Day 1 Linear Systems: Solve with Graphing

<table>
<thead>
<tr>
<th>System of Linear Equations</th>
<th>Solution of a System of Linear Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two or more linear equations</td>
<td>An ordered pair that makes all of the equations in a system true; the point of intersection</td>
</tr>
</tbody>
</table>

**One solution:** A system of linear equations has one solution when the graphs intersect at a point.

**No solution:** A system of linear equations has no solution when the graphs are parallel.

**Infinite solutions:** A system of linear equations has infinite solutions when the graphs are the exact same lines.

**Example:** Graph to find solution:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>a)</td>
<td>$y = 3x + 1$</td>
</tr>
<tr>
<td>b)</td>
<td>$2y = -4x - 8$</td>
</tr>
</tbody>
</table>

**Step 1:** Graph the lines.

**Methods:**
1. In **slopes intercept form**; graph using the y-intercept ($b$) and slope ($m$).
2. Put equations into slope intercept form:
   * Add or subtract the $x$-term.
   * Divide all terms by # in front of $y$.
3. Graph using y-intercept ($b$) and slope ($m$).
4. Make a table and find points to plot.
5. Find the $x$- and $y$-intercepts.

**Step 2:** Identify the solutions.
(Ordered pair where the lines intersect)

**Step 1:**

**Equation a:** $y = 3x + 1$

*is in Slope-Intercept form.*

Use method 1

**Equation b:** $2y = -4x - 8$

*is not in Slope-Intercept form.*

Use method 2, 3, or 4.
Select the correct system of equations for each graph:

1. 
   A) \( y = x + 2 \)
   B) \( y = 3x - 3 \)
   C) \( y = \frac{1}{3}x - 2 \)

2. 
   A) \( y = -1 \)
   B) \( y = \frac{1}{2}x - 4 \)
   C) \( y = \frac{1}{3}x - 4 \)

3. 
   A) \( y + x = 3 \)
   B) \( y = 2 \)
   C) \( y + x = -3 \)

4. 
   A) \( y = x + \frac{1}{2} \)
   B) \( y = 3x + \frac{1}{2} \)
   C) \( y = \frac{1}{2}x + 1 \)

5. 
   A) \( 2x + 3y = 15 \)
   B) \( y = \frac{2}{3}x + 5 \)
   C) \( y = x - 2 \)

6. 
   A) \( y = x + 4 \)
   B) \( y = -x - 4 \)
   C) \( y = -x + 4 \)
**DAY 2 Linear Systems: Substitution Method**

**Example 1:**

A) \( y = 2x - 1 \)

B) \( 3x + 2y = 26 \)

<table>
<thead>
<tr>
<th>Step 1: Isolate one of the variables</th>
<th>Step 1: Equation (A) already has ( y ) isolated.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 2: Substitute the expression from Step 1 into the OTHER equation.</td>
<td>Step 2: ( 3x + 2y = 26 ) [ 3x + 2(2x - 1) = 26 ]</td>
</tr>
<tr>
<td>- The resulting equation should have only one variable, not both ( x ) and ( y ).</td>
<td>Step 3: ( 3x + 4x - 2 = 26 ) [ 7x - 2 = 26 ] [ +2 + 2 ] [ 7x = 28 ] [ 7 ] [ x = 4 ]</td>
</tr>
<tr>
<td>Step 3: Solve the new equation.</td>
<td>Step 4: ( y = 2 \times 4 - 1 )</td>
</tr>
<tr>
<td>- This will give you one of the coordinates.</td>
<td>( y = 7 )</td>
</tr>
<tr>
<td>Step 4: Substitute the result from Step 3 into either of the original equations.</td>
<td>Step 5: ( y = 7 )</td>
</tr>
<tr>
<td>Step 5: Solve for the other coordinate.</td>
<td>Step 6: ( (4, 7) )</td>
</tr>
<tr>
<td>Step 6: Write the solution as an ordered pair. ((x, y))</td>
<td></td>
</tr>
</tbody>
</table>

**Example 2:**

A) \(-4x + y = 6\)

B) \(-5x - y = 21\)

