AP Calculus Readiness Test

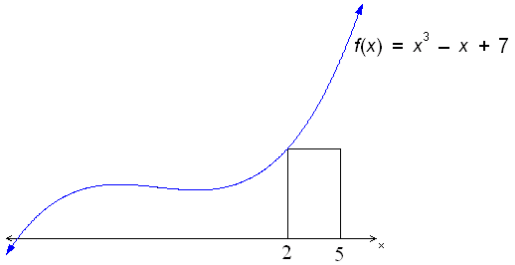
The following test consists of topics from Algebra through Pre-Calculus, as well as some problems that will test your general problem solving ability. Complete all problems **without** a calculator. This packet will not be collected or graded, but you are expected to understand all of this material prior to the start of the school year.

1. Find all solutions to the equation $3x^2 = 4x + 1$.

2. If 2^{13} is approximately equal to 8000, approximate 2^{26}.

3. Solve $x + \sqrt{x} - 6 = 0$.

4. What is the area of the rectangle shown in the figure below? (Note: The figure is not drawn to scale.)



5. If $f(x) = 3x + 3$ and $g(y) = 2y + 5$, what is $g(f(2))$?

6. Solve the inequality $|x - 4| < 8$.

7. Money in a bank triples every 8 years. If \$100 is deposited today, what will its value be after 32 years?

8. If $f(x) = 2^{-x}$, find $f(4)$.

9. Solve $\log_2(x - 6) = 6$ for x.

10. Simplify $2^{-5} \cdot 64^{\frac{2}{3}}$

11. What is the y-coordinate of the point of intersection of the graph of $-x + 4y = -50$ and $x + y = 20$?

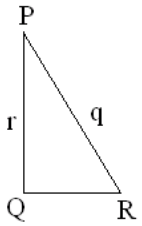
12. If f(x) = $\frac{5x+3}{2x+3}$, find f(n + 1).

13. The length of a certain rectangle is 6 meters more than twice its width. What is the perimeter of the rectangle if the area of the rectangle is 260 square meters?

14. What is the radian measure of an angle whose degree measure is 240?

15. Find the point of intersection in the first quadrant of the line $y = 4x + 2$ and the curve $y = x^2$.

16. In the figure below, if $\sin R = 5/8$ and r = 2, then what is q?



17. Simplify.

$$\frac{1}{1+\frac{1}{x+1}}$$

18. Let f(x) = $\frac{6x+3}{3x+k}$. If f(–2) = 5, find k.

19. A rectangular box has a square base and a closed top. The height is twice the length of one side of the base. If a side of the base is x, find the surface area of the rectangular box in terms of x.

20. Solve $\log_2 t + \log_2(t + 1) = 1$ for t.

21. The line $y = 3x + 12$ intersects the ellipse $3x^2 + y^2 = 36$ at only one point. Find the y- coordinate of that point.

22. Simplify.

$$\left(\frac{8x^{12}y^{-3}}{y^6z^3}\right)^{-\frac{4}{3}}$$

23. $\csc(30°)$ =

24. If $f(x) = 3x + 3$ what is $f(f(2))$?

25. Simplify completely.

$$\frac{x}{\sqrt{x+1}-1}$$

26. Solve $8^{2-x} = 4^{3x}$.

27. If the length x of a rectangle is doubled and the width y is tripled, find the amount the area of the rectangle is increased by in terms of x and y.

28. If $\frac{(2x-3)(x+5)}{x-7} = 0$ then x =

a) 5, 7, or $-\frac{3}{2}$ b) 5 or $\frac{3}{2}$ c) –5, 7, or $\frac{3}{2}$ d) –5 or $\frac{3}{2}$

29. The quantity $a - b$ is a factor of how many of the following?

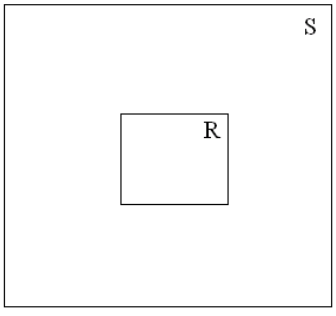
$a^2 - b^2$ $a^2 + b^2$ $a^3 - b^3$ $a^3 + b^3$

a) one only b) two only c) three only d) four

30. For which values of x in the interval $0 \le x \le 2\pi$ does $(\sin x - 1)(\sin x - 5) = 0$?

a) $\frac{\pi}{2}$ only b) 1 and 5 c) π d) 0 and 2π

31. A rectangle R has width x and length y. A rectangle S is formed from R by multiplying each of the sides of the rectangle R by 4 as shown in the figure below. In terms of x and y, what is the area of the portion of S lying outside R? (Note: The figure is not drawn to scale.)



