Skills Practice

Three-Dimensional Figures

Draw the back view and corner view of a figure given each orthogonal drawing.

1. 
   
<table>
<thead>
<tr>
<th>top view</th>
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2. 
   
<table>
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</table>

Identify each solid. Name the bases, faces, edges, and vertices.

3. 
   
   [Diagram of a cube]

4. 
   
   [Diagram of a square pyramid]

5. 
   
   [Diagram of a cone]
12-1 Practice

Three-Dimensional Figures

Draw the back view and corner view of a figure given each orthogonal drawing.

1.  

2.  

Identify each solid. Name the bases, faces, edges, and vertices.

3.  

4.  

5. **MINERALS** Pyrite, also known as fool’s gold, can form crystals that are perfect cubes. Suppose a gemologist wants to cut a cube of pyrite to get a square and a rectangular face. What cuts should be made to get each of the shapes? Illustrate your answers.
Models for Three-Dimensional Figures One way to relate a three-dimensional figure and a two-dimensional drawing is to use isometric dot paper. Another way is to make a flat pattern, called a net, for the surfaces of a solid.

**Example 1** Use isometric dot paper to sketch a triangular prism with 3-4-5 right triangles as bases and with a height of 3 units.

**Step 1** Draw $\overline{AB}$ at 3 units and draw $\overline{AC}$ at 4 units.

**Step 2** Draw $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$, each at 3 units.

**Step 3** Draw $\overline{BC}$ and $\triangle DEF$.

**Example 2** Match the net at the right with one of the solids below.

![Net](image)

The six squares of the net can be folded into a cube. The net represents solid c.

**Exercises**

Sketch each solid using isometric dot paper.

1. cube with edge 4
2. rectangular prism 1 unit high, 5 units long, and 4 units wide

Draw a net for each solid.

3.
4.

5.
6.
**Surface Area** The surface area of a solid is the sum of the areas of the faces of the solid. Nets are useful in visualizing each face and calculating the area of the faces.

**Example**

Find the surface area of the triangular prism.

First draw a net using rectangular dot paper. Using the Pythagorean Theorem, the hypotenuse of the right triangle is $\sqrt{3^2 + 4^2} = 5$.

Surface area $= A + B + C + D + E$

$= \frac{1}{2}(4 \cdot 3) + 4 \cdot 4 + 4 \cdot 3 + 4 \cdot 5 + \frac{1}{2}(4 \cdot 3)$

$= 60$ square units

**Exercises**

Find the surface area of each solid. Round to the nearest tenth if necessary.

1. [Diagram of a prism with dimensions 5 in., 12 in., and 8 in.]

2. [Diagram of a rectangular prism with dimensions 10 cm, 24 cm, and 5 cm.]

3. [Diagram of a cube with side length 5 m.]

4. [Diagram of a triangular prism with dimensions 6, 9, and 12.]

5. [Diagram of a pyramid with dimensions 10, 8, 8, and 10.]

6. [Diagram of a solid with dimensions 18, 15, 20, 40, and 25.]
12-2 Skills Practice

Nets and Surface Area

Sketch each solid using isometric dot paper.

1. cube 2 units on each edge
2. rectangular prism 2 units high, 5 units long, and 2 units wide

For each solid, draw a net and find the surface area.

3.

4.
**Practice**

**Nets and Surface Area**

Sketch each solid using isometric dot paper.

1. rectangular prism 3 units high, 3 units long, and 2 units wide
2. triangular prism 3 units high, whose bases are right triangles with legs 2 units and 4 units long

3. For the solid, draw a net and find the surface area.

4. **SHIPPING** Rawanda needs to wrap a package to ship to her aunt. The rectangular package measures 2 inches high, 10 inches long, and 4 inches wide. Draw a net of the package. How much wrapping paper does Rawanda need to cover the package?
Surface Areas of Prisms

The surface area of a prism is the lateral area of the prism plus the areas of the bases.

If the total surface area of a prism is $T$ square units, its height is $h$ units, and each base has an area of $B$ square units and a perimeter of $P$ units, then $T = L + 2B$.

Example

Find the surface area of the triangular prism above.

Find the lateral area of the prism.

$L = Ph$

$L = (18)(10)$

$L = 180\text{ cm}^2$

Find the area of each base. Use the Pythagorean Theorem to find the height of the triangular base.