<table>
<thead>
<tr>
<th>Step 1: Isolate one of the variables</th>
<th>Step 1: Isolate Equation (A) because ( y ) has a coefficient of positive 1.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 2: Substitute the expression from Step 1 into the OTHER equation.</td>
<td>(-4x + y = 6) [ +4x ] [ +4x ] [ y = 6 + 4x ]</td>
</tr>
<tr>
<td>- The resulting equation should have only one variable, not both ( x ) and ( y ).</td>
<td>Step 2: (-5x - y = 21) [ -5x - (6 + 4x) = 21 ]</td>
</tr>
<tr>
<td>Step 3: Solve the new equation.</td>
<td>Step 3: (-5x - 6 - 4x = 21) [ -9x = 27 ] [ -9 = -9 ] [ x = -3 ]</td>
</tr>
<tr>
<td>- This will give you one of the coordinates.</td>
<td></td>
</tr>
<tr>
<td>Step 4: Substitute the result from Step 3 into either of the original equations.</td>
<td>Step 4: ( y = 6 + 4(-3) )</td>
</tr>
<tr>
<td>Step 5: Solve for the other coordinate.</td>
<td>( y = -6 )</td>
</tr>
<tr>
<td>Step 6: Write the solution as an ordered pair. ((x, y))</td>
<td>Step 6: ((-3, -6))</td>
</tr>
</tbody>
</table>

**Try It Out:** (Solutions are after the Practice Problems)

A) \( \begin{cases} x = -2 \\ x + 3y = 4 \end{cases} \)

B) \( \begin{cases} x = 3y \\ x - 3y = 0 \end{cases} \)

C) \( \begin{cases} y = -3x + 4 \\ y = 4x - 10 \end{cases} \)

D) \( \begin{cases} -5x + y = -2 \\ -3x + 6y = -12 \end{cases} \)
Practice Problems:

1. \[
\begin{cases}
    x = 5 \\
    x + y = 12
\end{cases}
\]

2. \[
\begin{cases}
    y = 5 \\
    -3x + 4y = 8
\end{cases}
\]

3. \[
\begin{cases}
    y = 2x \\
    x + y = 9
\end{cases}
\]

4. \[
\begin{cases}
    y = -3x \\
    x + y = 4
\end{cases}
\]

5. \[
\begin{cases}
    y = 3x - 4 \\
    4x + 3y = 1
\end{cases}
\]

6. \[
\begin{cases}
    x = 3y + 1 \\
    2x + 4y = 12
\end{cases}
\]

7. \[
\begin{cases}
    -5x + y = -3 \\
    3x - 8y = 24
\end{cases}
\]

8. \[
\begin{cases}
    x + 3y = 1 \\
    -3x - 3y = -15
\end{cases}
\]

Try It Out Answers:

<table>
<thead>
<tr>
<th>A)</th>
<th>B)</th>
<th>C)</th>
<th>D)</th>
</tr>
</thead>
</table>
| \[
\begin{align*}
    x + 3y &= 4 \\
    -2 + 3y &= 4 \\
    +2 &+ 2
\end{align*}
\]
| \[
\begin{align*}
    x - 3y &= 0 \\
    3y - 3y &= 0 \\
    0 &= 0
\end{align*}
\]
| Since both equations tell us what y equals we will set them equal to each other and solve for x
| \[
\begin{align*}
    -3x + 4 &= 4x - 10 \\
    +3x &+3x
\end{align*}
\]
| \[
\begin{align*}
    y &= -2 + 5x \\
    y &= -2 + 5x
\end{align*}
\]
| \[
\begin{align*}
    -3x + 6(-2 + 5x) &= -12 \\
    -3x - 12 + 30x &= -12 \\
    +12 &+12
\end{align*}
\]
| \[
\begin{align*}
    -3x + 30x &= 0 \\
    27x &= 0 \\
    x &= 0
\end{align*}
\]
| \[
\begin{align*}
    -5(0) + y &= -2 \\
    Y &= -2 \\
    (0,-2)
\end{align*}
\]

(-2, 2)

Since the left side and right side are equal, the solution would be INFINITELY MANY SOLUTIONS

\[
\begin{align*}
    3y &= 6 \\
    \frac{3y}{3} &= \frac{6}{3} \\
    Y &= 2
\end{align*}
\]
DAY 3 Linear Systems: Elimination Method

Example: \[
\begin{align*}
2x + 5y &= 17 \\
6x - 5y &= -9
\end{align*}
\]

Step 1: Line up the x’s and y’s

Step 2: Look to see if one variable has opposite coefficients
  • Yes, move to step 3
  • No, multiply one or both equations by a (LCM) in order to make the coefficients of the x or y terms opposites

Step 3: Add the equations together to eliminate one of the variables

Step 4: Solve for the remaining variable

Step 5: Substitute the value you found into one of the original equations to solve for the other variable

Step 6: Write your answer as an ordered pair.