a) 16xy b) 15xy c) 4xy d) x^4y^4

32. If $\ln(x + 1) = a + \ln 3$, x =

a) $e^{3a} - 1$ b) $3a - 1$ c) $3e^a + 1$ d) $3e^a - 1$ e) $a + 2$

33. The remainder of dividing the polynomial $x^3 - x^2 + x - 1$ by $x - 1$ is
a) 2 b) 0 c) 3 d) x e) 1

34. Simplify $[a^4 - 4a^2 + 4]^{\frac{1}{2}}$
a) $a^2 + 2$ b) $|a^2 + 2|$ c) $a^2 + 2a + 2$ d) $|a^2 - 2|$ e) $a^2 - 2$

35. The expression $x^{-3} - y^{-3}$ is equivalent to
a) $(x - y)^{-3}$ b) $\frac{x^3y^3}{x^3 - y^3}$ c) $\frac{1}{(x-y)^3}$ d) $\frac{y^3 - x^3}{x^3y^3}$ e) None of these

36. If $\ln\sqrt{x+2} = 1$, then x =
a) $e^2 - 2$ b) e^2 c) –1 d) $e^2 + 2$ e) 0

37. Peter Alvaro made x dollars profit on each of the (x + 10) bicycles he sold. His total profit was 144 dollars. How many bicycles did he sell?
a) 19 b) 18 c) 20 d) 15 e) 21

38. Let $f(x) = 6x^2 - 3x + 3$. Let h be a number not equal to zero. Compute and simplify the quotient $\frac{f(x+h) - f(x)}{h}$ to obtain an expression of the form mx + nh + p, where m, n, and p do not depend on x or h. Find p.
a) – 4 b) –5 c) 0 d) –3 e) –6

39. The set of solution(s) of $\cos^2\theta - 8\cos\theta + 7 = 0$ that is(are) in the interval $(-\pi,\pi)$ is which of following?
a) $\left\{\frac{\pi}{2}\right\}$ b) $\left\{-\frac{\pi}{3}\right\}$ c) $\left\{-\frac{\pi}{3},\frac{\pi}{6}\right\}$ d) $\left\{\frac{\pi}{6},-\frac{\pi}{4}\right\}$ e) $\{0\}$

40. In right triangle ABC, where C is the right angle, AB = 5x and BC = 3x. If the perimeter of triangle ABC is 84, then x =
a) $\frac{21}{4}$ b) $\frac{28}{3}$ c) 4 d) 7 e) 12

41. The expression $\dfrac{\frac{1}{b^2}+a^2}{\frac{1}{a^2}+b^2}$ simplifies to:

a) $\dfrac{1+a^4}{1+b^4}$ b) $\dfrac{b^2+a^2}{a^2b^2}$ c) $\dfrac{b^2}{a^2}$ d) $\dfrac{a^2}{b^2}$ e) None of these

42. The function f(x) = cot$x$ is not defined for:

a) x = 0 b) x = $\dfrac{\pi}{4}$ c) x = $\dfrac{\pi}{2}$ d) x = $\dfrac{\pi}{3}$ e) None of these

43. If f(x) = ax + b and f(2) = f(4), then a =

a) $-\dfrac{b}{2}$ b) $-\dfrac{b}{4}$ c) 0 d) 2 e) $\dfrac{1}{2}$

44. In right triangle ABC, where C is the right angle, side BC = 2 and the radian measure of angle A is $\dfrac{\pi}{6}$, what is the length of AC?

a) 1 b) $2\sqrt{2}$ c) 4 d) $2\sqrt{3}$ e) $4\sqrt{3}$

45. If $x + a = \dfrac{b}{3}x$ and $b \neq 3$, then x =

a) $\dfrac{3a}{b}$ b) $\dfrac{a}{b-3}$ c) $\dfrac{3a}{b-3}$ d) $\dfrac{a}{b+3}$ e) $\dfrac{3a}{b+3}$