$h^2 + 3^2 = 6^2$

$h^2 = 27$

$h = 3\sqrt{3}$

$B = \frac{1}{2} \times \text{base} \times \text{height}$

$B = \frac{1}{2}(6)(3\sqrt{3})$ or $15.6\text{ cm}^2$

The total area is the lateral area plus the area of the two bases.

$T = 180 + 2(15.6)$

$T = 211.2\text{ cm}^2$

Exercises

Find the surface area of each prism. Round to the nearest tenth if necessary.

1. 

2. 

3. 

4. 

5. 

6.
Skills Practice

Surface Areas of Prisms

Find the lateral area of each prism.

1. 

2. 

3. 

4. 

Find the surface area of each prism. Round to the nearest tenth if necessary.

5. 

6. 

7. 

8.
Surface Areas of Cylinders The surface area of a cylinder is the lateral area of the cylinder plus the areas of the bases.

### Surface Area of a Cylinder

If a cylinder has a surface area of \( T \) square units, a height of \( h \) units, and the bases have radii of \( r \) units, then \( T = 2\pi rh + 2\pi r^2 \).

### Example

Find the surface area of the cylinder.

Find the lateral area of the cylinder. If the diameter is 12 centimeters, then the radius is 6 centimeters.

\[
L = Ph
\]
\[
= (2\pi r)h
\]
\[
= 2\pi (6)(14)
\]
\[
= 527.8 \text{ Simplify.}
\]

Find the area of each base.

\[
B = \pi r^2
\]
\[
= \pi (6)^2
\]
\[
= 113.1 \text{ Simplify.}
\]

The total area is the lateral area plus the area of the two bases.

\[
T = 527.8 + 113.1 + 113.1 \text{ or 754 square centimeters.}
\]

### Exercises

Find the surface area of each cylinder. Round to the nearest tenth.

1. 2.

3. 4.

5. 6.
Skills Practice

Surface Areas of Cylinders

Find the surface area of a cylinder with the given dimensions. Round to the nearest tenth.

1. \( r = 10 \text{ in.}, h = 12 \text{ in.} \) 
2. \( r = 8 \text{ cm}, h = 15 \text{ cm} \)
3. \( r = 5 \text{ ft}, h = 20 \text{ ft} \)
4. \( d = 20 \text{ yd}, h = 5 \text{ yd} \)
5. \( d = 8 \text{ m}, h = 7 \text{ m} \)
6. \( d = 24 \text{ mm}, h = 20 \text{ mm} \)

Find the surface area of each cylinder. Round to the nearest tenth.

7. \( \) 
8. \( \)

Find the radius of the base of each cylinder.

9. The surface area is 603.2 square meters, and the height is 10 meters.
10. The surface area is 100.5 square inches, and the height is 6 inches.
11. The surface area is 226.2 square centimeters, and the height is 5 centimeters.
12. The surface area is 1520.5 square yards, and the height is 14.2 yards.
Find the surface area of each regular pyramid. Round to the nearest tenth if necessary.

1. 
   - Base: 7 cm
   - Height: 4 cm

2. 
   - Base: 20 in.
   - Height: 8 in.

3. 
   - Base: 9 m
   - Height: 10 m

4. 
   - Base: 12 ft
   - Height: 14 ft

5. 
   - Base: 9 mm
   - Height: 6 mm

6. 
   - Base: 7 yd
   - Height: 6 yd

7. 
   - Base: 12 m
   - Height: 18 m

8. 
   - Base: 20 in.
   - Height: 16 in.
Find the surface area of each regular pyramid. Round to the nearest tenth if necessary.

1. 2.

3. 4.

5. 6.

7. 8.

9. GAZEBOS The roof of a gazebo is a regular octagonal pyramid. If the base of the pyramid has sides of 0.5 meters and the slant height of the roof is 1.9 meters, find the area of the roof.
Skills Practice
Surface Areas of Cones

Find the surface area of each cone. Round to the nearest tenth if necessary.