Example:

Step 1: \[
\begin{align*}
2x + 5y &= 17 \\
6x - 5y &= -9
\end{align*}
\]

Step 2: Yes

Step 3: \[
\begin{align*}
8x &= 8 \\
8 &= 8
\end{align*}
\]

Step 4: \[x = 1\]

Step 5: \[
\begin{align*}
2(1) + 5y &= 17 \\
2 + 5y &= 17 \\
-2 &= -2
\end{align*}
\]

\[5y = 15\]

\[y = 3\]

Step 6: (1,3)

Example:

Step 1: Line up the x’s and y’s

Step 2: Look to see if one variable has opposite coefficients
  • Yes, move to step 3
  • No, multiply one or both equations by a (LCM) in order to make the coefficients of the x or y terms opposites

Step 3: Add the equations together to eliminate one of the variables

Step 4: Solve for the remaining variable

Step 5: Substitute the value you found into one of the original equations to solve for the other variable

Step 6: Write your answer as an ordered pair.

Example:

Step 1: \[
\begin{align*}
3x + y &= 9 \\
5x + 4y &= 22
\end{align*}
\]

Step 2: No, multiply one or both equations by a constant (LCM) in order to make the coefficients of the x or y terms opposites.

Step 3: \[
\begin{align*}
-12x - 4y &= -36 \\
5x + 4y &= 22
\end{align*}
\]

\[-7x = -14\]

\[-7 = -7\]

Step 4: \[x = 2\]

Step 5: \[
\begin{align*}
5(2) + 4y &= 22 \\
10 + 4y &= 22 \\
-10 &= -10
\end{align*}
\]

\[4y = 12\]

\[y = 3\]

Step 6: (2,3)

Try It Out: (Solutions are after the Practice Problems)

A) \[
\begin{align*}
3y + 2x &= 6 \\
5y - 2x &= 10
\end{align*}
\]

B) \[
\begin{align*}
x + 3y &= 18 \\
-x - 4y &= -25
\end{align*}
\]

C) \[
\begin{align*}
4x + 2y &= 8 \\
16x - y &= 14
\end{align*}
\]
Practice Problems

1. \( \begin{align*}
2x + 2y &= -2 \\
3x - 2y &= 12
\end{align*} \)

2. \( \begin{align*}
4x - 2y &= -1 \\
-4x + 4y &= -2
\end{align*} \)

3. \( \begin{align*}
6x + 5y &= 4 \\
6x - 7y &= -20
\end{align*} \)

4. \( \begin{align*}
3x + y &= -21 \\
-x - y &= 5
\end{align*} \)

5. \( \begin{align*}
-x + 9y &= -5 \\
x - 5y &= 1
\end{align*} \)

6. \( \begin{align*}
-2x + y &= 10 \\
4x - y &= -14
\end{align*} \)

7. \( \begin{align*}
3x + 2y &= 0 \\
x - 5y &= 17
\end{align*} \)

8. \( \begin{align*}
-4x - 15y &= -17 \\
x + 5y &= -13
\end{align*} \)

Try It Out Answers:

A) \( \begin{align*}
3y + 2x &= 6 \\
5y - 2x &= 10
\end{align*} \)

\[ \begin{align*}
8y &= 16 \\
y &= 2
\end{align*} \]

\( \begin{align*}
3(2) + 2x &= 6 \\
6 + 2x &= 6 \\
-6 &= -6 \\
2x &= 0 \\
x &= 0
\end{align*} \)

\((0, 2)\)

B) \( \begin{align*}
x + 3y &= 18 \\
-x - 4y &= -25
\end{align*} \)

\[ \begin{align*}
y &= 7 \\
x &= -7
\end{align*} \]

\( \begin{align*}
x + 3(7) &= 18 \\
x + 21 &= 18 \\
-21 &= -21 \\
x &= -3
\end{align*} \)

\((-3, 7)\)

C) Since neither variable has opposite coefficients, we must multiply the top equation by -4:

\( \begin{align*}
-4(4x + 2y &= 8) \\
16x - y &= 14
\end{align*} \)

\[ \begin{align*}
-16x - 8y &= -32 \\
16x - y &= 14
\end{align*} \]

\[ \begin{align*}
-y &= -18 \\
-9 &= -9 \\
y &= 2
\end{align*} \]

\( \begin{align*}
4x + 2(2) &= 8 \\
4x + 4 &= 8 \\
-4 &= -4 \\
4x &= 4 \\
x &= 1
\end{align*} \)

\((1, 2)\)