AP Calculus - Readiness Test

(1) $3x^2-4x-1=0$

$x=\frac{4\pm\sqrt{16+12}}{6}$

$x=\frac{4\pm\sqrt{28}}{6}$

$x=\frac{4\pm2\sqrt{7}}{6}$

$x=\frac{2\pm\sqrt{7}}{3}$

(2) $2^{13}\approx8000$

$(2^{13})^2=2^{26}\approx64{,}000{,}000$

(3) $(\sqrt{x}+3)(\sqrt{x}-2)=0$

$\sqrt{x}+3=0$ | $\sqrt{x}-2=0$

$\sqrt{x}=-3$ (no real answer) | $\sqrt{x}=2$

$x=4$

(4) $b=5-2=3$

$h=f(2)=f(5)=13$

$A=3(13)=39$

(5) $g[f(2)]$

$g(9)$

23

(6) $|x-4|<8$

$x-4<8$ and $x-4>-8$

$x<12$ and $x>-4$

(7) $100(3)^4$

$100(81)$

$\$8100$

(8) $f(4)$

$\frac{1}{2^4}$

$\frac{1}{16}$

(9) $\log_2(x-6)=6$

$2^6=x-6$

$64=x-6$

$x=70$

(10) $\frac{1}{2^5}\cdot4^2$

$\frac{16}{32}$

$\frac{1}{2}$

(11) $-x+4y=-50$

$x+y=20$

$5y=-30$

$y=-6$

(12) $f(n+1)$

$\frac{5(n+1)+3}{2(n+1)+3}$

$\frac{5n+5+3}{2n+2+3}$

$\frac{5n+8}{2n+5}$

(13) $L=2W+6$

$LW=260\rightarrow W=\frac{260}{L}$

$L=2\left(\frac{260}{L}\right)+6$

$L=\frac{520}{L}+6$

$L^2=520+6L$

$L^2-6L-520=0$

$(L+20)(L-26)=0$

$L=-20$ $L=26$

$W=\frac{260}{26}=10$

$P=2(10)+2(26)=72$ in

(14) $240\left(\frac{\pi}{180}\right)$

$\frac{4\pi}{3}$

(15) $x^2=4x+2$

$x^2-4x-2=0$

$x=\frac{4\pm\sqrt{16+8}}{2}$

$x=\frac{4\pm\sqrt{24}}{2}$

$x=\frac{4\pm2\sqrt{6}}{2}$

$x=2\pm\sqrt{6}$

first quadrant, so $x=2+\sqrt{6}$

$y=4(2+\sqrt{6})+2$

$y=8+4\sqrt{6}+2$

$y=10+4\sqrt{6}$

$(2+\sqrt{6},10+4\sqrt{6})$

(16) $\frac{5}{8}=\frac{\text{opp.}}{\text{hyp.}}=\frac{r}{q}$

$\frac{5}{8}=\frac{2}{q}$

$5q=16$

$q=\frac{16}{5}$

(17) $\dfrac{1}{1+\frac{1}{x+1}}\Big/\dfrac{1}{\frac{x+1+1}{x+1}}\Big/\dfrac{1}{\frac{x+2}{x+1}}$

$1\div\frac{x+2}{x+1}$

$1\left(\frac{x+1}{x+2}\right)$

$\frac{x+1}{x+2}$

(18) $f(-2)=5=\frac{-12+3}{-6+k}$

$5=\frac{-9}{-6+k}$

$-30+5k=-9$

$5k=21$

$k=\frac{21}{5}$

(19)

$SA=2(x^2)+4(2x^2)$

$=2x^2+8x^2$

$=10x^2$

(20) $\log_3t(t+1)=1$

$\log_2(t^2+t)=1$

$2^1=t^2+t$

$t^2+t-2=0$

$(t+2)(t-1)=0$

$t=-2$ $t=1$

(21) $3x^2+(3x+12)^2=36$

$3x^2+9x^2+72x+144=36$

$12x^2+72x+108=0$

$12(x^2+6x+9)=0$

$12(x+3)^2=0$

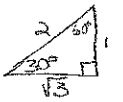
$x=-3\rightarrow y=3(-3)+12=3$

(22) $\left(\dfrac{y^6z^3}{8x^{12}y^{-3}}\right)^{\frac{4}{3}}\Big/\left(\dfrac{y^9z^3}{8x^{12}}\right)^{\frac{4}{3}}$