1. 
   \[ \text{Surface Area} = \pi r (r + l) \]
   \[ = \pi \times 7 \times (7 + 22) \]
   \[ = \pi \times 7 \times 29 \]
   \[ = 203.5 \text{ square meters} \]

2. 
   \[ \text{Surface Area} = \pi r (r + l) \]
   \[ = \pi \times 10 (10 + 25) \]
   \[ = \pi \times 10 \times 35 \]
   \[ = 1099.5 \text{ square feet} \]

3. 
   \[ \text{Surface Area} = \pi r (r + l) \]
   \[ = \pi \times 21 (21 + 8) \]
   \[ = \pi \times 21 \times 29 \]
   \[ = 1721.5 \text{ square inches} \]

4. 
   \[ \text{Surface Area} = \pi r (r + l) \]
   \[ = \pi \times 9 (9 + 17) \]
   \[ = \pi \times 9 \times 26 \]
   \[ = 713.5 \text{ square millimeters} \]

5. 
   \[ \text{Surface Area} = \pi r (r + l) \]
   \[ = \pi \times 7 (7 + 22) \]
   \[ = \pi \times 7 \times 29 \]
   \[ = 203.5 \text{ square centimeters} \]

7. Find the surface area of a cone if the height is 12 inches and the slant height is 15 inches.
   \[ \text{Surface Area} = \pi \times 12 (12 + 15) \]
   \[ = \pi \times 12 \times 27 \]
   \[ = 942.5 \text{ square inches} \]

8. Find the surface area of a cone if the height is 9 centimeters and the slant height is 12 centimeters.
   \[ \text{Surface Area} = \pi \times 9 (9 + 12) \]
   \[ = \pi \times 9 \times 21 \]
   \[ = 614.5 \text{ square centimeters} \]

9. Find the surface area of a cone if the height is 10 meters and the slant height is 14 meters.
   \[ \text{Surface Area} = \pi \times 10 (10 + 14) \]
   \[ = \pi \times 10 \times 24 \]
   \[ = 753.9 \text{ square meters} \]

10. Find the surface area of a cone if the height is 5 feet and the slant height is 7 feet.
    \[ \text{Surface Area} = \pi \times 5 (5 + 7) \]
    \[ = \pi \times 5 \times 12 \]
    \[ = 188.5 \text{ square feet} \]
Find the surface area of each cone. Round to the nearest tenth if necessary.

1. Find the surface area of a cone if the height is 8 feet and the slant height is 10 feet.

8. Find the surface area of a cone if the height is 14 centimeters and the slant height is 16.4 centimeters.

9. Find the surface area of a cone if the height is 12 inches and the diameter is 27 inches.

10. HATS Cuong bought a conical hat on a recent trip to central Vietnam. The basic frame of the hat is 16 hoops of bamboo that gradually diminish in size. The hat is covered in palm leaves. If the hat has a diameter of 50 centimeters and a slant height of 32 centimeters, what is the lateral area of the conical hat?
Surface Areas of Spheres

You can think of the surface area of a sphere as the total area of all of the nonoverlapping strips it would take to cover the sphere. If \( r \) is the radius of the sphere, then the area of a great circle of the sphere is \( \pi r^2 \). The total surface area of the sphere is four times the area of a great circle.

| Surface Area of a Sphere | If a sphere has a surface area of \( T \) square units and a radius of \( r \) units, then \( T = 4\pi r^2 \). |

**Example**

Find the surface area of a sphere to the nearest tenth if the radius of the sphere is 6 centimeters.

\[
T = 4\pi r^2 \\
= 4\pi \cdot 6^2 \\
= 4\pi \cdot 36 \\
= 452.4 \text{ Simplify.}
\]

The surface area is 452.4 square centimeters.

**Exercises**

Find the surface area of each sphere with the given radius or diameter to the nearest tenth.

1. \( r = 8 \text{ cm} \) 
2. \( r = 2\sqrt{2} \text{ ft} \)
3. \( r = \pi \text{ cm} \) 
4. \( d = 10 \text{ in.} \)
5. \( d = 6\pi \text{ m} \) 
6. \( d = 16 \text{ yd} \)

7. Find the surface area of a hemisphere with radius 12 centimeters.

8. Find the surface area of a hemisphere with diameter \( \pi \) centimeters.

9. Find the radius of a sphere if the surface area of a hemisphere is 192\( \pi \) square centimeters.
12-7 Skills Practice
Surface Areas of Spheres

In the figure, A is the center of the sphere, and plane T intersects the sphere in circle E. Round to the nearest tenth if necessary.

