair of triangles below, state the parts that are congruent and then state a congruency statement for the two triangles.

There are several ways of proving triangles.

<table>
<thead>
<tr>
<th>SSS</th>
<th>SAS</th>
<th>ASA</th>
<th>AAS</th>
<th>HL</th>
</tr>
</thead>
<tbody>
<tr>
<td>![SSS diagram]</td>
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<td>![ASA diagram]</td>
<td>![AAS diagram]</td>
<td>![HL diagram]</td>
</tr>
</tbody>
</table>

- **SSS**
  - Note that the congruent angles are located between the two sides that are marked congruent. The sides make up the sides of the angle.

- **SAS**
  - Note that the congruent sides are located between the two angles that are marked congruent.

- **ASA**
  - Note that in this case the congruent side is not between the two angles that are marked congruent.

- **AAS**
  - In this case, you must have a right triangle.
YOUR TURN: For each set of triangles shown below, determine if the triangles are congruent. If the triangles are congruent, state how you know they are congruent.

<table>
<thead>
<tr>
<th>1.</th>
<th>2.</th>
<th>3.</th>
<th>4.</th>
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<tbody>
<tr>
<td><img src="image1.png" alt="Triangle 1" /></td>
<td><img src="image2.png" alt="Triangle 2" /></td>
<td><img src="image3.png" alt="Triangle 3" /></td>
<td><img src="image4.png" alt="Triangle 4" /></td>
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<table>
<thead>
<tr>
<th>5.</th>
<th>6.</th>
<th>7.</th>
<th>8.</th>
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<tbody>
<tr>
<td><img src="image5.png" alt="Triangle 5" /></td>
<td><img src="image6.png" alt="Triangle 6" /></td>
<td><img src="image7.png" alt="Triangle 7" /></td>
<td><img src="image8.png" alt="Triangle 8" /></td>
</tr>
</tbody>
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**SIMILAR TRIANGLES**
Two shapes are Similar when one can become the other after a dilation (resize), reflection (flip), translation (slide) or a rotation (turn).

In each case, the triangles are similar.

<table>
<thead>
<tr>
<th>Dilation</th>
<th>Rotation</th>
<th>Reflection</th>
<th>Translation</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image9.png" alt="Dilation" /></td>
<td><img src="image10.png" alt="Rotation" /></td>
<td><img src="image11.png" alt="Reflection" /></td>
<td><img src="image12.png" alt="Translation" /></td>
</tr>
</tbody>
</table>

When two shapes are similar, then:
- Corresponding angles are equal, and
- Corresponding sides are in proportion

**Are Congruent Shapes also Similar?**
Yes, because corresponding angles are congruent and the sides are in proportion. The sides are actually in a 1 to 1 ratio.

Two triangle are similar if the only difference is size (and possibly the need to rotate or reflect.)

**Corresponding Sides**
In similar triangles, corresponding sides are always in the same ratio. For example:
Triangles R and S are similar. The equal angles are marked with the same number of arcs.
What are the corresponding lengths?
- The lengths 7 and a are corresponding (they face the angle marked with one arc)
- The lengths 8 and 6.4 are corresponding (they face the angle marked with two arcs)
- The lengths 6 and b are corresponding (they face the angle marked with three arcs)

Calculating the Lengths of Corresponding Sides
We can sometimes calculate lengths we don’t know yet.
Step 1: Find the ratio of the corresponding sides
Step 2: Use that ratio to find the unknown lengths

Example: Let’s find the value of a and b, in triangle S.
Step 1: Find the ratio.
- We know in triangle S that the side between the angle marked with one arc and the angle marked by 3 arcs is 6.4. Look for that same side in triangle R. Its value is 8.
  
  Now set up the ratio. We can either use \( \frac{8}{6.4} \) or \( \frac{6.4}{8} \). Once you decide which ratio you want to use, it’s a good idea to set up the word ratio as well so you remember where to put the other numbers. We will use \( \frac{8}{6.4} \), which is the side of R side of S.

Step 2: Use the ratio.
- To find a: \( \frac{8}{6.4} = \frac{7}{a} \) (a is side between the angle marked with two arcs and 3 arcs, so in triangle R we had to find the value of the side between the angle marked with two arcs and 3 arcs, which has a value of 7. Now solve the proportion.
  
  \( 8a = 7 \times 6.4 \)
  
  \( 8a = 44.8 \)

  \( a = 5.6 \)

  Now we’ll do the same thing to find b. \( \frac{8}{6.4} = \frac{6}{b} \Rightarrow 8b = 38.4 \Rightarrow b = 4.8 \)

YOUR TURN: Each pair of triangles are similar. Find the missing side.

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Just as you can prove Triangles congruent, you can prove triangles similar.
FINDING SIMILAR TRIANGLES

Remember: Triangles are similar if they have:
- All angles equal
- Corresponding sides are in the same ratio

Just like we didn’t need to know that all the parts of congruent triangles were congruent to know that they were congruent, we do not need to all three sides and all three angles to show that the triangles are similar. Usually if we know two or three of the six pieces, we have enough information to determine if the triangles are similar.

There are three way to show that two triangles are similar:

<table>
<thead>
<tr>
<th>AA</th>
<th>If two triangles have two of their angles equal, then the triangles will be similar. If two of their angles are equal, then the third angle must be equal because angles of a triangle always to make 180°.</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAS</td>
<td>If two triangles have two pairs of sides in the same ration and included angles are equal, the triangles are similar. In the triangles at right, we can see • One pair of sides is in the ratio of 21: 14 = 3: 2 • Another pair of sides is in the ratio of 15: 10 = 3: 3 • There is a matching angle of 75° in between them.</td>
</tr>
<tr>
<td>SSS</td>
<td>If two triangle have three pairs of sides in the same ratio, then the triangles are similar. In the triangles at right, the ratio of the sides are: ( \frac{8}{10} = \frac{4}{5} = \frac{6}{7.5} ), simplifies to ( \frac{4}{5} ) and ( \frac{6}{7} ) which simplifies to ( \frac{4}{5} ). All of the sides are in the same ratio, so the triangles are similar.</td>
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When we name similar triangles, we use the \( \sim \) to indicate that the triangle are similar. Just as with naming congruent triangles, the letters are important. (If you remember that the largest side is opposite the largest angle that will help you with similar triangles.) We know the triangles at right are similar. To write the similarity statement begin by naming one of the triangles by their letters and the character for similar. \( \Delta ABC \sim \), now look at angle A go across from the angle to the other side, notice is 6 which is smaller than 8 but larger than 4. So in the other triangle find the side which would be the middle side in terms of the lengths of the side, 7.5. Go across from that side, 7.5, to the angle which would be X. Now look at angle B, it’s across from the largest side. Go to the other triangle and find the largest side, 10, go across from it to get the letter of the angle, Y. So now we can write the similarity statement: \( \Delta ABC \sim \Delta XYZ \).
YOUR TURN: State if each triangle pair is similar. If so, name the similar triangles, and how they are similar.

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