$\frac{y^{12}z^4}{16x^{16}}$

(23) $\csc(30^\circ)$

$\frac{1}{\sin30^\circ}=\frac{1}{\frac{1}{2}}=2$



(24) $f[f(2)]$

$f(9)$

30

25 $\frac{x}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}$

$\frac{x(\sqrt{x+1}+1)}{x+1-1}$

$\frac{x(\sqrt{x+1}+1)}{x}$

$\sqrt{x+1}+1$

26 $8^{2\cdot x} = 4^{3x}$

$(2^3)^{2-x} = (2^2)^{3x}$

$2^{6-3x} = 2^{6x}$

$6-3x = 6x$

$6 = 9x$

$x = \frac{2}{3}$

27 $L_{new} = 2x$

$W_{new} = 3y$

$A_{new} = 6xy$

$A_{original} = xy$

increased by

$6xy - xy$

$5xy$

28 $2x-3=0$

$x=\frac{3}{2}$

$x+5=0$

$x=-5$

d

29 b

a^2-b^2

$(a+b)(a-b)$

a^2+b^2

prime

a^3-b^3

$(a-b)(a^2+ab+b^2)$

a^3+b^3

$(a+b)(a^2-ab+b^2)$

30

$\sin x - 1 = 0$	$\sin x - 5 = 0$
$\sin x = 1$	$\sin x = 5$
$x = \frac{\pi}{2}$	$\sin x$ will never $= 5$

a

31 $A_R = xy$

$A_s = 4x(4y) = 16xy$

$A_{s-R} = 15xy$

32 $\ln(x+1) = a + \ln 3$

$x+1 = e^{a+\ln 3}$

$x+1 = e^a \cdot e^{\ln 3}$

$x+1 = 3e^a$

$x = 3e^a - 1$

d

33

	1	-1	1	-1
1		1	0	1
	1	0	1	0

b

34 $[(a^2-2)^2]^{\frac{1}{2}}$

$\sqrt{(a^2-2)^2}$

$|a^2-2|$

d

35 $\frac{1}{x^3} - \frac{1}{y^3}$

$\frac{y^3-x^3}{x^3y^3}$

d

36 $\ln\sqrt{x+2} = 1$

$e^1 = \sqrt{x+2}$

$e^2 = x+2$

$x = e^2 - 2$

a

37 $x(x+10) = 144$

$x^2+10x-144=0$

$(x+18)(x-8)=0$

$x=-18$ $x=8$

$8+10=18$ bikes

b

38 $\frac{6(x+h)^2-3(x+h)+2-6x^2+3x-2}{h}$

$\frac{6x^2+12xh+6h^2-3x-3h-6x^2+3x}{h}$

$\frac{h(12x+6h-3)}{h}$

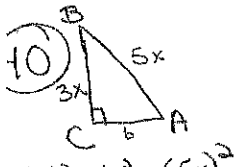
d

39 $(\cos\theta-1)(\cos\theta-1) =$

$\cos\theta = 1$	$\cos\theta = 1$
$\cos\theta$ will never $= 1$	$\theta = 0$

e

10



$(3x)^2 + b^2 = (5x)^2$

$9x^2 + b^2 = 25x^2$

$b^2 = 16x^2$

$b = 4x$ or special right $\triangle$ 3,4,5

$3x+4x+5x=84$

$12x=84$

$x=7$ d

41 $\frac{1+a^2b^2}{\frac{b^2}{1+a^2b^2}}$

$\frac{1+a^2b^2}{b^2} \cdot \frac{a^2}{1+a^2b^2}$

$\frac{a^2}{b^2}$

d

42 $f(x) = \cot x$

$f(x) = \frac{\cos x}{\sin x}$

$\sin x \neq 0$

a

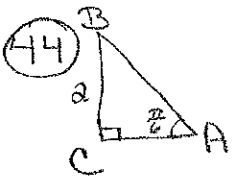
43 $2a+b = 4a+b$

$0 = 2a$

$a = 0$

c

44



$\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ angles

$1, \sqrt{3}, 2$ ratio of sides

$\frac{1}{\sqrt{3}} = \frac{2}{x}$

$x = 2\sqrt{3}$ d

45 $x + a = \frac{b}{3}x$

$a = \frac{b}{3}x - x$

$a = x\left(\frac{b}{3} - 1\right)$

$x = \frac{a}{\frac{b}{3}-1}$

$x = \frac{a}{\frac{b-3}{3}}$

$x = a \cdot \frac{3}{b-3}$

$x = \frac{3a}{b-3}$

c