1. If \( AE = 5 \) and \( DE = 12 \), find \( AD \).

2. If \( AE = 7 \) and \( DE = 15 \), find \( AD \).

3. If the radius of the sphere is 18 units and the radius of \( \odot E \) is 17 units, find \( AE \).

4. If the radius of the sphere is 10 units and the radius of \( \odot E \) is 9 units, find \( AE \).

5. If \( M \) is a point on \( \odot E \) and \( AD = 23 \), find \( AM \).

Find the surface area of each sphere or hemisphere. Round to the nearest tenth.

6. a hemisphere with a radius of the great circle 8 yards

7. a hemisphere with a radius of the great circle 2.5 millimeters

8. a sphere with the area of a great circle 28.6 inches
Write the letter for the correct answer in the blank at the right of each question.

1. Which of these is part of an orthogonal drawing?
   A. a perspective view  
   B. a corner view  
   C. a two-dimensional top view  
   D. a three-dimensional view

For Questions 2–4, use the figure.

2. Identify this solid figure.
   A. square pyramid  
   B. square prism  
   C. triangular pyramid  
   D. triangular prism

3. Name the base.
   A. △ABE  
   B. □ABCD  
   C. △CDE  
   D. E

4. How many edges does this figure have?
   A. 3  
   B. 4  
   C. 6  
   D. 8

5. This net could be folded into a _____.
   A. tetrahedron  
   B. square pyramid  
   C. square prism  
   D. triangular prism

6. Find the surface area of this solid.
   A. 9 in²  
   B. 27 in²  
   C. 36 in²  
   D. 54 in²

7. The areas of how many faces of a rectangular prism would be included in the lateral area?
   A. 2  
   B. 4  
   C. 6  
   D. 8

8. Find the surface area of a rectangular prism with a length of 8 inches, a width of 5 inches, and a height of 2 inches.
   A. 15 in²  
   B. 66 in²  
   C. 80 in²  
   D. 132 in²

9. The area of each face of a cube is 60 square centimeters. Find the surface area of the cube.
   A. 120 cm²  
   B. 240 cm²  
   C. 360 cm²  
   D. 3600 cm²

10. The lateral area of a right cylinder with a radius of 10 feet is $320\pi$ square feet. Find the surface area of the cylinder.
    A. $220\pi$ ft²  
    B. $360\pi$ ft²  
    C. $420\pi$ ft²  
    D. $520\pi$ ft²
Chapter 12 Test, Form 1 (continued)

For Questions 11 and 12, use the figure.

11. Find the lateral area to the nearest tenth.
   A. 75.4 ft²  B. 62.8 ft²  C. 50.3 ft²  D. 25.1 ft²

12. Find the surface area to the nearest tenth.
   A. 75.4 ft²  B. 62.8 ft²  C. 50.3 ft²  D. 25.1 ft²

13. The lateral area of a regular pyramid is 300 square units. The perimeter of its base is 100 units. Find the slant height of the pyramid.
   A. 3 units  B. 6 units  C. 12 units  D. 30 units

For Questions 14 and 15, use the figure.

14. Find the lateral area.
   A. 108 cm²  B. 144 cm²  C. 162 cm²  D. 324 cm²

15. Find the surface area.
   A. 108 cm²  B. 144 cm²  C. 162 cm²  D. 324 cm²

16. Find the surface area to the nearest tenth.
   A. 546.6 units²  B. 989.6 units²  C. 1017.9 units²  D. 1046.2 units²

17. The radius of a right circular cone is 6 inches and the height is 8 inches. Find the slant height.
   A. 2 in.  B. 4 in.  C. 10 in.  D. 14 in.

18. A cone has a radius 17 inches long and slant height 20 inches long. Find the surface area to the nearest tenth.
   A. 18,158.4 in²  B. 1976.1 in²  C. 1068.1 in²  D. 340 in²

19. Which could be the intersection of a sphere and a plane?
   A. line  B. square  C. oval  D. point

20. A sphere has a diameter 42 centimeters long. Find the surface area to the nearest tenth.
   A. 5541.8 cm²  B. 2770.9 cm²  C. 2167.1 cm²  D. 527.8 cm²

Bonus Find the amount of glass needed to cover the sides of the greenhouse shown. The bottom, front, and back are not glass.

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Write the letter for the correct answer in the blank at the right of each question.

1. Given the corner view of a figure, which is the top view?
   A. □ B. □ C. □ D. □ 1._____  
   ![Top View Diagram]

For Questions 2 and 3, use the figure.

2. Identify the figure.
   A. pyramid B. prism C. cone D. cylinder 2._____  
   ![Figure Diagram]

3. Name a base.
   A. □ B. N C. ＭＮ D. Ｍ 3._____  
   ![Base Diagram]

4. What name is given to a pyramid having seven faces?
   A. heptagonal pyramid B. hexagonal pyramid C. pentagonal pyramid D. octagonal pyramid 4._____  

5. This net could be folded into a ____.  
   A. rectangular pyramid B. rectangular prism C. triangular pyramid D. triangular prism 5._____  
   ![Net Diagram]

6. Find the surface area of the solid.
   A. 88 cm² B. 102 cm² C. 156 cm² D. 160 cm² 6._____  
   ![Solid Diagram]

7. Find the lateral area of an equilateral triangular prism if the area of each lateral face is 10 square centimeters.
   A. 10√3 cm² B. 30 cm² C. 50 cm² D. 100 cm² 7._____  

8. The surface area of a cube is 96 square inches. Find the length of an edge.
   A. √24 in. B. 4 in. C. 8 in. D. 16 in. 8._____  

9. The surface area of a rectangular prism is 190 square inches, the length is 10 inches, and the width 3 inches. Find the height.
   A. 30 in. B. 20 in. C. 10 in. D. 5 in. 9._____  

10. A right cylinder has a radius of 2 feet and a height of 5 feet. Find its surface area.
   A. 20π ft² B. 28π ft² C. 36π ft² D. 40π ft² 10._____  
   ![Cylinder Diagram]
For Questions 11 and 12, use a right cylinder with a radius of 5 centimeters and a height of 22 centimeters. Round to the nearest tenth.

11. Find the lateral area.
   A. 848.2 cm²  
   B. 769.7 cm²  
   C. 691.2 cm²  
   D. 345.6 cm²
   11. __________

12. Find the surface area.
   A. 848.2 cm²  
   B. 769.7 cm²  
   C. 691.2 cm²  
   D. 345.6 cm²
   12. __________

13. Find the lateral area of the conical hat to the nearest tenth.
   A. 942.5 in²  
   B. 408.4 in²  
   C. 204.2 in²  
   D. 188.5 in²
   13. __________

For Questions 14 and 15, use a tetrahedron that has edges of length 12 centimeters.

14. Find the lateral area.
   A. $48\sqrt{3}$ cm²  
   B. $96\sqrt{3}$ cm²  
   C. $108\sqrt{3}$ cm²  
   D. $144\sqrt{3}$ cm²
   14. __________

15. Find the surface area.
   A. $48\sqrt{3}$ cm²  
   B. $96\sqrt{3}$ cm²  
   C. $108\sqrt{3}$ cm²  
   D. $144\sqrt{3}$ cm²
   15. __________

For Questions 16 and 17, use the figure. Round to the nearest tenth.

16. Find the lateral area.
   A. 75.4 cm²  
   B. 103.7 cm²  
   C. 131.9 cm²  
   D. 150.8 cm²
   16. __________

17. Find the surface area.
   A. 75.4 cm²  
   B. 103.7 cm²  
   C. 131.9 cm²  
   D. 150.8 cm²
   17. __________

18. Find the surface area of the solid to the nearest tenth.
   A. 62.8 cm²  
   B. 56.5 cm²  
   C. 47.1 cm²  
   D. 37.7 cm²
   18. __________

19. Name a tangent to the sphere.
   A. $\overline{YU}$  
   B. $\overline{XZ}$  
   C. $\overline{UZ}$  
   D. $\overline{WV}$
   19. __________

20. The surface area of a sphere is 64π square centimeters. Find the radius.
   A. 16 cm  
   B. 8 cm  
   C. 4 cm  
   D. 2 cm
   20. __________

**Bonus** Find the surface area of the frustum of a square pyramid.

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